Popuniti odmah! PISATI JEDNOSTRANO!

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DATUM: 26.6.2012. VRIJEME: OD 8:30 DO .

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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Broj ↓ bodova

1. Zadan je skup linearnih jednadžbi:

$$\begin{aligned} 2x + 3y - 3z - w &= -5 \\ 3z - 2y &= 5 \\ 2y - x - w &= -1 \\ w - 4x + 3z &= 9 \end{aligned}$$

   (a) zapisati dani sustav matrično, 2

   (b) riješiti matrični sustav Gaussovom metodom 10

   (c) provjeriti izračunato rješenje matričnim množenjem 3

2. Riješiti u kompleksnim brojevima sljedeće jednadžbe:

   (a) $z^3 + |3 - 4i| = \frac{5}{i}$. 10

   (b) $-z + |z - 4i| = \overline{3 + 4i}$. 10

3. Za funkciju $f(x) = x - \sqrt{x^2 - x}$:

   (a) odrediti asimptote i 10

   (b) odrediti prvu derivaciju 10

4. Za funkciju $g(x) = \frac{e^{2x}}{x^2}$:

   (a) uz pomoć L'Hopitalovog pravila odrediti: $\lim_{x \to +\infty} g(x)$ 10

   (b) uz pomoć zaključka iz (a) diskutirati konvergenciju reda: $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^2}$ 10

5. Zadana je funkcija: $h(x) = \frac{x^2 - 3}{x^2 + 3}$. Na temelju ispitivanja toka funkcije:

   (a) diskutirati da li je funkcija globalno ograničena ili ne, 5

   (b) navesti sve lokalne ekstreme, 5

   (c) navesti sve točke infleksije i 5

   (d) napraviti skicu grafa funkcije. 10 5

1.)

$$\left(\begin{array}{cccc|c} 2 & 3 & -3 & -1 & -5 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array}\right) \sim \left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & -1 \\ 0 & -2 & 3 & 0 & 5 \\ 2 & 3 & -3 & -1 & -5 \\ -4 & 0 & 3 & 1 & 9 \end{array}\right) \begin{array}{l} \\ \\ /2_3 + 2 \cdot 2_1 \\ /2_4 - 4 \cdot 2_1 \end{array}$$

VIDI RJEŠENJE 1

$$\left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & -1 \\ 0 & -2 & 3 & 0 & 5 \\ 0 & 9 & -3 & -3 & -7 \\ 0 & -8 & 3 & 5 & 13 \end{array}\right) /2_3 + 2_4 \quad \left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & -1 \\ 0 & -2 & 3 & 0 & 5 \\ 0 & 1 & 0 & 2 & 6 \\ 0 & -8 & 3 & 5 & 13 \end{array}\right)$$

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4.)

a) $\lim_{x\to+\infty} \frac{e^{2x}}{x^2} \overset{L'H}{=} \lim_{x\to+\infty} \frac{2e^{2x}}{2x} \overset{L'H}{=} \lim \frac{e^{2x}}{x} \overset{L'H}{=} \lim \frac{2e^{2x}}{1} = \lim 2e^{2x} = \lim_{x\to+\infty} 2e^{2x} = ?$

DERIVIRANO

b) $\lim_{n=1} \frac{e^{2n}}{n^2} = \lim_{n=1} \frac{2e^{2n}}{2n} = \frac{e^{2n}}{n} = \frac{e^{2\cdot 1}}{1} = 7.4$

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nastavak

1) zad.

$$\left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & -1 \\ 0 & -2 & 3 & 0 & 5 \\ 0 & 1 & 0 & 2 & 6 \\ 0 & -8 & 3 & 5 & 13 \end{array}\right) \begin{matrix} \\ \\ \\ /\cdot 4 - 4\cdot 2 \end{matrix} \left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & -1 \\ 0 & -2 & 3 & 0 & 5 \\ 0 & 1 & 0 & 2 & 6 \\ 0 & 0 & -9 & 5 & -7 \end{array}\right) \begin{matrix} \\ \\ \\ /2+3 \end{matrix}$$

