MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: DINO KURIĆ **Broj indeksa:** 56192-2008

Vrijeme: od _____ **do** _____ ♣4 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(15)* Integriraj

$$\int \frac{1+\sin(3x)}{\cos^2(3x)}dx$$

2. ~~(15)~~ 15 Integriraj

$$\int_{-1}^{1} \frac{x}{(x+2)(x^2+1)}dx$$

3. ~~(15)~~ 15 Odredi površinu koju zatvaraju krivulja $y^2 = 2x+1$ i pravac $y = x+1$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = xy + 4x^2 - 3y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x^2+y^2)$$

5. ~~(20+15)~~ Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' - 5y = x^4$$

b)

$$y'' + 6y' + 9y = 2\cos x$$

VIDI RJEŠENJE 2

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME!

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5) b) $y'' + 6y' + 9y = 2\cos x$

$r^2 + 6r + 9 = 0$

$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2a}$

$r_{1,2} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 9}}{2}$

$r_{1,2} = \frac{-6 \pm \sqrt{36 - 36}}{2}$

$r_{1,2} = \frac{-6 \pm \sqrt{0}}{2c}$ $\quad r_1 = r_2 = -6$

$y = y_h + y_p$

$y = e^{rx}(C_1 + C_2 \cdot x)$

$y_h = e^{-6x}(C_1 + C_2 \cdot x) =$

4) $z = x \cdot y + 4x^2 - 3y^2$

$z_x = 8x^2 + y$

$z_y = x - 6y$

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5. a) $xy' - 5y = x^4$ $\quad g(x) = x^4$, $f(x) = -5$

TREBALO JE PRVO PREPISATI JEDNADŽBU U OBLIKU

$$y' - \frac{5}{x}y = x^3$$

$$\Rightarrow f(x) = -\frac{5}{x},\ g(x) = x^3$$

TEK ZATIM UVRSTITI U FORMULU

$$y = e^{-\int f(x)dx}\left[\int e^{\int f(x)dx} \cdot g(x) + C\right]$$

$$y = e^{\int -5\,dx} = e^{-5\int dx} = e^{-5x}$$

$$e^{-\int -5\,dx} = e^{\int 5\,dx} = e^{5x}$$

$$y = e^{5x} \cdot \left[\int e^{-5x} \cdot x^4\,dx + C\right]$$

$$\int -\frac{5}{x}dx = -5\ln|x| + C = \ln x^{-5} + C$$

$$\int e^{\ln x^{-5}} \cdot x^3\,dx = \int x^{-5} \cdot x^3\,dx = \int x^{-2}dx = \frac{x^{-1}}{-1} = -\frac{1}{x}$$

$$\Rightarrow y = x^5\left[-\frac{1}{x} + C\right]$$

$$I = \int e^{-5x} \cdot x^4\,dx = u \cdot v - \int v \cdot du$$

$x^4 = u \mid d$ $\quad v = \int e^{-5x}dx$ $\quad -5x = t \mid d$

$4x^3dx = du$ $\quad V = -\int e^t \cdot \frac{dt}{5}$ $\quad -5dx = dt \mid :-5$

$V = -\frac{1}{5}\cdot\int e^t\,dx$ $\quad dx = -\frac{dt}{5}$

$V = -\frac{1}{5}\cdot e^{-5x} + C$

$$I = x^4 \cdot -\frac{1}{5}e^{-5x} - \int -\frac{1}{5}\cdot e^{-5x} \cdot 4x^3\,dx = -x^4 \cdot \frac{1}{5}e^{-5x} - \left(-\frac{1}{5}\right)\int e^{-5x} \cdot 4x^3\,dx$$

$$= -x^4 \cdot \frac{1}{5}e^{-5x} + \frac{1}{5}\int e^{-5x} \cdot 4x^3\,dx$$

$$= -x^4 \cdot \frac{1}{5}e^{-5x} + \frac{4}{5}\int e^{-5x} \cdot x^3\,dx$$

$x^3 = u \mid d$ $\quad V = \int e^{-5x}dx$

$3x^2dx = du$ $\quad V = -\frac{1}{5}\cdot e^{-5x}$

$$= -x^4 \cdot \frac{1}{5}e^{-5x} + \frac{4}{5}\left(u \cdot V - \int v \cdot du\right)$$

