

Popuniti odmah! PISATI JEDNOSTRANO!
IME I PREZIME: KRISTINA TISSAUER

BROJ INDEKSA: 17-2-0129-2011

DATUM: 26.6.2012. VRIJEME: OD DO

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

37

Broj ↓
bodova

1. Zadan je skup linearnih jednadžbi:

$$2x + 3y - 3z - w = -5$$

$$3z - 2y = 5$$

$$2y - x - w = -1$$

$$w - 4x + 3z = 9$$

(a) zapisati dani sustav matrično,

2

(b) riješiti matrični sustav Gaussovom metodom

10

(c) provjeriti izračunato rješenje matričnim množenjem

3

2. Riješiti u kompleksnim brojevima sljedeće jednadžbe:

(a) $z^3 + |3 - 4i| = \frac{5}{i}$.

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(b) $-z + |z - 4i| = \overline{3 + 4i}$.

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3. Za funkciju $f(x) = x - \sqrt{x^2 - x}$:

(a) odrediti asimptote i

10

(b) odrediti prvu derivaciju

10

4. Za funkciju $g(x) = \frac{e^{2x}}{x^2}$:

(a) uz pomoć L'Hopitalovog pravila odrediti: $\lim_{x \rightarrow +\infty} g(x)$

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(b) uz pomoć zaključka iz (a) diskutirati konvergenciju reda: $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^2}$

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5. Zadana je funkcija: $h(x) = \frac{x^2 - 3}{x^2 + 3}$. Na temelju ispitivanja toka funkcije:

(a) diskutirati da li je funkcija globalno ograničena ili ne,

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(b) navesti sve lokalne ekstreme,

5

(c) navesti sve točke infleksije i

5

(d) napraviti skicu grafa funkcije.

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VIDI RJEŠENJE 1

⑤ $h(x) = \frac{x^2-3}{x^2+3}$

① Domena

$$x^2+3=0$$

$$x^2=-3$$

$\Delta f x \in \mathbb{R}$ ✓

② NUL TOČKE

$$x^2-3=0$$

$$x^2 = \overset{\ominus}{-3} \underset{\oplus}{}$$

Nema nul točaka ✗

③ PARNOST I NEPARNOST

$$f(-x) = \frac{(-x)^2-3}{(-x)^2+3} = f(x) = \frac{x^2-3}{x^2+3}$$

funkcija je parna ✓

ASIMPTOTE

Nema vertikalne asimptote jer nema točaka presjeka x na rubu domene

Horizontalna

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x^2+3}{x^2+3} = \lim_{x \rightarrow -\infty} \frac{(-x)^2-3}{(-x)^2+3} =$$

$$\lim_{x \rightarrow \infty} \frac{x^2-3}{x^2+3} \begin{matrix} | : x^2 \\ 1 - \frac{3}{x^2} \\ 1 + \frac{3}{x^2} \end{matrix} = \frac{1 - \frac{3}{\infty}}{1 + \frac{3}{\infty}} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x^2-3}{x^2+3} \begin{matrix} | : x^2 \\ 1 - \frac{3}{x^2} \\ 1 + \frac{3}{x^2} \end{matrix} = \frac{1 - \frac{3}{\infty}}{1 + \frac{3}{\infty}} = \frac{1}{1} = 1$$

$\boxed{y=1}$ ✓

Nema kose asimptote kad nema horizontalne.

→ nastavak

③ $f(x) = x - \sqrt{x^2 - x}$

ASIMPTOTE

horizontalna

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} x - \sqrt{x^2 - x} \cdot \frac{-x + \sqrt{x^2 - x}}{-x + \sqrt{x^2 - x}} =$$

$$\lim_{x \rightarrow \infty} \frac{(-x) - \sqrt{x^2 + x}}{1} \cdot \frac{(-x) + \sqrt{x^2 + x}}{-x + \sqrt{x^2 + x}} =$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - (x^2 + x)}{-x + \sqrt{x^2 + x}} = \lim_{x \rightarrow \infty} \frac{\cancel{x^2} - \cancel{x^2} - x}{-x + \sqrt{x^2 + x}} \cdot \frac{1 : x}{1 : x}$$

$$\lim_{x \rightarrow \infty} \frac{-1}{-1 + \sqrt{1 + \frac{1}{x}}} = \lim_{x \rightarrow \infty} \frac{-1}{-1 + 1} = \frac{-1}{0} = -\infty \quad \checkmark$$