$$\left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & -1 \\ 0 & -1 & 3 & 2 & 11 \\ 0 & 1 & 0 & 2 & 6 \\ 0 & 0 & -9 & 5 & -7 \end{array}\right) \begin{matrix} \\ /3+2 \\ \\ \\ \end{matrix} \left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & -1 \\ 0 & -1 & 3 & 2 & 11 \\ 0 & 0 & 3 & 4 & 17 \\ 0 & 0 & -9 & 5 & -7 \end{array}\right) \begin{matrix} \\ 2-3 \\ \\ /4-3 \end{matrix}$$

$$\left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & -1 \\ 0 & -1 & 0 & -2 & -6 \\ 0 & 0 & 3 & 4 & 17 \\ 0 & 0 & -12 & 1 & -24 \end{array}\right) \begin{matrix} /1+2 \\ \\ \\ \\ \end{matrix} \left(\begin{array}{cccc|c} -1 & 1 & 0 & -3 & -7 \\ 0 & -1 & 0 & -2 & -6 \\ 0 & 0 & 3 & 4 & 17 \\ 0 & 0 & -12 & 1 & -24 \end{array}\right) \begin{matrix} /1+2 \\ \\ \\ \\ \end{matrix}$$

$$\left(\begin{array}{cccc|c} -1 & 0 & 0 & -5 & -13 \\ 0 & -1 & 0 & -2 & -6 \\ 0 & 0 & 3 & 4 & 17 \\ 0 & 0 & -12 & 1 & -24 \end{array}\right)$$

$-1x - 5w = -13 \Rightarrow -1x = -13 + 5w \times -1 \Rightarrow 13 - 5w$

$-1y - 2w = -6 \Rightarrow -1y = -6 + 2w \Rightarrow y = 6 - 2w$

$3z + 4w = 17 \Rightarrow 3z = 17 - 4w$

$-12z + 1w = -24 \Rightarrow w = -24 + 12z$

$w = z \quad z = 4$

$$z\begin{pmatrix} -5 \\ -2 \\ -4 \\ 0 \end{pmatrix} + \begin{pmatrix} 13 \\ 6 \\ 17 \\ -24 \end{pmatrix} = \begin{pmatrix} 13 \\ 6 \\ 17 \\ -24 \end{pmatrix} \to \begin{pmatrix} 13 \\ +5 \\ 0 \\ 0 \end{pmatrix}$$

$\to -12 + 18 = 5$

$-17 + 34 - 17 = 0$

$96 - 72 - 24 = 0$

$26 - 13 = 13$

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$\frac{x^2-3}{x^2+3}$ $x_2+3\neq 0$, $x^2\neq -3$ $D=\mathbb{R}$ ✓

NUL TOČKE

$x^2-3=0$

$x^2=3$

$x=\pm\sqrt{3}$ ✓ $x_1=\sqrt{3}$, $x_2=-\sqrt{3}$ } SJEČE x-OS

V.A NEMA.

H.A.

$\lim_{x\to\pm\infty}\frac{x^2-3\,/:x^2}{x^2+3\,/:x^2}=\frac{1}{1}=1$ $y=1$ H.A. ✓

K.A $y=kx+l$

$\lim_{x\to\infty}\frac{x^2-3}{x^2+3}=\lim_{x\to\infty}\frac{x^2-3\,/:x^3}{x^3+3x\,/:x^3}=\frac{1}{0}=\infty$ K.A. NEMA. ✓

$f'(x)=\frac{(x^2-3)'\cdot(x^2+3)-(x^2-3)\cdot(x^2+3)'}{(x^2+3)^2}=\frac{2x(x^2+3)-(x^2-3)\cdot 2x}{(x^2+3)^2}$

$=\frac{2x^3+6x-2x^3+6x}{(x^2+3)^2}=\frac{12x}{(x^2+3)^2}$ $12x=0$, $x=0$

	$-\infty$	$-\sqrt{3}$	$\sqrt{3}$	$+\infty$
	-2	1	2	
f'	−	+	+	
f	↓	↑	↑	

$m\left(-\sqrt{3},\ \ \right)$ ✗

$f''(x)=\frac{(12x)'\cdot(x^2+3)^2-(12x)\cdot 2(x^2+3)\cdot 2x}{(x^2+3)^4}$ (4x)

