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Has logic any ontology?

1. Ontological preamble

Problem of identity is one of the basic problems of ontology regardless of whether its "ontological content" is expressed as being, object, thing, existence, entity, or most generally as "what is".[2]

Determination of the kinds of ontological objects almost compulsory led to assignment of its identities.[3]

It is possible to notice the universality of the problem of identity even from the beginings of the ontological investigations. This problem Aristotle separated as particular problem:

" Again, some things are one in respect of number, some in respect of form, some in respect of genus, some in respect of analogy: in number things whose matter is one, in form things whose formula is one, in genus things whose figure of predication is the same, in respect of analogy any things related as are two further things." (Metaphysica Δ, 1016[b])[4]

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[2] "Content" of ontology is expressed in different assignments of ontology. I. e. Gocklenius: "ontologia or philosophia of being", Kant: "ontology of objects of immanent thought", Crusius: "a science of the general essence of things", Jacoby: "the theory of the most general formal relations of reality", Ingarden: "the a priori science of possible orders of existence", Quine: "knowledge of what a given doctrine says there is", Heidegger: "ontology is to deal with the being (Sein) of all beings on the basis of the ontology of human being (Dasein) which understands being (Sein)".

[3] This determination is difficult to decline even for the case of abstainment from detailed assignment of ontological "content". Strawson's refutation of traditional ontological narrow determination of identity of ontological objects with his assignment of "something which is present", does not precludes the question about the modus of presence. I. e. question about the modus of presence of the concept of ether in physical theories.

[4] C. Kirwan's (1971) translation. J. H. Mc Mahon's translation: "And, moreover, some things are one according to number, but others according to species, and others according to genus, and others according to analogy. Those things are one in number of which the matter is one, but in species of which the definition is one, but in genus of which there is the same figure of predication; but according to analogy are things one as many as are disposed as one thing in relation to another."
W. Charlton (1994) very succesfully hinted at Aristotle's idea (Met. I, 1052[a] 22-5 and 1016[b] 35-6) according which that things which have the full unity of continuity must each have "some one form" which makes them one in form with itself.

Aristotle's foundation of identity on unity in first order refers on "unity through continuity", but it keeps thoroughness of the ontological assignment of identity with itself. I think that (separately from the problem of substantiality) this foundation is acceptible for the case of abstract mathematical entities. Moreover, I think that Leibniz-Russel's standpoint, which includes the assignment of abstract mathematical entities through their properties, is not in contradiction with this part of Aristotle's foundation.[5]

I will try to show that ontological concept of identity (unity with itself) stands in unavoidable relation with the basic ideas in contemporary logic.

2. Putnam's denial

Basic assignments of Putnam's variant of philosophy of scientific realism results with his argumentation in benefit of claims: a) Mathematical entities have only relative and conditional role in mathematical theories. b) It is possible to infer meaning of any concept excusively from the external physical reality.[6]

Here, I will show that the above claims lead Putnam to the standpoint that logic has no ontology.[7]

Philosophical "residuum" of his basic assignments Putnam shows through his standpoint according to which mathematical experience confirms that mathematics becomes true owing to its

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[5] M. Black's objection (from 1954) according which Leibniz-Russell's standpoint can not be at the core of identity because the identity of indiscernibles is not a necessary truth is hardly applicable to the case of abstract mathematical entities. That is to say, it is hardly uncontradictory to imagine such a world in which there are two mutualy different indiscernible mathematical entities, and we know about them excusively on their same properties.

H. Reichenbach's objection (from 1958) concerning ontological circularity (only irreflexive or asymmetrical relational properties can determine identity) is hardly applicable on logical principle of identity, and on mathematical reflexive law of identity.

[6] Basic assignments of Putnam's "external" realism are: realism not only in relation with materiality but in relation with physical magnitudes and fields, realism in relation with mathematical necessity, refutation of any kind of unconditional apriorism, refutation of any idea about the possibility of reduction of reality on sense data.

[7] "The idea that 'ontology' (i.e. the domain of the bound variables) in a mathematically true statement is a domain of sets or numbers or functions or other 'mathematical objects' , and (moreover) that *this* is what distinguishes mathematics from other sciences is a widespread one. On this view, mathematics is distinguished from marine biology by the difference in the objects studied. This idea lives on in a costant tension with the other idea, familiar since Frege and Russell, that there is no sharp separation to be made between *logic* and mathematics. Yet logic, as such, has no 'ontology'! It is precisely the chief characteristics of the principles and inference rules of logic that *any* domain of abjects may be selected, and that any

application on physical reality, and physical reality confirms reality of this interpretation. So far mathematics becomes a part of natural sciences, and logic (which is unseparable from mathematics) by means of mathematics becomes a part of the same physical reality.[8] In other words, a sentence of classical mathematics becomes true owing to its successful application to physical reality. Empirical premises of this interpretation are aproximately true, and rules of logic preserve that truth.

