

MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: Jovan Fröhwirth Broj indeksa: _____

Vrijeme: od 8:20 do 9:55 ♣4

Broj bodova: 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. ~~(15)~~ Integriraj

$$\int \frac{1 + \sin(3x)}{\cos^2(3x)} dx$$

2. ~~(15)~~ Integriraj

$$\int_{-1}^1 \frac{x}{(x+2)(x^2+1)} dx$$

3. ~~(15)~~ Odredi površinu koju zatvaraju krivulja $y^2 = 2x + 1$ i pravac $y = x + 1$.

4. ~~(10+10)~~

- a) Ispitaj ekstreme funkcije

$$f(x, y) = xy + 4x^2 - 3y^2$$

- b) Odredi domenu funkcije:

$$f(x, y) = \ln(x^2 + y^2)$$

5. ~~(20+15)~~ Riješi sljedeće diferencijalne jednačbe:

- a)

$$xy' - 5y = x^4$$

- b)

$$y'' + 6y' + 9y = 2 \cos x$$

VIDI RJEŠENJE 2

PISATI JEDNOSTRANO!

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NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME!

(4.) $f(x,y) = xy + 4x^2 - 3y^2$

$f(x,y)_x = y + 8x$

$A(0,0)$

$f(x,y)_y = x - 6y$

$$\begin{array}{r} y + 8x = 0 \\ x - 6y = 0 \quad / \cdot (-8) \end{array} \qquad \begin{array}{r} y + 8x = 0 \\ x = 0 \end{array}$$

$$\begin{array}{r} y + 8x = 0 \\ -8x + 48y = 0 \end{array}$$

$y = 0$

$f(x,y)_{xx} = 8$

$f(x,y)_{xy} = 0$ ~~X~~

$f(x,y)_{yx} = 0$ ~~X~~

$f(x,y)_{yy} = -6$

$H_A = \begin{vmatrix} 8 & 0 \\ 0 & -6 \end{vmatrix} = -48 < 0$

Točka nije ekstrem!



NEMA BODOVA JER
NISU TOČNE SVE PARCIJALNE DERIVACIJE.

IME I PREZIME: Ivan Frihwirth $\left| \frac{x+2=z}{dx=dz} \right| = \int \frac{1}{z} dz = \ln|x+2| + C$

BROJ INDEKSA:

$\int \frac{1}{1+x^2} dx = \arctan \frac{x}{1} + C$

2. $\int_{-1}^1 \frac{x}{(x+2)(x^2+1)} dx = \int_{-1}^1 \frac{-1}{x+2} dx + \int_{-1}^1 \frac{1}{x^2+1} dx = \left(-\ln|x+2| + \arctan x \right) \Big|_{-1}^1$

$= (-1,09 + 45) - (-0,45 - 45) =$
 $= (-43,91 + 45) = 88,91$

$\frac{x}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

$x = A(x^2+1) + [Bx+C(x+2)]$

$x = Ax^2 + A + [Bx^2 + 2Bx + Cx + 2C]$

$x = x^2(A+B) + x(2B+C) + (A+2C)$

$A+B=0 \Rightarrow B=0$ ~~X~~

$2B+C=1 \Rightarrow C=1$

$A+2C=1 \Rightarrow A+2=1 \Rightarrow A=-1$

1. $\int \frac{1 + \sin(3x)}{\cos^2(3x)} dx = \int \frac{1}{\cos^2(3x)} dx + \int \frac{\sin(3x)}{\cos^2(3x)}$

$\int \frac{1 + \sin(3x)}{\cos^2(3x)} dx = \left| \frac{3x=z}{dx = \frac{dz}{3}} \right| = \int \frac{1 + \sin(z)}{\cos^2(z)} \frac{dz}{3} = \int \frac{1}{\cos^2(z)} \frac{dz}{3} + \int \frac{\sin z}{\cos^2(z)} \frac{dz}{3}$

$= \frac{1}{3} \int \frac{1}{\cos^2 z} dz + \frac{1}{3} \int \frac{\sin z}{\cos^2(z)} dz = \frac{1}{3} \tan(3x)$

$\int \frac{\sin z}{\cos^2 z} dz =$

IME I PREZIME:

Jean Trübenwirth

x	0	1	2	3	5
y	1	3	5	7	9

BROJ INDEKSA:

x	1	2	1	1
y	0	1	2	1

5. a) $xy' - 5y = x^4$

$$x \frac{dy}{dx} - 5y = 0$$

$$x \frac{dy}{dx} = 5y \quad | :dy$$

$$\frac{x}{dx} = \frac{5y}{dy}$$

$$\frac{dx}{x} = \frac{5y}{5y} \quad | \int$$

$$\int \frac{1}{x} dx = \int \frac{1}{y} dy \cdot \frac{1}{5}$$

$$\ln|x| + \ln|C| = 5 \ln|y|$$

$$\ln|x \cdot C| = \ln|y^5|$$

$$x \cdot C = y^5 \Rightarrow x \cdot C = \sqrt[5]{y}$$

3. $y^2 = 2x + 1$

$$y = x + 1$$

MULTIPLIKACIJA

$$\sqrt{2x+1} = 0$$

$$2x+1 = 0$$

$$2x = -1$$

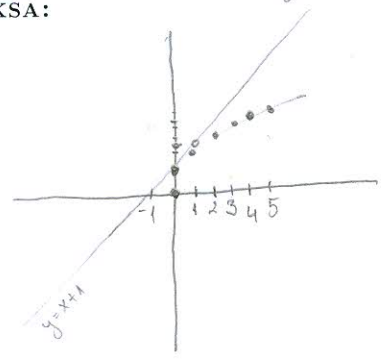
$$x = -\frac{1}{2}$$

SPECIŠTA:

$$x+1 = \sqrt{2x+1} \quad | ^2$$

$$x^2 + 2x + 1 = 2x + 1$$

$$x^2 = 0$$



?

b) $y'' + 6y' + 9y = 0$

$$x_{1,2} = \frac{-6 \pm \sqrt{36 - 4(1)(9)}}{2}$$

$$x_{1,2} = \frac{-6 \pm 0}{2} \Rightarrow -3$$

$$y_0 = C_1 e^{-3x} + C_2 e^{-3x}$$

$$y_0 = e^{-3x} (C_1 + C_2)$$

$$y = A \cos x + B \sin x$$

$$y' = A' \sin x + B' \cos x$$

$$y'' = -A'' \cos x + B'' \sin x$$

TREBALO JE RIJESITI NEHOMOGENU ODJ:

$$y'' + 6y' + 9y = 2 \cos x$$

⊙