$=\frac{12x^2+36x-48x^2}{(x^2+3)^3}=\frac{-36x^2+36}{(x^2+3)^3}$ ✓ $\Rightarrow -36x^2+36x=0$ ✗

$x=0$ $\frac{-36\pm\sqrt{36^2-4\cdot(-36)\cdot 0}}{-72}=\frac{-36+36}{-72}$ ✗

$x_1=\frac{0}{-72}=0$

$x_2=\frac{-72}{-72}=1$

	$-\infty$	0	1	$+\infty$
	-1	1.5	4	
f''	+	−	+	
f	∪	∩	∪	

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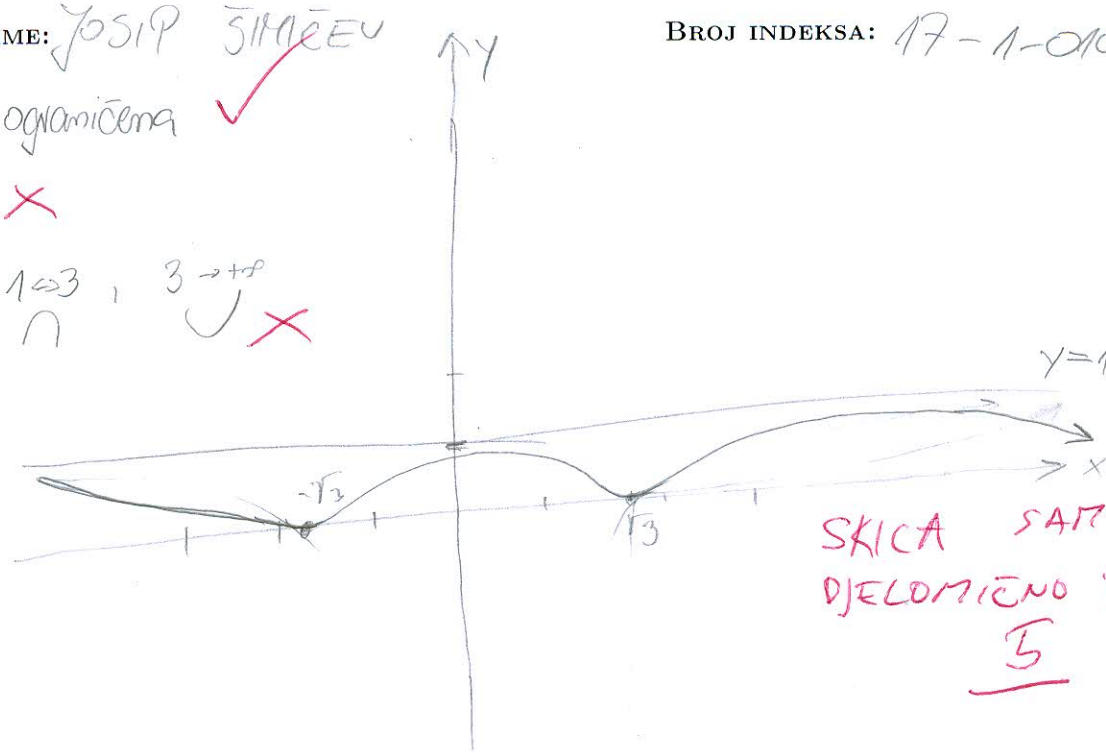
a) funkcija je ograničena ✓

b) $m(\sqrt{3})$ ✗

c) $-\infty \to 1$ ∩, $1 \to 3$ ∩, $3 \to +\infty$ ∪ ✗

d) →



SKICA SAMO DJELOMIČNO TOČNA

5

2.)

a) $z^3 + |3-4i| = \frac{5}{i}$

$|3-4i| = \sqrt{3^2+(-4)^2} = 5$

$\frac{5}{i} = \frac{5}{i}\cdot\frac{-i}{-i} = \frac{-5i}{-i^2} = \frac{-5i}{1} = -5i$

$z^3 + 5 = -5i$ ✓

$z^3 = -5i - 5$ ✓

$z = \sqrt[3]{-5-5i}$

$x = -5$, $y = -5i$ $\Big\}$ $r = \sqrt{x^2+y^2} = \sqrt{(5)^2+(-5)^2} = \sqrt{50}$

