MATEMATIKA 2
14. lipnja 2012.

Ime i prezime: Nikolina Burazin **Broj indeksa:** 17-2-0082-2011

Vrijeme: od ____________ **do** ________ ♣5 **Broj bodova:** $\frac{30}{80} = 37.5\,\%$

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(15)* Integriraj

$$\int_0^1 x \tan(x^2+1)\,dx$$

2. *(20)* Integriraj

$$\int \frac{x^2+1}{(x+1)^2(x-1)}\,dx$$

3. *(20)* Odredi površinu koju zatvaraju krivulje $y = 1 - x^2$, $y = 3 + 2x - x^6$ i os apscisa. IZBAČENO

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^2 + y^2 + xy - 3x - 6y$$

b) Odredi domenu funkcije:

$$f(x,y) = x - \sqrt{x+y}$$

5. *(10+15)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' = -\frac{y}{\text{[illegible]}}$$

b)

$$y'' + y' + \frac{1}{4}y = 2$$

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME!

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Broj indeksa:

4. a) $f(x,y)=x^2+y^2+xy-3x-6y$

$\frac{df}{dx}=2x+y-3$

$\frac{df}{dy}=2y+x-6$

$2x+y-3=0 \rightarrow y=3-2x$

$2y+x-6=0$

$2\cdot(3-2x)+x-6=0$

$\cancel{6}-4x+x-6=0$

$-3x=0$

$\underline{x=0}$ $\quad y=3-2\cdot 0$

$\underline{y=3}$ $\quad T(0,3)$ ✓

$\frac{d^2f}{d^2x}=2$

$\Delta=\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}=4-1=3$

$\frac{d^2f}{d^2y}=2$

$\Delta>0$ $\quad \Delta>0$

$3>0$ $\underline{\min.}$ ✓ $\quad \underline{10}$

$\frac{d^2f}{dx\,dy}=1$

$\frac{d^2f}{dy\,dx}=1$

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2.)
$$\int \frac{x^2+1}{(x+1)^2(x-1)}\,dx \qquad \frac{x^2+1}{(x+1)^2(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-1)} \Big/ (x+1)^2(x-1)$$

$$x^2+1 = A(x+1)(x-1) + B(x-1) + C(x+1)^2$$

$$x^2+1 = A(x^2\cancel{-x}\cancel{+x}-1) + Bx - B + C(x^2+2x+1)$$

$$x^2+1 = Ax^2 - A + Bx - B + Cx^2 + 2Cx + C$$

$$1 = A + C \to A = 1 - C$$

$$0 = B + 2C \to B = -2C$$

$$\underline{1 = -A - B + C}$$

$$1 = -(1-C) - (-2C) + C$$

$$1 = -1 + C + 2C + C$$

$$2 = 4C$$

$$C = \frac{1}{2} \qquad A = \frac{1}{2} \qquad B = -1$$

$$\int \frac{x^2+1}{(x+1)^2(x-1)} = \frac{\frac{1}{2}}{(x+1)} + \frac{-1}{(x+1)^2} + \frac{\frac{1}{2}}{(x-1)}$$

$$= \int \frac{\frac{1}{2}}{(x+1)}dx - \int \frac{1}{(x+1)^2}dx + \frac{1}{2}\int \frac{1}{(x-1)}dx$$

$$= \frac{1}{2}\cdot \ln|x+1| - \left(-\frac{1}{x+1}\right) + \frac{1}{2}\ln|x-1|$$

$$= \frac{1}{2}\ln|x+1| + \frac{1}{x+1} + \frac{1}{2}\ln|x-1| + C$$

✓ 20

$$\int \frac{dx}{x+1} = \begin{bmatrix} x+1 = t \\ dx = dt \end{bmatrix} = \int \frac{dt}{t} = \ln|t| = \ln|x+1| + C$$

$$\int \frac{dx}{(x+1)^2} = \begin{bmatrix} x+1 = t \\ dx = dt \end{bmatrix} = \int \frac{dt}{t^2} = \int t^{-2}\,dt = \frac{t^{-1}}{-1} = -\frac{1}{t} = \frac{-1}{x+1} + C$$

$$\int \frac{dx}{x-1} = \begin{bmatrix} x-1 = t \\ dx = dt \end{bmatrix} = \int \frac{dt}{t} = \ln|t| = \ln|x-1| + C$$

1.)
$$\int_0^1 x\tan(x^2+1)\,dx = \begin{bmatrix} x^2+1 = t \\ 2x\,dx = dt \\ x\,dx = \frac{dt}{2} \end{bmatrix} = \int_0^1 \tan t\,\frac{dt}{2} = \frac{1}{2}\int_0^1 \tan t\,dt = \frac{1}{2}\cdot(-\ln|\cos t|)\Big|_0^1 = -\frac{1}{2}\ln\cos(x^2+1)\Big|_0^1 =$$

$$= -\frac{1}{2}\ln\cos(1^2+1) - \left(-\frac{1}{2}\ln\cos(0^2+1)\right) = -\frac{1}{2}\ln\cos 2 + \frac{1}{2}\ln\cos 1 + C = \emptyset$$

$\cos 2 = 0.999 \simeq 1$

$\cos 1 = 0.9998 \simeq 1$

$$= -\frac{1}{2}\ln + \frac{1}{2}\ln + C = 0 + C$$

REZULTAT NE MOŽE BITI DRUGO DO CI BROJ

KOD ODREĐENIH INTEGRALA NEMA NEODREĐENIH KONSTANTI – TO SU POVRŠINE

IME I PREZIME: Nikolina Burazin BROJ INDEKSA:

5) b) $y'' + y' + \frac{1}{4}y = 2$

$y'' + y' + \frac{1}{4}y - 2 = 0$

$\lambda^2 + \lambda + \frac{1}{4} \cdot 1 - 2 = 0$

$\lambda^2 + \lambda + \frac{1}{4} - 2 = 0 \quad / 4$

$4\lambda^2 + 4\lambda + 1 - 8 = 0$

$4\lambda^2 + 4\lambda - 7 = 0$

$\lambda_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 4 \cdot (-7)}}{2 \cdot 4}$

$\lambda_{1,2} = \frac{-4 \pm \sqrt{16 + 112}}{8}$

$\lambda_{1,2} = \frac{-4 \pm \sqrt{128}}{8}$

$\lambda_{1,2} = \frac{-4 \pm 4\sqrt{8}}{8}$ $\lambda_1 = \frac{-4 + 4\sqrt{8}}{8}$

$\lambda_2 = \frac{-4 - 4\sqrt{8}}{8}$

$y(x) = C_1 e^{\frac{-4+4\sqrt{8}}{8} x} + C_2 e^{\frac{-4-4\sqrt{8}}{8} \cdot x}$

$C_1, C_2 \in \mathbb{R}$

a) $y' = -\frac{y}{x}$

$\frac{dy}{dx} = \frac{-y}{x} \quad / \int$

$\int \frac{dy}{}$