At first sight, it seems possible to conclude that "rules of logic which preserves truths" has particular status in comparation with the Putnam's "mathematical/physical" assignments. In other words, it seems possible to infer (according to the rules of logic) a mathematical sentence from the "mathematical/physical" reality, and to keep the logical rule of inference as a distinct part of particular human ability that is not reducible to external reality. But this exception should jeopardize universality of Putnam's proposals.

So, Putnam tried implicitly to reduce even logic on external reality. This reduction is taken in the two steps: 1. It is possible to reduce logic on external reality because it is not possible to separate logic from mathematics, and it is possible to reduce mathematics on physical reality. In the core of this reduction there is possibility of relativisation of the role of abstract mathematical entities in mathematical theories. 2. It is possible to show, by the construction of "Twin Earth" examples, that the meaning of any concept is inferred from external reality.

Here, I will try to deny the plausibility of Putnam's reductions.

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expressions may be instantiated for the predicate letters and sentential letters that they contain." Putnam, H. *"Mathematics, Matter and Method"* Cambridge University Press, Cambridge 1975. pp. 1-2.

[8] Putnam's "external realism" is only one variant of reductionism in contemporary philosophy of science. E. g. significantly different kind of reductionism is Kitcher's proposal in his book "*The Nature of Mathematical Knowledge*" . Namely, trying to preclude the loss of certainty in the case of extremely long procedures of mathematical inference, Kitcher proposed its ensurance by the reliable psychological processes (results of psycho-physiological processes). I think that this proposal is not able to stand the test of criticism form these points: a) "Reliable" psychological processes are not yet discovered. b) If "reliable" psychological processes should ever be discovered it should be only by the methodological procedures of sciences. c) Methodological procedures of sciences are not more reliable than mathematical procedures are. Putnam's reduction of the truths of mathematical statements is its reduction on physical statements. Kitcher's reduction of the truths of mathematical statements is its reduction on psychological statements.

Amongs Putnam's arguments that should ensure his standpoint about the impossibility of separation between logic and mathematics there is the following one:

"There are two apples on the desk. There are two apples on the table. The apples on the desk and table are all the ones in this room. No apple is both on the desk and on the table. Two plus two equals four. / (*therefore*) There are four apples in this room."

According to Putnam, the logicist account of above inference is achieved by definition of formulas "there are two *A*s" and "there are four *A*s" on such a manner that one can prove that "there are two *A*s" is equivalent to a statement of pure quantification theory (with identity), namely: "There is an x and there is a y such that x is an A and y is an A and $x \neq y$ and such that for every z, if z is an A then either $z = x$ or else $z = y$ ". Similarly, "There are four *A*s" is equivalent to a formula in quantification theory with identity. Thus the entire inference is equivalent (except for "two plus two equals four") to an inference in pure logic. In other words, the logicist account is achieved by the narrovest standard – quantification theory with identity. But, if we omit "two plus two equals four" above inference still remains valid. Moreover, the logicist translation of "two plus two equals four" is equivalent, not to a formula of first order logic (quantification theory), but to a formula of second order logic; in fact, to a formula: "For every *A*, *B*, *C*, is the union of *A* and *B* and *A* and *B* are disjoint and *A* has two members and *B* has two members then *C* has four members". So, the formula "two plus two equals four" is not an added premiss but the principle by which the conclusion is derived from the other premisses. The principle is essenttially a first order principle: since the initial universal quantifiers "for every *A*, *B*, *C*" can be inserted in front of every valid first order principle. Because of this facts Putnam concluded on arbitrarity of division between logic and mathematics.

I will try to show that the same facts do not preclude the possibility of some differentiation between logical and mathematical sentences. Nemely, it is possible to express "two plus two equals four" in formal number theory: with the logical postulates (axioms and logical rule of inference) in addition of some "non-logical" axioms, as follows:

For axiom shema (**1**) x is any variable, $A(x)$ is any formula, and $A(0)$, $A(x')$ are the results of substituting 0, x' respectively for the free occurences of x in $A(x)$. Open "non-logical" axioms can be closed by introduction of universal quantifier, i.e. axiom (**6**) is equal with its closure: $\forall a \forall b(a = b \rightarrow a' = b')$.

Alphabet: $=, +, \cdot, x$, $A, B, C,.., a, b, c,..0, '$,
Axioms: (**1**) $A(0) \wedge \forall x\, (A(x) \rightarrow A(x')) \rightarrow A(x)$. (**2**) $a' = b' \rightarrow a = b$.
(3) $a = b \rightarrow (a = c \rightarrow b = c)$. (**4**) $a + 0 = a$. (**5**) $\neg a' = 0$. (**6**) $a = b \rightarrow a' = b$.
(**7**) $a + b' = (a + b)'$. (**8**) $a \cdot 0 = 0$. (**9**) $a \cdot b' = a \cdot b + a$.