$$\lim_{x \rightarrow \infty} \frac{x - \sqrt{x^2 - x}}{1} \cdot \frac{x + \sqrt{x^2 - x}}{x + \sqrt{x^2 - x}} =$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - (x^2 - x)}{x + \sqrt{x^2 - x}} = \lim_{x \rightarrow \infty} \frac{\cancel{x^2} - \cancel{x^2} + x}{x + \sqrt{x^2 - x}} \cdot \frac{1 : x}{1 : x}$$

$$\lim_{x \rightarrow \infty} \frac{1}{1 + \sqrt{1 - \frac{1}{x}}} = \frac{1}{1 + 1} = \frac{1}{2} \quad \boxed{y = \frac{1}{2}} \quad \checkmark \quad \underline{5}$$

Domena

$$x^2 - x \geq 0$$

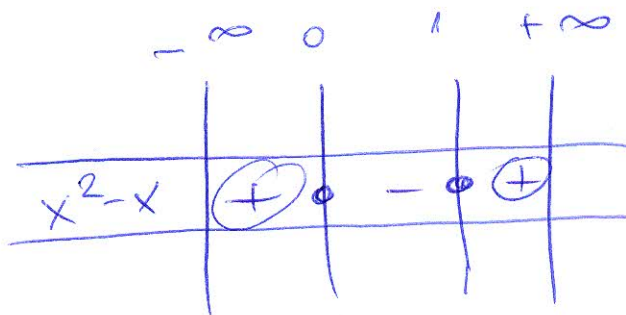
$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x_1 = 0$$

$$x - 1 = 0$$

$$x_2 = 1$$



$$\Delta f(x) \quad x \in \langle -\infty, 0 \rangle \cup$$

$$[1, +\infty) \quad \checkmark$$

③ nastavak

Nema kosih asimptota jer ~~prejela~~ ~~ima~~ ~~horizontalne~~ i nema V.A jer nema točaka prejelja.

NEMA LIJEVU H.A. PA MOŽE IMATI L.K.A.

PRVA DERIVACIJA

$$f(x) = x - \sqrt{x^2 - x}$$

$$f'(x) = x - (x^2 - x)^{\frac{1}{2}}$$

$$f'(x) = 1 - \frac{1}{2} (x^2 - x)^{-\frac{1}{2}} \cdot (2x - 1)$$

$$f'(x) = 1 - \frac{1}{\sqrt{x^2 - x}} \cdot (2x - 1)$$

$$f'(x) = 1 - \frac{2x - 1}{\sqrt{x^2 - x}} \quad \checkmark \quad 10$$

$$-2x(x^2 - 3)$$

⑤ nastavak

STACIONARNE TOČKE

$$f(x) = \frac{x^2 - 3}{x^2 + 3}$$

$$f'(x) = \frac{(x^2 - 3)' \cdot (x^2 + 3) - (x^2 - 3) \cdot (x^2 + 3)'}{(x^2 + 3)^2}$$

$$f'(x) = \frac{2x \cdot (x^2 + 3) - (x^2 - 3) \cdot 2x}{(x^2 + 3)^2}$$

$$f'(x) = \frac{2x^3 + 6x - 2x^3 + 6x}{(x^2 + 3)^2}$$

$$f'(x) = \frac{12x}{(x^2 + 3)^2}$$

$$12x = 0 \mid :12$$

$$x = 0$$

$-\infty \quad 0 \quad +\infty$

| | | | |
|---------|----|----|--|
| | -1 | +1 | |
| $f'(x)$ | - | + | |
| $f(x)$ | ↘ | ↗ | |

$$\frac{-12}{(1+3)^2} = -\frac{12}{8}$$

MN $(0, 0)$

MN $(0, -1)$

VIDI SE SA SLIKE

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100% INFLERSI

NETA T.I. \emptyset

$$f'(x) = \frac{12x}{(x^2+3)^2}$$

$$f''(x) = \frac{(12x)' \cdot ((x^2+3)^2)' - 12x \cdot ((x^2+3)^2)'}{((x^2+3)^2)^2}$$

