Popuniti odmah! PISATI JEDNOSTRANO!

IME I PREZIME: ALEKSANDAR BEADER BROJ INDEKSA: 17-1-0027-2010

DATUM: 26.6.2012. VRIJEME: OD 8:20 DO 9:50

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

23

Broj ↓ bodova

1. Zadan je skup linearnih jednadžbi:

$$\begin{aligned} 2x + 3y - 3z - w &= -5 \\ 3z - 2y &= 5 \\ 2y - x - w &= -1 \\ w - 4x + 3z &= 9 \end{aligned}$$

   (a) zapisati dani sustav matrično, 2

   (b) riješiti matrični sustav Gaussovom metodom 10

   (c) provjeriti izračunato rješenje matričnim množenjem 3

2. Riješiti u kompleksnim brojevima sljedeće jednadžbe:

   (a) $z^3 + |3 - 4i| = \frac{5}{i}$. 10

   (b) $-z + |z - 4i| = \overline{3 + 4i}$. 10

3. Za funkciju $f(x) = x - \sqrt{x^2 - x}$:

   (a) odrediti asimptote i 10

   (b) odrediti prvu derivaciju 10

4. Za funkciju $g(x) = \frac{e^{2x}}{x^2}$:

   (a) uz pomoć L'Hopitalovog pravila odrediti: $\lim_{x \to +\infty} g(x)$ 10

   (b) uz pomoć zaključka iz (a) diskutirati konvergenciju reda: $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^2}$ 10

5. Zadana je funkcija: $h(x) = \frac{x^2 - 3}{x^2 + 3}$. Na temelju ispitivanja toka funkcije:

   (a) diskutirati da li je funkcija globalno ograničena ili ne, 5

   (b) navesti sve lokalne ekstreme, 5

   (c) navesti sve točke infleksije i 5

   (d) napraviti skicu grafa funkcije. 10

1.

$$\left[\begin{array}{cccc|c} 2 & 3 & -3 & -1 & -5 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array}\right] \cdot \frac{1}{2} \sim \left[\begin{array}{cccc|c} 1 & 3/2 & -3/2 & -1/2 & -5/2 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array}\right] \begin{array}{l} /(+1) \\ \\ \leftarrow + \\ \leftarrow + \end{array} \cdot 4 \sim$$

$$\left[\begin{array}{cccc|c} 1 & 3/2 & -3/2 & -1/2 & -5/2 \\ 0 & -2 & 3 & 0 & 5 \\ 0 & 7/2 & -3/2 & -3/2 & -7/2 \\ 0 & 6 & -3 & -1 & -1 \end{array}\right] \cdot \left(-\frac{1}{2}\right) \sim \left[\begin{array}{cccc|c} 1 & 3/2 & -3/2 & -1/2 & -5/2 \\ 0 & 1 & -3/2 & 0 & -5/2 \\ 0 & 7/2 & -3/2 & -3/2 & -7/2 \\ 0 & 6 & -3 & -1 & -1 \end{array}\right] \begin{array}{l} \leftarrow + \\ \\ \\ \leftarrow + \end{array} \cdot \left(-\frac{3}{2}\right) \cdot \left(-\frac{7}{2}\right) \cdot 6$$

VIDI RJEŠENJE 1

IME I PREZIME: ALEKSANDAR BEADER BROJ INDEKSA: 17-1-0027-2019

$$\sim \begin{bmatrix} 1 & 0 & -3/4 & -1/2 & 5/4 \\ 0 & 1 & -3/2 & 0 & -5/2 \\ 0 & 0 & 15/4 & -3/2 & 21/4 \\ 0 & 0 & 6 & -1 & 14 \end{bmatrix} \Big| \cdot \frac{4}{15}$$

$$\begin{bmatrix} 1 & 0 & -3/4 & -1/2 & 5/4 \\ 0 & 1 & -3/2 & 0 & -5/2 \\ 0 & 0 & 1 & -2/5 & 21/15 \\ 0 & 0 & 6 & -1 & 14 \end{bmatrix} \begin{matrix} \leftarrow + \\ \leftarrow + \\ \cdot 3/4 \,/\, \cdot \frac{3}{2} \,/\, \cdot (-6 \\ \leftarrow \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -4/5 & 46/20 \\ 0 & 1 & 0 & 3/5 & -2/5 \\ 0 & 0 & 1 & -2/5 & 21/15 \\ 0 & 0 & 0 & 7/5 & 28/5 \end{bmatrix} \Big| \cdot \frac{5}{7}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -4/5 & 46/20 \\ 0 & 1 & 0 & 3/5 & -2/5 \\ 0 & 0 & 1 & -2/5 & 21/5 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \begin{matrix} \leftarrow + \\ \leftarrow + \\ \leftarrow + \\ \cdot 4/5 \,/\, \cdot (-\frac{3}{5}) \,/\, \cdot \frac{2}{5} \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 11/2 \\ 0 & 1 & 0 & 0 & -14/5 \\ 0 & 0 & 1 & 0 & 29/5 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$A^{-1}$

$$A \cdot A^{-1} = I \quad \times$$

$$\begin{bmatrix} 2 & 3 & -3 & -1 & 11/2 \\ 0 & -2 & 3 & 0 & -14/5 \\ -1 & 2 & 0 & -1 & 29/5 \\ -4 & 0 & 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IME I PREZIME: ALEKSANDAR BEADER