Demonstration:

(A^2_0) $2 + 0 = 2$ [axiom **4**]
(A^2_1) $2 + 1 = 2 + 0' = (2 + 0)'$ [axiom **7**] $= 2'$ [A^2_0] $= 3$
(A^2_2) $2 + 2 = 2 + 1' = (2 + 1)'$ [axiom **7**] $= 3'$ [A^2_1] $= 4$

Usage of different simbols is irrelevant (i.e. we can use 0, $(0)'$, $((0')')'$, $(((0)')')'$ … instead of 1, 2, 3, …). So, it is possible to keep some diferentiation between logical postulates and "non-logical" axioms. Usage of the closure does not preclude this possibility.

Putnam's argument that should ensure his general standpoint about the possibility of relativisation of the role of abstract mathematical entities in mathematical theories is as follows:[9]

It is realized Turing machine T, and P_1, P_2, …, P_n are predicates in ordinary "thing language" which describe its states (i.e. P_1 might be: "ratchet G_1 of T is pressing against bar T_4, etc."). An atomic instruction might be: "If P_2 (T) and T is scanning the letter "*y*" , T will erase the "*y*" , print "z" in its stead, shift one square left on the tape (more tape will be adjoined if T ever reaches the end), and then adjust itself so that P_6 (T)". Such an instruction is wholly in "nominalistic language" and does not quantify over abstract "entities". T is completely characterized by a finite set of such instructions, I_1, I_2, …, I_k . So, the statement: "As long as I_1 and I_2 and … and I_k , then T

[9] Putnam's general standpoint about the role of abstract mathematical entities is: ".. I conclude that, whatever may be essential to mathematics, reference to abstract 'entities' is not… What seems to

does not halt" could be a mathematically true statement, and quantifies only over physical objects. This realized Turing machine *T* deals with the Fermat's "last theorem". It is possible to use the simbols I, II, III, … to designate the numbers one, two, three, … (i.e. the name of the number *n* is a string of *n*'s "I's"). The sum of two numbers can be obtained by merely concatenating the numerals: *nm* is always the sum of *n* and *m*. Written $x = y*$ mean "*x* equals *y* cubed", *Nx* mean "*x* is a number", and ! indicates absurdity. It is rudimentary formal system whose axioms can be seen to be correct on this interpretation:

System *E.S.*

Alphabet: I , . , = , * , ! , *N*

Axioms:
(1) $N\,\mathrm{I}$
(2) $N\,x \rightarrow N\,x\,\mathrm{I}$
(3) $N\,x \rightarrow x = x$
(4) $N\,x \rightarrow x\,.\,\mathrm{I} = x$
(5) $x\,.\,y = z \rightarrow x\,.\,y\,\mathrm{I} = zx$
(6) $x\,.\,x = y,\; x\,.\,y = z \rightarrow z = x*$
(7) $z_1 = x_1*,\; z_2 = x_2*,\; z_3 = x_3*,\; z_1 = z_2\,z_3 \rightarrow\, !$

Thus the following is true: "If *X* is any finite sequence of inscriptions in the alphabet I , . , = , * , ! , *N* and each member of *X* is either an inscription of *N* I, or of a substitution instance of one of the remaining above axioms, or comes from two preceding terms in the sequence by Detachment, then *X* does not contain!". Furthermore, a finite sequence of inscriptions $I_1, \ldots, I_n$ can itself be identified with an inscription – say, with $I_1 \# I_2 \# \ldots \# I_n$, where # is a symbol not in the alphabet I , . , = , * , ! , *N* , and it is employed as a "spacer". Putnam concluded once again that previous true statement is an example of mathematically true statement that refers only to physical objects (inscriptions). For the case of possible criticism that, even if some mathematically true statements quantify only over physical objects, still the proofs of these statements would refer at least to numbers, Putnam answered that this premise is false. The principle needed to prove previous mathematically true statement is the principle of mathematical induction. This can be stated

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characterize mathematics is a certan style of reasoning; but that style of reasoning is not essentially connected with an 'ontology'." Putnam, H. ibid. pp. 4 –5.

directly for finite inscriptions. Namely, it is not the case that it is necessary to state the principle first for numbers and to derive the principle for inscriptions via goedel numbering. Every inscription possesses a goedel number, which cannot be proved without assuming the principle for inscriptions.

I think that the possibility of further criticism of Putnam's standpoint still remains despite of his trial to preclude any kind of it.