$$f''(x) = \frac{12((x^2+3)^2)' - 12x \cdot 2(x^2+3) \cdot 2x}{(x^2+3)^4}$$

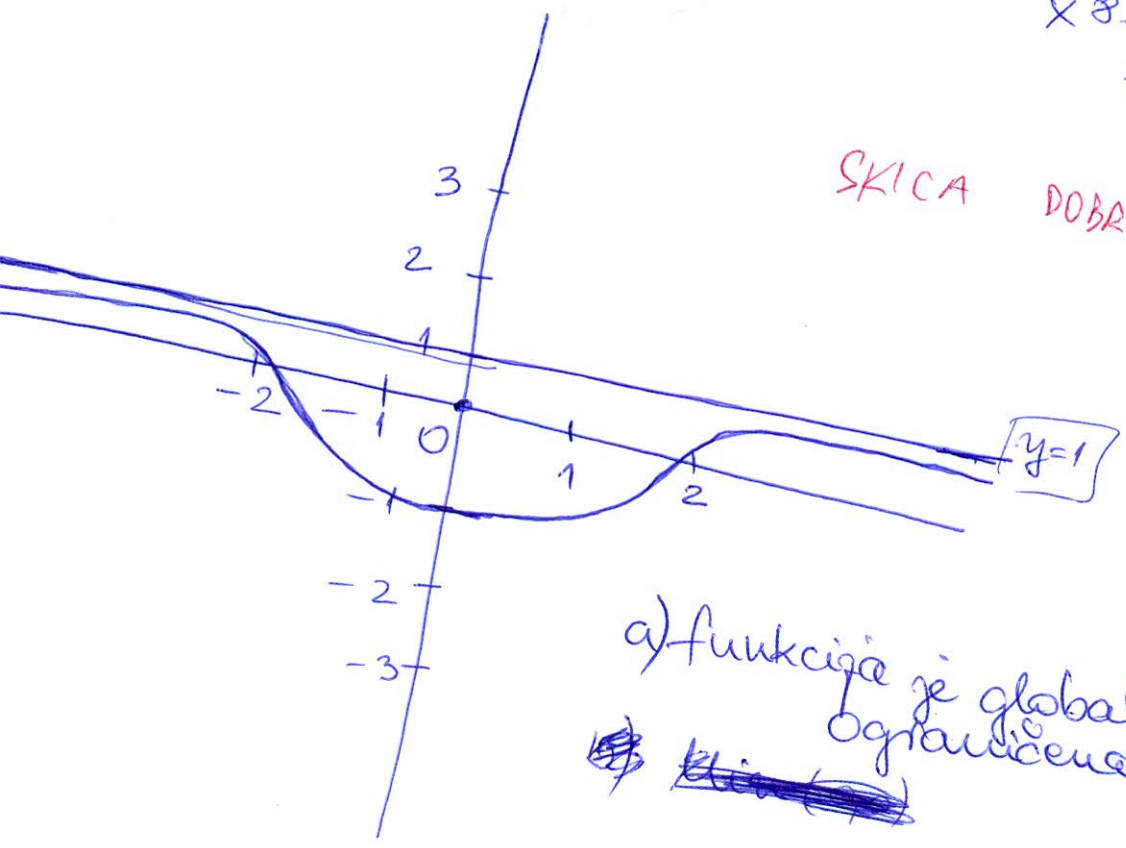
$$f''(x) = \frac{12((x^2+3)^2)' - 24x \cdot 2x^2 + 6}{(x^2+3)^4}$$

$$f''(x) = \frac{12(x^2+3)^4 - 48x^3 + 6}{(x^2+3)^4}$$

GRAF

$$f''(x) = \frac{-36x^4 - 72x^2 + 108}{x^8 + 12x^6 + 54x^4 + 108x^2 + 81}$$

SKICA DOBRA $\sqrt{15}$



a) funkcija je globalno ograničena $\sqrt{3}$

$$\begin{aligned} \textcircled{1} \quad & 2x + 3y - 3z - w = -5 \\ & 3z - 2y = 5 \\ & 2y - x - w = -1 \\ & w - 4x + 3z = 9 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 2 & 3 & -3 & -1 & 5 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array} \right) \begin{array}{l} 1 \cdot 2 \\ \downarrow + \end{array}$$

~~$$\left(\begin{array}{cccc|c} 2 & 3 & -3 & -1 & 5 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ 0 & 6 & -3 & -1 & 19 \end{array} \right) \begin{array}{l} \uparrow \\ 1 \cdot 2 \\ \sim \end{array}$$~~

$$\left(\begin{array}{cccc|c} 0 & 7 & -3 & 2 & -10 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ 0 & 6 & -3 & -1 & 19 \end{array} \right) \begin{array}{l} 1 \cdot 3 \\ \downarrow \sim \end{array}$$