BROJ INDEKSA: 17-1-0027-2015

4. $g(x) = \frac{e^{2x}}{x^2} = \lim_{x \to \infty} \frac{e^{2x}}{x^2} = \lim_{x \to \infty} \frac{\frac{1}{2x} e^{2x}}{x^2} = \frac{\frac{1}{2x^3} e^{2x}}{1} = +\infty$

5. $\frac{x^2-3}{x^2+3}$

$D_f: x \in (-\sqrt{3}, +\infty)$

$x^2+3 \neq 0$

$x^2 = -3$

$x_{1,2} = \sqrt{3}$

$a = 1 > 0 \cup$ - konveksna

IME I PREZIME: ALEKSANDAR BEADER BROJ INDEKSA: 171-0027-2017

2. (a)

$$z^3 + |3 - 4i| = \frac{5}{i}$$

$$z^3 + \sqrt{3^2+4^2} = \frac{5}{i}$$

$$z^3 + \sqrt{9+16} = \frac{5}{i}$$

$$z^3 + 5 = \frac{5}{i}$$

$$z^3 = \frac{5}{i} - 5$$

$$z^3 = \frac{5 - 5i}{i} \cdot \frac{-i}{-i}$$

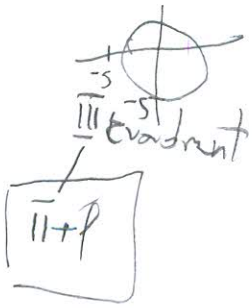
$$z^3 = \frac{-5i + 5i^2}{1}$$

$$z^3 = -5 - 5i$$

$x$ $y$

$$\tan\varphi = \frac{-5}{5} = 1$$

$$\varphi' = 0,79$$




$$\varphi = \Pi + \varphi'$$

$$\varphi = 3,14 + 0,79$$

$$\varphi = 3,93$$

$$r = \sqrt{x^2+y^2} = \sqrt{25+25} = \sqrt{50} = 7,07$$

$$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\varphi + 2k\Pi}{n} + i\sin\frac{\varphi + 2k\Pi}{n}\right)$$

$k = 0, 1, 2$

$k=0$

$$z_1 = \sqrt[3]{7,07}\left(\cos\frac{3,93}{3} + i\sin\frac{3,93}{3}\right) = 1,92\cdot(\cos 1,31 + i\sin 1,31)$$

$$z_1 = 0,49 + 1,85i$$

$k=1$

$$z_2 = 1,92\left(\cos\frac{3,93+2\Pi}{3} + i\sin\frac{3,93+2\Pi}{3}\right) = z_2 = -1,85 - 0,49i$$

3,4 3,4

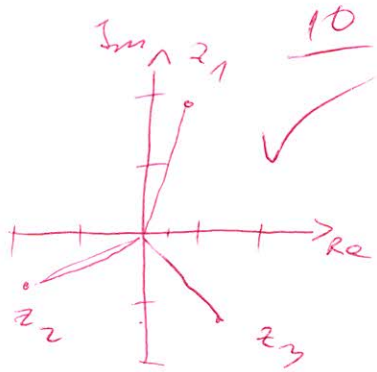
$k=2$

$$z_3 = 1,92\left(\cos\frac{3,93+4\Pi}{3} + i\sin\frac{3,93+4\Pi}{3}\right)$$

16,5 16,5

$$z_3 = -1,35 - 1,36i$$

10



IME I PREZIME: ALEKSANDAR BEĆADER BROJ INDEKSA: 17 - 1 - 0027 - 2015

2.

b) $-z + |z-4i| = \overline{3+4i}$

$-z + \sqrt{x^2+y^2} = 3-4i$

$-1 + \sqrt{x^2+y^2} = 3$

$\boxed{y=-4}$

$\sqrt{x^2+16} = 4 \,/^2$

$x^2+16 = 16$

$x^2 = 0 \,/\sqrt{\ }$

$x = 0$

$-z + \sqrt{0+16} = 3-4i$

$-z + 4 = 3-4i$

$-z = -1-4i \,/\cdot(-1)$

$z = 1+4i$

$x=1 \qquad y=4$

3. b) $f(x) = x - \sqrt{x^2-x}$

$f'(x) = 1 - \frac{1}{2\sqrt{x^2-x}} \cdot (2x-1)$

$f'(x) = 1 - \frac{(2x-1)}{2\sqrt{x^2-x}}$ ✓

a) $x - \sqrt{x^2-x}$

$x^2 - x \geq 0$

$x^2 \geq x$

$x \geq 1$

$D\ [1, +\infty\rangle$

V. A.

$\lim_{x\to 1} x - \sqrt{x^2-x} = 1 - \sqrt{1-1} = 1$ // V.A.?

H. A

$\lim_{x\to\pm\infty} x - \sqrt{x^2-x} =$ NEMA ✗

a) kosa A.

$k = \lim_{x\to-\infty} \frac{x-\sqrt{x^2-x}}{x} \,/:x^2 = \frac{\frac{1}{x} - \sqrt{\frac{1}{x^2} - \frac{1}{x^3}}}{\frac{1}{x}}$

$k = 0$ NEMA K.A. ✗