First of all, in the above Putnam's example we have three kinds of "inscriptions". First kind of "inscriptions" is the set of instructions $I_1, I_2, \ldots, I_k$, second kind are the strings of symbols I, II, III, … , and third kind is the identification $I_1\# I_2\#\ldots\# I_n$ of the first kind of "inscriptions". Putnam's first candidate for the true sentence, which could quantify only over physical states of realized Turing machine, and still remains mathematically true sentence, is: " $I_1, I_2, \ldots, I_k$ ". However, it is so only if realized Turing machine deals with the mathematical theory (not for example with the marine biology). In above Putnam's example for this purpose serves axiomatized Fermat's "last theorem". The strings of simbols I, II, III, … should have double purpose. They serve as designations for the numbers one, two, three, … (in system *E.S.*), and they simultaneuosly serve as parts of physical states (physical objects) of the realized Turing machine *T*. Putnam's second candidate for the mathematically true statement which refers only to physical objects (inscriptions I, II, III, …) is: "If *X* is any finite sequence of inscriptions in the alphabet **I** , **.** , = , * , ! , *N* and each member of *X* is either an inscription of *N* **I**, or of a substitution instance of one of the remaining above axioms, or comes from two preceding terms in the sequence by Detachment, then *X* does not contain !". Both of the candidates for mathematically true sentences that should refer (quantify) only to phyisical objects depend on the "interpretation" of the strings of symbols: I, II, III… This "interpretation" is denoted in axioms of the formal system *E.S.* (e.g. in axiom **(1)** as *N* **I**). Designator "*N*" means "number". Number means some kind of abstract mathematical entity. Now, I have to ask: "What will happen if we should omit "*N*" from the axioms of the formal system *E.S.*?" In this case "axioms" should become completely meaningless (they are not even w.f.f.'s). Furthermore: "What will happen if we should omit "*N*" from the content of the Putnam's second candidate for the mathematically true statement which should refer only to physical objects?" In this case the "true statement" should become meaningless too. So, the string of symbols (inscriptions I, II, III, …) remains symbols for numbers even as the "parts of physical states" (physical objects) of the realized Turing machine *T*. "Inscriptions", of course, as a symbols for numbers, posseses a goedel number. If I wrote: 1+2 = 3, or I + II = III, or

| I | II | III | | , would I quantify over the abstract mathematical entities, or over the physical states?

Putnam's example that should corroborate his standpoint about the relative role of abstract mathematical entities in mathematical theories colapsed to the question about the usage of symbols for abstract mathematical entities.

Well known constructions of "Twin Earth" examples should ensure Putnam's standpoint that the meaning of any concept is inferred excusively from external physical reality.[10]

I will try to get one counterexample. Let us suppose that there is no difference between the people on the Earth and the people on the Twin Earth. Mutually they are molecule for molecule "identical", they speak the same languages, think the same verbalized thoughts, have the same sense data, the same dispositions, etc. Water on the Twin Earth is H_2O as it is on the Earth. When people from the Earth "mean" beech they says "beech", when they "mean" elm they says "elm". When people from the Twin Earth "mean" beech they says "beech", when they "mean" elm they says "elm". There is only one difference between them. When people from the Earth say "modus ponens" they "mean": $A \rightarrow B, A \vdash B$. When people from the Twin Earth say "modus ponens"

[10] "I suppose I have a Doppelgänger on Twin Earth who is molecule for molecule 'identical' (in the sense in which two neckties can be 'identical'). If you are dualist, then also suppose my Doppelgänger thinks the same verbalized thoughts I do, has the same sense data, the same dispositions, etc. It is absurd to think his psychological state is one bit different from mine: yet he 'means' beech when he says 'elm' and I 'mean' elm when I say elm. Cut the pie any way you like, 'meanings' , just ain't in the head!" Putnam, H. "*Mind, Language and Reality*" Cambridge University Press, Cambridge 1975. p. 227

they "mean": $A \rightarrow B, B \vdash A$. People from the Earth and people from the Twin Earth have always made conclusions according to "modus ponens", form times immemorial. They have done it from the times even they did not call it "modus ponens" and did not discover its logical form. They have done it in the same way as they drank water from the times even they did not call it "water" and did not discover its chemical formula. When they landed on the Twin Earth people from the Earth prepared a celebrated meeting with the people from the Twin Earth. They marked the place of meeting under a big shady beech tree. Because of a dust, they sprayed with water narrow surrounding area. Delegation of people from the Twin Earth came with opened umbrellas. They did not close their umbrellas even they reached the place under a big shady beech tree. It was fine bright day. On the question about the role of the oppened umbrellas people from the Twin Earth answered that they were leaded with the "modus ponens", which applied give this conclusion: "If it is raining, a land is wet. Land is wet. Therefore, it is raining." People from the Earth sent to the Earth a message: "People from the Twin Earth use affirmation of consequence, and call it "modus ponens".[11]

The difference between the meaning of Earth people's understanding of modus ponens, and the meaning of Twin Earth people's understanding of "modus ponens" should be immidiately recognized. It should not be possible to interprete it as the difference that is "not in the head". Namely, the Twin Earth, as a planet, is molecule for molecule identical with the planet Earth.

