Popuniti odmah! PISATI JEDNOSTRANO!

IME I PREZIME: LUKA STIPIĆ **BROJ INDEKSA:** 17-20083-2011

DATUM: 26.6.2012. VRIJEME: OD DO

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova: 25

1. Zadan je skup linearnih jednadžbi:

$$\begin{aligned} 2x + 3y - 3z - w &= -5 \\ 3z - 2y &= 5 \\ 2y - x - w &= -1 \\ w - 4x + 3z &= 9 \end{aligned}$$

   (a) zapisati dani sustav matrično, 2

   (b) riješiti matrični sustav Gaussovom metodom 10

   (c) provjeriti izračunato rješenje matričnim množenjem 3

2. Riješiti u kompleksnim brojevima sljedeće jednadžbe:

   (a) $z^3 + |3 - 4i| = \frac{5}{i}$. 10

   (b) $-z + |z - 4i| = \overline{3 + 4i}$. 10

3. Za funkciju $f(x) = x - \sqrt{x^2 - x}$:

   (a) odrediti asimptote i 10

   (b) odrediti prvu derivaciju 10

4. Za funkciju $g(x) = \frac{e^{2x}}{x^2}$:

   (a) uz pomoć L'Hopitalovog pravila odrediti: $\lim_{x \to +\infty} g(x)$ 10

   (b) uz pomoć zaključka iz (a) diskutirati konvergenciju reda: $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^2}$ 10

5. Zadana je funkcija: $h(x) = \frac{x^2 - 3}{x^2 + 3}$. Na temelju ispitivanja toka funkcije:

   (a) diskutirati da li je funkcija globalno ograničena ili ne, 5

   (b) navesti sve lokalne ekstreme, 5

   (c) navesti sve točke infleksije i 5

   (d) napraviti skicu grafa funkcije. 10

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1.
$$\left[\begin{array}{cccc|c} 2 & 3 & -3 & -1 & -5 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array}\right] \sim \left[\begin{array}{cccc|c} -1 & 2 & 0 & -1 & -1 \\ 0 & -2 & 3 & 0 & 5 \\ 2 & 3 & -3 & -1 & -5 \\ -4 & 0 & 3 & 1 & 9 \end{array}\right] /\cdot(+1)$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 1 \\ 0 & -2 & 3 & 0 & 5 \\ 2 & 3 & -3 & -1 & -5 \\ -4 & 0 & 3 & 1 & 9 \end{array}\right] \begin{array}{l} \\ \\ /+1R\cdot 2 \\ /+1R\cdot 4 \end{array} \quad \left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 1 \\ 0 & -2 & 3 & 0 & 5 \\ 0 & 7 & -3 & -3 & -7 \\ 0 & -8 & 3 & 5 & 13 \end{array}\right] /\cdot(-\tfrac{1}{2})$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 1 \\ 0 & 1 & -\tfrac{3}{2} & 0 & -\tfrac{5}{2} \\ 0 & 7 & -3 & -3 & -7 \\ 0 & -8 & 3 & 5 & 13 \end{array}\right] \begin{array}{l} /+2R\cdot 2 \\ \\ /-2R\cdot 7 \\ /+2R\cdot 8 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -4 \\ 0 & 1 & -\tfrac{3}{2} & 0 & -\tfrac{5}{2} \\ 0 & 0 & -\tfrac{1}{2} & -3 & 21/2 \\ 0 & 0 & -9 & 5 & -7 \end{array}\right] /\cdot\tfrac{2}{15}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -4 \\ 0 & 1 & -\tfrac{3}{2} & 0 & -\tfrac{5}{2} \\ 0 & 0 & 1 & -\tfrac{2}{5} & \tfrac{7}{5} \\ 0 & 0 & -9 & 5 & -7 \end{array}\right] \begin{array}{l} /+3R\cdot 3 \\ /+3R\cdot \tfrac{3}{2} \\ \\ /+3R\cdot 9 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\tfrac{1}{5} & \tfrac{1}{5} \\ 0 & 1 & 0 & -\tfrac{3}{5} & -\tfrac{2}{5} \\ 0 & 0 & 1 & -\tfrac{2}{5} & \tfrac{7}{5} \\ 0 & 0 & 0 & \tfrac{7}{5} & \tfrac{28}{5} \end{array}\right] /\cdot\tfrac{5}{7}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\tfrac{1}{5} & \tfrac{1}{5} \\ 0 & 1 & 0 & -\tfrac{3}{5} & -\tfrac{2}{5} \\ 0 & 0 & 1 & -\tfrac{2}{5} & \tfrac{7}{5} \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \begin{array}{l} /+4R\cdot\tfrac{1}{5} \\ /+4R\cdot\tfrac{3}{5} \\ /+4R\cdot\tfrac{2}{5} \\ \\ \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$2\cdot 1 + 3\cdot 2 - 3\cdot 3 - 4 = -5$