$r = \sqrt[3]{\sqrt{50}}$

$\varphi = \pi + \arctan\left(\frac{y}{x}\right) = \pi + \arctan\left(\frac{-5}{5}\right) = 2.35$

$k = 0, 1, 2 \ldots$

$z_1 = \sqrt[6]{50}\cdot\left(\cos\left(\frac{2.35}{3}\right) + i\sin\left(\frac{2.35}{3}\right)\right) = \sqrt[6]{50}\cdot(0{,}708 + 0{,}705i)$

$z_2 = \sqrt[6]{50}\cdot\left(\cos\frac{2.35+2\pi}{3} + i\sin\frac{2.35+2\pi}{3}\right) = \sqrt[6]{50}\cdot(-0.96 + 0.26)$

$z_3 = \sqrt[6]{50}\cdot\left(\cos\frac{2.35\cdot 4\pi}{3} + i\sin\frac{2.35+4\pi}{3}\right) = \sqrt[6]{50}\cdot(0.26 - 0.96)$

$z_1 = 1.35 + 1.35i$

$z_2 = -1.84 + 0.5i$ ✓

$z_2 = 0.5 - 1.84i$

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b) $-2 + |2 - 4i| = \overline{3+4i}$

$(x+yi) + \sqrt{x^2+(y^2-4)^2} = 3-4i$

$\sqrt{x^2+(y-4)^2} - x = 3$ ✓

$y = -4$ ✗

$\sqrt{x^2+(-4-4)^2} = 3+x \;/^2$

$64 + x^2 = 9 + 6x + x^2$

$-6x = 9 - 64$

$-6x = -55$

$x = \frac{55}{6}$ ∅

$|x+yi-4i| = |x+i(y-4)|$

$= \sqrt{x^2+(y-4)^2}$

$\sqrt{x^2+y^2+(4)^2}$

$(y+4)^2 = y^2 - 8y + 16 = 64$

$(-4-4)^2 = 16+32+16$

$= 64$

$z = ?$

3. $x - \sqrt{x^2-x} \Rightarrow f'(x) = 1 - \frac{2x}{2\sqrt{x^2-x}} = 1 - \frac{x}{\sqrt{x^2-x}} = \frac{\sqrt{x^2-x} - x}{\sqrt{x^2-x}}$ //

$x^2 - x \geq 0$

$x^2 = 1$

$x = \pm 1$

V.A.

$\lim_{x \to \pm 1} x - \sqrt{x^2-x} \cdot \frac{x+\sqrt{x^2-x}}{x+\sqrt{x^2-x}} = \frac{x^2 - x^2 - x \;/:x}{x+\sqrt{x^2-x}\;/:x} = \frac{-1}{1+\sqrt{1}} = -\frac{1}{2}$

NEMA V.A. ✓

$\lim_{x \to \pm\infty} x - \sqrt{x^2-x} \cdot \frac{x+\sqrt{x^2-x}}{x+\sqrt{x^2-x}} = \frac{-x \;/:x}{x+\sqrt{x^2-x}\;/:x} = \frac{-1}{1+\sqrt{1}} = \frac{-1}{2}$ ✗

V.A. $y = kx + c$

$\lim_{x \to +\infty} \frac{x - \sqrt{x^2-x}}{x} \cdot \frac{-x}{-x} = \frac{-x^2 + (x\sqrt{x^2-x})\;/:x^2}{-x^2\;/:x^2} = \frac{-1 \cdot (1 - \sqrt{\frac{x^2}{x^2} - \frac{x}{x^2}})}{-1} = \frac{-1}{2}$ H.A.

$= \frac{-1+1}{-1} = \frac{0}{-1} = 0$

$y = 1x$

V.A. ?

H.A.

$\lim_{x \to \infty} x - \sqrt{x^2-x} - x = -\sqrt{x^2-x} \cdot \frac{\sqrt{x^2-x}}{\sqrt{x^2-x}} = \frac{-x^2 - x \;/:x^2}{\sqrt{x^2-x}\;/:x^2} \to +\infty$