Counterexample shows that the "Twin Earth" constructions have no universal validity. Constructions failed for the case of the logical rule of inference.

3. Ontological concept of identity and logical rule of inference

Ontological concept of unity or identity with itself, for the case of abstract mathematical entities, is recognizable in formal number theory, e.g. in the proof for reflexive law of equality. Namely, the concept of unity or identity with itself includes equality of properties for any abstract

[11] It is possible to put objection from the e.g. standpoints of the evolutionary epistemology. Such a people should not be able to survive. Of course this is true, but this objection does not preclude the possibility of

mathematical entity. Contrary, the concept of unity or identity with itself becomes empty, and the concept of equality becomes contradictory in itself. The following is a formal proof in **N**.

Alphabet: 0, *a*, *b*, *c*, = , + , A, B, C, ∀, → .
Axioms: (**1**) $A \to (B \to A)$. (**2**) $a = b \to (a = c) \to b = c)$. (**3**) $a + 0 = a$.
(**4**) $\forall x\ A(x) \to A(r)$.
Rules: (**I**) $A, A \to B \vdash B$. (**II**) *If* $\vdash C \to A(x)$ *then* $C \to \forall x\ A(x)$.

Demonstration:[12]

1. $a = b \to (a = c \to b = c)$ Axiom (**2**).
2. $0 = 0 \to (0 = 0 \to 0 = 0)$ Axiom shema (**1**).
3. $\{a = b \to (a = c \to b = c)\} \to \{[0 = 0 \to (0 = 0 \to 0 = 0)] \to [a = b \to (a = c \to b = c)]\}$ Axiom shema (**1**).
4. $[0 = 0 \to (0 = 0 \to 0 = 0)] \to [a = b \to (a = c \to b = c)]$ Rule (**I**)1, 3.
5. $[0 = 0 \to (0 = 0 \to 0 = 0)] \to \forall c\ [a = b \to (a = c \to b = c)]$ Rule (**II**) 4.
6. $[0 = 0 \to (0 = 0 \to 0 = 0)] \to \forall b \forall c\ [a = b \to (a = c \to b = c)]$ Rule (**II**) 5.
7. $[0 = 0 \to (0 = 0 \to 0 = 0)] \to \forall a \forall b \forall c\ [a = b \to (a = c \to b = c)]$ Rule (**II**) 6.
8. $\forall a \forall b \forall c\ [a = b \to (a = c \to b = c)]$ Rule (**I**) 2, 7.
9. $\forall a \forall b \forall c\ [a = b \to (a = c \to b = c)] \to \forall b \forall c\ [a + 0 = b \to (a + 0 = c \to b = c)]$ Axiom shema (**4**).
10. $\forall b \forall c[a + 0 = b \to (a + 0 = c \to b = c)]$ Rule (**I**) 8, 9.
11. $\forall b \forall c\ [a + 0 = b \to (a + 0 = c \to b = c)] \to \forall c\ [a + 0 = a \to (a + 0 = c \to a = c)]$ Axiom shema (**4**).
12. $\forall c\ [a + 0 = a \to (a + 0 = c \to a = c)]$ Rule (**I**)10, 11.
13. $\forall c\ [a + 0 = a \to (a + 0 = c \to a = c)] \to [a + 0 = a \to (a + 0 = a \to a = a)]$ Axiom shema (**4**).
14. $[a + 0 = a \to (a + 0 = a \to a = a)]$ Rule (**I**) 12, 13
15. $a + 0 = a$ Axiom (**3**).
16. $a + 0 = a \to a = a$ Rule (**I**) 15, 14.
17. $a = a$ Rule (**I**) 15, 16.

I think that mathematical reflexive law of identity is in relation with the ontologically understood concept of identity. From ontological point of view, this relation could be "subordinative". When we speak about any entity (or being) in general, we could speak about its unity or identity with itself. When we speak about special mathematical abstract entity, we could speak about its unity or identity with itself too. Unity or identity of an entity with itself should be compared through its properties. For spatio/temporal entities (or beings) comparisons refer to unity or identity with

the "Twin Earth" construction for the case of the logical rule of inference.
[12] Kleene 1967.

themeselves through continuity. For abstract mathematical entities comparisons refer to unity or identity with itself out of spatio/temporal continuity.

In above demonstration interpreted logical axioms (**2**) and (**3**) enable the usage of logical rules (**I**) and (**II**). Formal properties of abstract mathematical entities include ontologically understood concept of identity. Does it mean that only interpreted logical axioms are in relation with the ontologically understood concept of identity?

Here, I will try to show that it is not unconditionally so.