~~$$\left(\begin{array}{cccc|c} 0 & 7 & -3 & 2 & -10 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ 0 & 0 & 6 & -1 & 34 \end{array} \right) \begin{array}{l} 1 \cdot (-1) \\ \sim \end{array}$$~~

$$\left(\begin{array}{cccc|c} 0 & 7 & -3 & 2 & -10 \\ 0 & -2 & 3 & 0 & 5 \\ +1 & -2 & 0 & +1 & +1 \\ 0 & 0 & 6 & -1 & 34 \end{array} \right) \begin{array}{l} 1 \cdot (-1) \\ \downarrow \sim \end{array}$$

~~$$\left(\begin{array}{cccc|c} 0 & 7 & -3 & 2 & -10 \\ 0 & -2 & 3 & 0 & 5 \\ 0 & 0 & -3 & 1 & -4 \\ 0 & 0 & 6 & -1 & 34 \end{array} \right) \begin{array}{l} \uparrow + \\ \sim \end{array}$$~~

$$\left(\begin{array}{cccc|c} 0 & 7 & -3 & 2 & -10 \\ 1 & -2 & 0 & 1 & 5 \\ 1 & 0 & -3 & 1 & -4 \\ 0 & 0 & 6 & -1 & 34 \end{array} \right) \begin{array}{l} \downarrow \sim \\ 1 \end{array}$$

~~$$\left(\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -4 \\ 1 & -2 & 0 & 1 & 5 \\ 0 & 7 & -3 & 2 & -10 \\ 0 & 0 & 6 & -1 & 34 \end{array} \right)$$~~

$$\textcircled{1} \left(\begin{array}{cccc|c} 2 & 3 & -3 & -1 & 5 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array} \right) \xrightarrow{1 \cdot (1)} \left(\begin{array}{cccc|c} 2 & -1 & 0 & -1 & 10 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array} \right)$$

OVO JE PROŠIRENA
MATRICA SUSTAVA
O.K. 2

$$\left(\begin{array}{cccc|c} 2 & -1 & 0 & -1 & 10 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 0 & 3 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array} \right) \xrightarrow{1 \cdot (-1)} \left(\begin{array}{cccc|c} 2 & -1 & 0 & -1 & 10 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 0 & 3 & -1 & -1 \\ -3 & 0 & 0 & 2 & 10 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 2 & -1 & 0 & -1 & 10 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -6 \\ -3 & 0 & 0 & 2 & 10 \end{array} \right) \xrightarrow{1 \cdot (-1)} \left(\begin{array}{cccc|c} 2 & -1 & 0 & -1 & 10 \\ 0 & -2 & 3 & 0 & 5 \\ 1 & -2 & 0 & 1 & 6 \\ -3 & 0 & 0 & 2 & 10 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 2 & -1 & 0 & -1 & 10 \\ 0 & -2 & 3 & 0 & 5 \\ 1 & 0 & -3 & 1 & -1 \\ -3 & 0 & 0 & 2 & 10 \end{array} \right) \xrightarrow{1 \cdot (3)} \left(\begin{array}{cccc|c} 2 & -1 & 0 & -1 & 10 \\ 0 & -2 & 3 & 0 & 5 \\ 1 & 0 & -3 & 1 & -1 \\ 0 & 0 & 6 & 5 & -7 \end{array} \right)$$

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BROJ INDEKSA:

$$\left(\begin{array}{cccc|c} 2 & -1 & 0 & -1 & 10 \\ 0 & -2 & 3 & 0 & 5 \\ 1 & 0 & -3 & 1 & -1 \\ \underline{2} & \underline{0} & \underline{0} & \underline{7} & \underline{-9} \end{array} \right) \begin{array}{l} \uparrow + \\ \\ \\ \downarrow 1 \cdot (-1) \end{array} \quad \begin{array}{l} \\ \\ \\ 2 \end{array}$$

$$\left(\begin{array}{cccc|c} 0 & -1 & 0 & -8 & 19 \\ 0 & -2 & 3 & 0 & 5 \\ 1 & 0 & -3 & 1 & -1 \\ 2 & 0 & 0 & 7 & -9 \end{array} \right) \begin{array}{l} \\ \\ \\ \updownarrow + \end{array}$$

$$\left(\begin{array}{cccc|c} 0 & -3 & 3 & -8 & 24 \\ 1 & 0 & -3 & 1 & -1 \\ 2 & 0 & 0 & 7 & -9 \end{array} \right)$$