MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: LUKA-DRAGAN BILAFER **Broj indeksa:** 0269032363

Vrijeme: od _____ **do** _____ ♣5 **Broj bodova:** $\frac{50}{80} = 62\%$

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(15)* 10 Integriraj

$$\int_0^1 x \tan(x^2+1)\,dx$$

2. *(20)* Integriraj

$$\int \frac{x^2+1}{(x+1)^2(x-1)}dx$$

3. *(20)* Odredi površinu koju zatvaraju krivulje $y = 1 - x^2$, $y = 3 + 2x - x^6$ i os apscisa. IZBAČENO

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 + y^2 + xy - 3x - 6y$$

b) Odredi domenu funkcije:

$$f(x,y) = x - \sqrt{x+y}$$

5. *(10+15)* Riješi sljedeće diferencijalne jednadžbe:

a) $y' = -\frac{y}{x}$

b)

$$y'' + y' + \frac{1}{4}y = 2$$

VIDI RJEŠENJE 1

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME!

(3)

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5 b) $y'' + y' + \frac{1}{4}y = 2$ | a) $y' = -\frac{y}{x}$ → $y' + \frac{1}{x} \cdot y = 0$ Homogena

$\lambda^2 + \lambda + \frac{1}{4} = 2$ ✗

$\lambda^2 + \lambda + \frac{1}{4} - 2 = 0$

$\lambda^2 + \lambda + \frac{1-8}{4} = 0$

$\lambda^2 + \lambda - \frac{7}{4} = 0$

$\lambda_{1,2} = \frac{-1 \pm \sqrt{1+7}}{2}$

$\lambda_{1,2} = \frac{-1 \pm \sqrt{8}}{2}$

$\lambda_{1,2} = \frac{-1 \pm 2\sqrt{2}}{2}$

$\lambda_1 = \frac{-1-2\sqrt{2}}{2}$

$\lambda_2 = \frac{-1+2\sqrt{2}}{2}$

$\lambda_1 = -\frac{1}{2} - \sqrt{2}$

$\lambda_2 = -\frac{1}{2} + \sqrt{2}$

$\frac{dy}{dx} = -\frac{1}{x} \cdot y \;/\cdot dx$

$dy = -\frac{dx}{x} \cdot y \;/:y$

$\frac{dy}{y} = -\frac{dx}{x} \;/\int$

$\int \frac{dy}{y} = -\int \frac{dx}{x}$

$\ln|y| = -\ln|x| + \ln|c|$

$\ln|y| = \ln\left|\frac{c}{x}\right| \;/ y = \frac{c}{x}$

✓ 10

VIDI RJEŠENJE 1

$\lambda_1 \neq \lambda_2$; $\lambda_1, \lambda_2 \in \mathbb{R}$

$y(x) = C_1 \cdot e^{\lambda_1 \cdot x} + C_2 \cdot e^{\lambda_2 \cdot x}$

$y(x) = C_1 \cdot e^{(-\frac{1}{2}-\sqrt{2})\cdot x} + C_2 \cdot e^{(-\frac{1}{2}+\sqrt{2})\cdot x}$

(1)

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① $\int_0^1 x \cdot tg(x^2+1)\,dx$ $\left| \begin{array}{l} x^2+1 = t \\ 2x\,dx = dt \\ x\,dx = \frac{1}{2}dt \end{array} \right| = \int \frac{1}{2} \cdot tg(t) \cdot dt = \frac{1}{2}\int tg(t)\,dt$

$= \frac{1}{2} \cdot -\ln|\cos(t)| + C = \left| -\frac{1}{2}\ln|\cos(x^2+1)| \right|_0^1$

$\int_0^1 x \cdot tg(x^2+1)\,dx = -\frac{1}{2}\ln|\cos(x^2+1)|\Big|_0^1 = -\frac{1}{2}\left(\ln|\cos(1+1)| - \ln|\cos(0+1)|\right)$

PARAMETAR TRIC. FJA GRANICAMA

$= -\frac{1}{2}\left(\ln|\cos(2)| - \ln|\cos(1)|\right) = -\frac{1}{2} \cdot \left(-4{,}57 \cdot 10^{-4}\right) = 2{,}285 \cdot 10^{-4}$

NEPRAVI INTEGRAL KOJI NE POSTOJI 10

② $\int \frac{x^2+1}{(x+1)^2(x-1)}\,dx$, $\frac{x^2+1}{(x+1)^2 \cdot (x-1)} = \frac{A}{(x+1)^2} + \frac{B}{(x+1)} + \frac{C}{(x-1)} \Big| \cdot (x+1)^2 \cdot (x-1)$ ✓