Purely logical axiom (**1**), rule of inference, includes formal logical implication assigned with the symbol "$\rightarrow$". Formal logical implication is defined with its truth table. To each entry (or line) of its basic truth table for the propositional calculus, the following four corresponding deducibility relationship respectively holds: (**L_1**) $A, B \vdash (A \rightarrow B)$. (**$L_2$**) $A, \neg B \vdash \neg(A \rightarrow B)$. (**$L_3$**) $\neg A, B \vdash (A \rightarrow B)$. (**$L_4$**) $\neg A, \neg B \vdash (A \rightarrow B)$.

Alphabet: Γ, A, B, C, $\vdash$, $\rightarrow$, $\neg$, $\geq$, *r*, *n*, 0.
Theorems: (α) (i) *For* $n \geq 1$: $A_1, \ldots, A_n \vdash A_1, A_1, \ldots A_n \vdash A_n$.(ii) *For* $n, r \geq 0$: *If* $A_1, \ldots A_n \vdash B_1$ *and* $A_1, \ldots A_n \vdash B_r$ i $B_1, \ldots B_r \vdash C$ *then* $A_1, \ldots A_n \vdash C$.
(β) *If* $\Gamma, A \vdash B$ *then* $\Gamma \vdash A \rightarrow B$.
(χ) $A, A \rightarrow B \vdash B$.
(δ) *If* $\Gamma, A \vdash B$ *and* $\Gamma, A \vdash \neg B$ *then* $\Gamma \vdash \neg A$.
(ε) $A, \neg A \vdash B$.

Demonstration:

1. $A, B, A \vdash B$	(α)
2. $A, B, \vdash A \rightarrow B$	(β), 1.
3. $A, \neg B, A \rightarrow B \vdash B$	(χ)
4. $A, \neg B, A \rightarrow B \vdash \neg B$	(α)
5. $A, \neg B, \vdash \neg(A \rightarrow B)$	(δ), 3, 4.
6. $\neg A, B, A \vdash B$	(α), (ε)
7. $\neg A, B \vdash (A \rightarrow B)$	(β), 6.
8. $\neg A, \neg B, A \vdash B$	(ε), (α).
9. $\neg A, \neg B \vdash (A \rightarrow B)$	(β), 8.

The fourth line (**L_4**) of the truth table of implication (eighth and nineth steps of above demonstration) introduces implication as true implication when its antecedens and its consequence are false. This introduction of true implication is made in the eighth step of above demonstration in accordance with the weak negation elimination (ε) and in the ninth step in accordance with the introduction of implication (β). The eighth step in list Γ contains {¬A,¬B, A}. This allows additional inference (*) as follows:

10. ¬A, ¬B, A ├ ¬B (α),
11. ¬A, ¬B ├ ¬A (δ), 8, 10.

Namely, in eight step of above inference ├ B is introduced from the list of formulas Γ{¬A,¬B, A} by weak negation elimination (ε). Further step introduced by deduction theorem (Herbrand 1930) set implication as true implication when its antecedent and its consequence are false. However, from the same list of formulas Γ it is possible to infer ├ ¬B by (α). Here, it is done in the tenth step of above inference. From the eighth and the tenth steps it is possible trivially to infer ├ ¬A by (δ). Here, it is done in the eleventh step of above inference. Denial of antecedens (i.e. ├ ¬A → [A → B]) deductively entails the following statement: "False sentence implies any sentence". But, it is not a matter of a mere convention to set implication as true implication when its antecedent and its consequence are false.

Corresponding determination to the reflexive law of identity in formal number theory is the principle of identity in elementary propositional calculus:

├ A → A

Alphabet: A, B, C, →, ├ .
Axioms: (**1**) A → (B → A). (**2**) (A → B) → ((A → B → C)) → (A → C)).
Rules: (**I**) A, A → B ├ B.

Demonstration:

1. A → (A → A) Axiom schema (1)
2. {A → (A → A)} → {[A → ((A → A) → A)] → [(A → A)]} Axiom schema (2)
3. [A → ((A → A) → A)] → [(A → A)] Rule (**I**) 1, 2
4. A → ((A → A) → A) Axiom schema (1)
5. A → A Rule (**I**) 4, 3

To each line of its truth table the following two corresponding deducibility relationships respectively hold: (**L₁**) A, A $\vdash$ A $\rightarrow$ A, and (**L₂**) $\neg$A, $\neg$A $\vdash$ A $\rightarrow$ A.

Demonstration:

1. A, A, A $\vdash$ A (α)
2. A, A, $\vdash$ (A $\rightarrow$ A) (β), 1.
3. $\neg$A, $\neg$A, A $\vdash$ (A $\rightarrow$ A) (ε).