$$= -x^4 \cdot \frac{1}{5}e^{-5x} + \frac{4}{5}\left(-x^3 \cdot \frac{1}{5}e^{-5x} - \int -\frac{1}{5}e^{-5x} \cdot 3x^2\,dx\right)$$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} + \frac{4}{5} \cdot \left(-\frac{1}{5}\right)x^3e^{-5x} + \frac{4}{5} \cdot \frac{3}{5}\int e^{-5x} \cdot x^2\,dx$$

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$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 e^{-5x} + \frac{12}{25}\int e^{-5x} \cdot x^2\,dx$$

$u = x^2 \quad V = -\frac{1}{5}e^{-5x}$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 e^{-5x} + \frac{12}{25}\int u \cdot v - \int v \cdot du \qquad du = 2\,dx$$

$$= -\frac{1}{5}\cdot x^4 \cdot e^{-5x} - \frac{1}{25}x^3 \cdot e^{-5x} + \frac{12}{25}\left(x^2 \cdot \left(\frac{-1}{5}\right)e^{-5x} - \int -\frac{1}{5}e^{-5x} \cdot 2x\,dx\right)$$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 \cdot e^{-5x} + \frac{12}{25}\left(-\frac{1}{5}\cdot x^2 e^{-5x} + \frac{2}{5}\int e^{-5x} x\,dx\right)$$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 \cdot e^{-5x} + \frac{12}{25}\cdot\left(-\frac{1}{5}\right)x^2 e^{-5x} + \frac{2}{5}\cdot\frac{12}{25}\int e^{-5x} x\,dx$$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 \cdot e^{-5x} - \frac{12}{125}x^2 e^{-5x} + \frac{24}{125}\int e^{-5x} \cdot x\,dx$$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 e^{-5x} - \frac{12}{125}x^2 e^{-5x} + \frac{24}{125}\left(u \cdot v - \int v \cdot du\right) \quad u = x \,/d \quad V = -\frac{1}{5}e^{-5x}$$

$du = dx$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 e^{-5x} - \frac{12}{125}x^2 e^{-5x} + \frac{24}{125}\left(x \cdot \left(-\frac{1}{5}\right)e^{-5x} - \int -\frac{1}{5}e^{-5x}\,dx\right)$$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 e^{-5x} - \frac{12}{125}x^2 \cdot e^{-5x} + \frac{24}{125}\left(-\frac{1}{5}x \cdot e^{-5x} + \frac{1}{5}\int e^{-5x}\,dx\right)$$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 e^{-5x} - \frac{12}{125}x^2 \cdot e^{-5x} + \frac{24}{125}\left(-\frac{1}{5}x \cdot e^{-5x} + \frac{1}{5}\cdot\left(-\frac{1}{5}\right)e^{-5x}\right)$$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 e^{-5x} - \frac{12}{125}x^2 \cdot e^{-5x} + \frac{24}{125}\cdot\left(-\frac{1}{5}x \cdot e^{-5x} - \frac{1}{25}e^{-5x}\right)$$

$$= -\frac{1}{5}x^4 \cdot e^{-5x} - \frac{1}{25}x^3 e^{-5x} - \frac{12}{125}x^2 \cdot e^{-5x} - \frac{24}{125}x \cdot e^{-5x} - \frac{24}{625}e^{-5x} + C$$

$$y = e^{5x} \cdot \left[-\frac{1}{5}x^4 e^{-5x} - \frac{1}{25}x^3 e^{-5x} - \frac{12}{125}x^2 \cdot e^{-5x} - \frac{24}{625}x \cdot e^{-5x} - \frac{24}{625}e^{-5x} + C\right]$$

ŽAO MI JE ZBOG SILNOG TRUDA.

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$$\int_{-1}^{1} \frac{x}{(x+2)(x^2+1)}$$