$x^2+1 = A \cdot (x-1) + B(x+1)(x-1) + C(x+1)^2$

$x^2+1 = Ax - A + B(x^2-1) + C(x^2+2x+1)$

$x^2+1 = Ax - A + Bx^2 - B + Cx^2 + 2xC + C$

$x^2+1 = x^2(B+C) + x(A+2C) + (C-B-A)$

$B+C=1 \rightarrow B = 1-C$

$A+2C=0 \rightarrow A = -2C \rightarrow A = -2 \cdot \frac{1}{2} = -1$

$C-B-A=1 \rightarrow C = 1+B+A$

$C = 1+(1-C)-2C \qquad B = 1-\frac{1}{2}$

$C = 1+1-C-2C \qquad B = \frac{1}{2}$

$C = 2-3C$

$4C = 2 \rightarrow C = \frac{1}{2}$

(2)

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(2) - NASTAVAK

$$\frac{x^2+1}{(x+1)^2\cdot(x-1)} = \frac{-1}{(x+1)^2} + \frac{\frac{1}{2}}{(x+1)} + \frac{\frac{1}{2}}{(x-1)}$$

$$\int \frac{x^2+1}{(x+1)^2(x-1)}\,dx = \int \left(\frac{-1}{(x+1)^2} + \frac{\frac{1}{2}}{(x+1)} + \frac{\frac{1}{2}}{(x-1)} \right) dx$$

$$= -\int \frac{dx}{(x+1)^2} + \frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{dx}{x-1}$$

$$= \boxed{\frac{1}{x+1} + \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C}$$ ✓

$I_1 = -\int \frac{dx}{(x+1)^2}$ | $x+1=t\,/'$, $dx=dt$

$= -\int \frac{dt}{t^2} = -\int t^{-2}\,dt = -\frac{t^{-1}}{-1}$

$= \frac{1}{t} = \frac{1}{x+1}$

$I_2 = \frac{1}{2}\int \frac{dx}{x+1}$ | $x+1=t\,/'$, $dx=dt$

$= \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| = \frac{1}{2}\ln|x+1|$

$I_3 = \frac{1}{2}\int \frac{dx}{x-1}$ | $x-1=t\,/'$, $dx=dt$

$= \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| = \frac{1}{2}\ln|x-1|$

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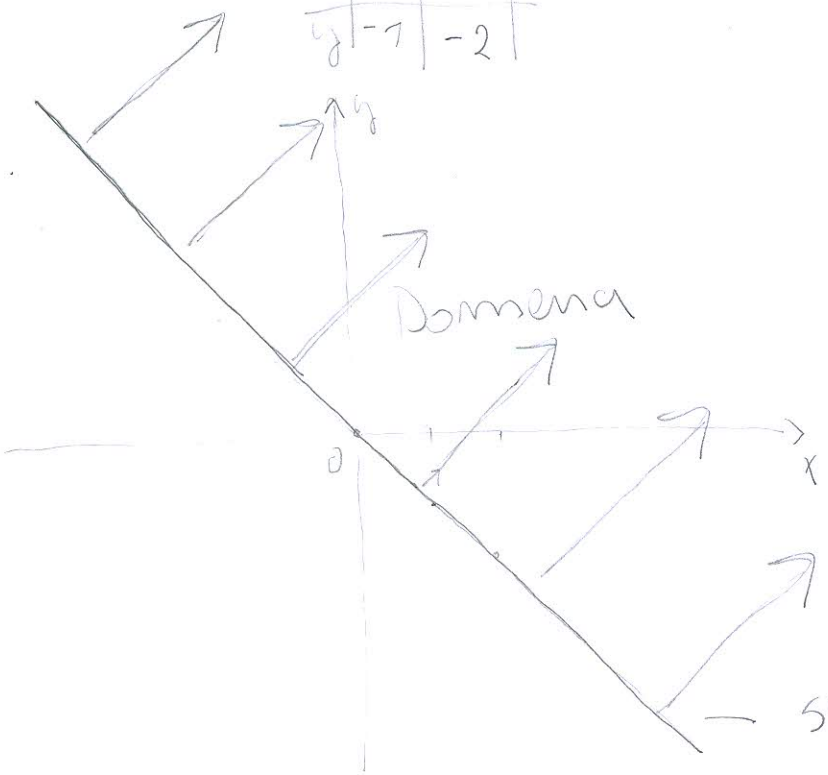
(4) b) $f(x;y) = x - \sqrt{x+y}$

$x+y \geq 0$

$\boxed{x \geq -y}$ → Pravac $x=-y$

x	1	2
y	-1	-2



✓

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— spadanja i točke na pravcu ravni zmaka (2)