The proof of inference of deducible relation to the first line of truth table of the logical principle of identity is shown in first two steps of above demonstration. It is done by Herbrand's theorem whose "descriptive" statement could be: "What is inferential (deducible) from truth is truly implied". Proof of inference of deducible relation to the second line of truth table of the logical principle of identity is shown in the third step of above demonstration. It is done by the weak negation elimination whose "descriptive" statement could be:"What is contradictory true and false it could imply anything".

Once again, it is possible only trivially to apply *reductio ad absurdum* [theorem (δ)], as it is possible in additional inference (*) as follows:[13]

4. $\neg$A, $\neg$A, A $\vdash$ $\neg$A (α)
5. $\neg$A, $\neg$A, A $\vdash$ A (α)
6. $\neg$A, $\neg$A $\vdash$ $\neg$A (δ), 4, 5.

So, for the case of Γ{$\neg$A,$\neg$A, A} application of (δ) gives $\vdash$ $\neg$A. This application trivially turns "A" into "$\neg$A".

The only possible nontrivial additional inference (**) is as follows:

7. $\neg$A, $\neg$A, A $\vdash$ A (α)
8. $\neg$A, $\neg$ A $\vdash$ (A $\rightarrow$ A). (β) 7.

So, for the list of formulas Γ{$\neg$A,$\neg$A, A}the only application of theorems (ε) and (β) gives nontrivially inference $\vdash$ (A $\rightarrow$ A). "What is contradictory true and false it could imply anything", and "What is inferential (deducible) from truth is truly implied" are both founded on ontological concept of unity or identity with itself by the formal property of truth. Theorem (δ) is nontrivially applicable to the list of formulas $\Gamma\{A,\neg B,\ A \rightarrow B\}$. For the case of logical principle of identity theorem (δ) should be applicable for the list of formulas $\Gamma\{A,\neg A,\ A \rightarrow A\}$, but, of course, such a contradictory list of formulas is not a part of the proof of deducible relationships of its truth table. The statement: "What contradictory entails truth and falsity is false" preserves truth, and it is in accordance with ontological concept of unity or identity with itself by means of the formal property of truth, too. Furthermore, when a compound (molecular) proposition contains repetitions of a constituent proposition in a suitable manner, it can be known to be true without our having to know the truth or falsehood of any constituent. The instance of logical principle of inference: $(A \rightarrow A) \equiv A|(A|A)$ means: "A is incompatible with the incompatibility of A with itself". Logic does not ought to have any ontology when it deals with uncompound (atomic) proposition, because its possible ontological status depends on its content. But the ontological concept of unity or identity with itself enables the truth of compound propositions of suitable form independent of any particular content of its constituents. Even in the case of atomic proposition, regardless of a possible ontological status of its possible content, ontological concept of unity or identity with itself must be obeyed by the formal property of truth.[14]

It is not the case that ontological concept of unity or identity with itself is transferred from reflexive law of equality to principle of identity. On the contrary, reflexive law of equality is inferred from principle of identity. Namely, if a proposition trully implies a function (any possible satisfactory argument of that function), then a proposition trully implies all cases of satisfactory

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[13] See previous demonstration: corresponding deducibility relationship for the fourth line (**L_4**) of the truth table of formal implication, steps 10 – 11, additional inference.

[14] Ludwig Wittgenstein in *"Tractatus Logico-Philosophicus"* [5 and 5.01] determines compound proposition as truth functions of elementary propositions, and elementary propositions as truth functions of itself.

argument of that function (e.g. rule (**II**), demonstration for reflexive law of equality). Any functional determination that containes a variable could be assigned by a proposition according to which any possible values of a variable satisfy functional property to which a variable is an argument. Identity is expressed by satisfaction of the same properties according to their implicative relations - that are expressed by equivalence of its properties.[15]

Gentzen's system has seven structural rules and initial sequents of the form A → A. Generally, any sequent is an expression of the form $X_1 \ldots X_n \rightarrow Y_1 \ldots Y_k$. Proof of any formula is a construction starting excusively with the initial sequent.

[15] Bertrand Russell this fact determines several times. E.g. in demonstration of ∗**13·11** [*Principia Mathematica*, p. 169. *vol.* I]: ($\vdash :. x = y \equiv : \phi ! x. \equiv_{\phi} . \phi ! y$)

Axioms and basic definitions:

∗**1·7** If p is an elementary proposition, $\neg p$ is an elementary proposition. ∗**2·08** $\vdash p \supset p$. ∗**3·2** $\vdash :. p. \supset : q. \supset. p . q$. ∗**4·2** $\vdash p \equiv p$. ∗**9·13** In any assertion containing a real variable, this real variable may be turned into an apparent variable of which all possible values are asserted to satisfy the function in question. ∗**10·11** If ϕy is true whatever possible argument y may be, then $(x). \phi x$ is true. In other words, whenever the propositional function ϕy can be asserted, so can the proposition $(x). \phi x$. ∗**10·02** $\phi x \supset_x \psi x. \equiv . (x). \phi x \supset \psi x$. ∗**10·2** $\vdash :. (x). p \vee \phi x . \equiv : p . \vee . (x). \phi x$. ∗**10·21** $\vdash :. (x). p \supset \phi x . \equiv : p . \supset. (x). \phi x$. ∗**10·22** $\vdash :. (x) \phi x . \psi x \equiv : (x). \phi x : (x). \psi x$. ∗**13·01** $x = y. = : (\phi) : \phi ! x. \supset. \phi ! y$. ∗**13·1** $\vdash :. x = y. \equiv : \phi ! x. \supset_{\phi} . \phi ! y$. **Transp.** In an implication the two sides may be interchanged by turning negative into positive and positive into negative. **Comp.** If a proposition implies each of two propositions, then it implies their logical product. **Hp.** Hypothesis. $\supset \vdash$ **Prop.** corresponds to Q.E. D.

Demonstration:

1.	$\vdash$ ∗**10·22.**	$\supset \vdash :. \phi ! x. \equiv_{\phi} \phi ! y: \supset: \phi ! x. \supset_{\phi} . \phi ! y$:	
2.	[∗**13·1**]	$\supset : x = y$	(1)
3.	$\vdash$.∗**13·101.**	$\supset \vdash :. x = y. \supset . \phi ! x \supset \phi ! y$	(2)
4.	$\vdash$. ∗**13·101.** ∗**1·7.**	$\supset \vdash :. x = y. \supset. \neg\phi ! x \supset \neg\phi ! y$.	
5.	[**Transp.**]	$\supset . \phi ! y \supset \phi ! x$	(3)
6.	$\vdash$. (2) . (3). **Comp.**	$\supset \vdash : x = y. \supset . \phi ! x \equiv \phi ! y$	
7.	[∗**10·11·21**]	$\supset \vdash :. x = y. \supset . \phi ! x \equiv_{\phi} \phi ! y$	(4)
8.	$\vdash$. (1) . (4) .	$\supset \vdash$. **Prop.**	

In step (3.) of above demonstration the following statement is stated: "If x and y are identical, any property of x implies property of y". Namely:

∗**13·101**. ($\vdash :. x = y. \supset . \phi ! x \supset \phi ! y$)

Demonstration:

3_i	$\vdash$. ∗**12·1**.	$\supset \vdash :. (\exists \phi) :. \psi x. \equiv. \phi ! x: \psi y. \equiv . \phi ! y$	(1)
3_{ii}	$\vdash$. ∗**13·1**.	$\supset \vdash ::$ **Hp**. $\supset :. \phi ! x. \supset_{\phi} . \phi ! y :.$	
3_{iii}	[∗**4·84·85.**∗**10·27**]	$\supset :. \psi x. \equiv . \phi ! x: \psi y . \equiv . \phi ! y : \supset_{\phi} : \psi x. \supset. \psi y :.$	
3_{iiii}	[∗**10·23**]	$\supset :. (\exists \phi): \psi x. \equiv . \phi ! x: \psi y . \equiv . \phi ! y : \supset : \psi x. \supset. \psi y$	(2)
3_{iiiii}	$\vdash$. (1) . (2) .	$\supset \vdash$. **Prop.**	

4. Philosophical remark

It is possible to criticize a relativisation of ontological status of abstract mathematical entities. Furthermore, it is possible to criticize reduction of mathematical statements on physical reality, or on psychological processes. It is not the case that the meaning of any possible concept is inferred from physical reality. Ontological concept of unity or identity with itself indepedently stands in relation with the basic logical rules of inference. It is not the case that logic "chooses" ontology, but logic "provokes" one form of ontology, or ontologically understood concept of identity is in the foundations of logic.

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Summary

This article deals with the problem of relations between logic and ontology. It is shown that it is possible to criticize reduction of logic on physical reality. It is shown that ontologically understood concept of identity indepedently stands in relation with the basic logical rules of inference.

In step (3_{ii}) of above demonstration it is reference to ∗**13·1** and **Hp**. Demonstration of ∗**13·1** is founded on ∗**4·2** ($\vdash p \equiv p$) and on definitions as follow: ∗**13·01** ($\vdash x = y . = : (\phi) : \phi ! x . \supset . \phi ! y$), ∗**10·02** ($\phi x \supset_x \psi x . \equiv . (x) . \phi x \supset \psi x$). Demonstration of ∗**4·2** are founded on ∗**2·08** ($\vdash p \supset p$) and ∗**3·2** $\vdash :. p . \supset : q . \supset . p . q$.

Deductively follow: $\vdash x = y$ on $\vdash p \equiv p$, and $\vdash p \equiv p$ on $\vdash p \supset p$.