$3\cdot 3 - 2\cdot 2 = 5$

$2\cdot 2 - 1 - 4 = -1$

$4 - 4\cdot 1 + 3\cdot 3 = 9$

$$\left[\begin{array}{cccc} 2 & 3 & -3 & -1 \\ 0 & -2 & 3 & 0 \\ -1 & 2 & 0 & -1 \\ -4 & 0 & 3 & 1 \end{array}\right] \cdot \left[\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}\right] \checkmark$$

$$\left[\begin{array}{l} 2\cdot 1 + 3\cdot 2 + (-3)\cdot 3 + (-1)\cdot 4 \\ 0\cdot 1 + (-2)\cdot 2 + 3\cdot (3) + 0\cdot 4 \\ (-1)\cdot 1 + 2\cdot 2 + 0\cdot 3 + (-1)\cdot 4 \\ (-4)\cdot 1 + 0\cdot 2 + 3\cdot 3 + 1\cdot 4 \end{array}\right] \Rightarrow \left[\begin{array}{c} -5 \\ 5 \\ -1 \\ 9 \end{array}\right]$$

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3) $f(x) = x - \sqrt{x^2 - x} \Rightarrow f(x) = x - (x^2 - x)^{\frac{1}{2}}$

$f'(x) = 1 - \frac{1}{2}(x^2 - x)^{-\frac{1}{2}} \cdot (2x - 1)$

$f'(x) = 1 - \frac{1}{\sqrt{x^2 - x}} \cdot (2x - 1)$

$f'(x) = 1 - \frac{2x - 1}{\sqrt{x^2 - x}}$ ✓ 10

ASIMPTOTE

V.A. $x^2 - x = 0 \quad x(x - 1) = 0 \quad x_1 = 0, \; x_2 = 1$ V.A = 1 ✗

HORIZONTALNA I KOSA

$y = k \cdot x + l$

$k = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x - \sqrt{x^2 - x}}{x} = \frac{\infty}{\infty} = \lim_{x \to \infty} \frac{1 - \frac{2x - 1}{\sqrt{x^2 - x}}}{1} = \lim_{x \to \infty} \frac{1 - 2x + 1}{\sqrt{x^2 - x}} = \lim_{x \to \infty} \frac{-2}{\frac{1}{2\sqrt{x^2 - x}} \cdot (2x - 1)} = \frac{-2}{\infty}$ NEMA K.A. ✗

$l = \lim_{x \to \infty} [f(x) - k \cdot x] = \lim_{x \to \infty} x - \sqrt{x^2 - x} = [\infty - \infty] = \lim_{x \to \infty}$

0

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4. $g(x) = \frac{e^{2x}}{x^2}$

a) $\lim_{x \to \infty} \frac{e^{2x}}{x^2} = \lim_{x \to \infty} \frac{e^{2x} \cdot 2}{2x} = \lim_{x \to \infty} = \frac{1 \cdot 2}{2} = \frac{2}{2} = 1$ ✓ ✗ ∅