Popuniti odmah! PISATI JEDNOSTRANO!

IME I PREZIME: RJEŠENJE 1 **BROJ INDEKSA:**

DATUM: 26.6.2012. VRIJEME: OD DO .

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓ bodova

1. Zadan je skup linearnih jednadžbi:

$$\begin{aligned} 2x + 3y - 3z - w &= -5 \\ 3z - 2y &= 5 \\ 2y - x - w &= -1 \\ w - 4x + 3z &= 9 \end{aligned}$$

   (a) zapisati dani sustav matrično, 2

   (b) riješiti matrični sustav Gaussovom metodom 10

   (c) provjeriti izračunato rješenje matričnim množenjem 3

2. Riješiti u kompleksnim brojevima sljedeće jednadžbe:

   (a) $z^3 + |3 - 4i| = \frac{5}{i}$. 10

   (b) $-z + |z - 4i| = \overline{3 + 4i}$. 10

3. Za funkciju $f(x) = x - \sqrt{x^2 - x}$:

   (a) odrediti asimptote i 10

   (b) odrediti prvu derivaciju 10

4. Za funkciju $g(x) = \frac{e^{2x}}{x^2}$:

   (a) uz pomoć L'Hopitalovog pravila odrediti: $\lim_{x \to +\infty} g(x)$ 10

   (b) uz pomoć zaključka iz (a) diskutirati konvergenciju reda: $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^2}$ 10

5. Zadana je funkcija: $h(x) = \frac{x^2 - 3}{x^2 + 3}$. Na temelju ispitivanja toka funkcije:

   (a) diskutirati da li je funkcija globalno ograničena ili ne, 5

   (b) navesti sve lokalne ekstreme, 5

   (c) navesti sve točke infleksije i 5

   (d) napraviti skicu grafa funkcije. 10

2.(b) $z = x + iy \Rightarrow -(x+iy) + |x+iy-4i| = 3-4i$

$-x - yi + \sqrt{x^2+(y-4)^2} = 3 - 4i$

za $x \geq 0$, $\sqrt{x^2} = x$

za $x < 0$, $\sqrt{x^2} = -x$

$\mathrm{Re}(x+iy-4i) = x$

$\mathrm{Im}(x+iy-4i) = y-4$

$|x+iy-4i| = \sqrt{x^2+(y-4)^2}$

$\Rightarrow$ Realni dio: $-x + \sqrt{x^2+(y-4)^2} = 3$

Imag. dio: $-y = -4$

$\Rightarrow y = 4$

$\Rightarrow -x + \sqrt{x^2+(4-4)^2} = 3$

$-x + \sqrt{x^2} = 3 \Rightarrow$ za $x \geq 0$: $-x + x = 3 \Rightarrow 0 = 3$ ↯

za $x \leq 0$: $-x - x = 3$

$-2x = 3$

$x = -\frac{3}{2}$

$\Rightarrow z = -\frac{3}{2} + 4i$

IME I PREZIME: ALEKSANDAR BEADER BROJ INDEKSA: 171-0027-2017

2. (a)

$$z^3 + |3-4i| = \frac{5}{i}$$

$$z^3 + \sqrt{3^2+4^2} = \frac{5}{i}$$

$$z^3 + \sqrt{9+16} = \frac{5}{i}$$

$$z^3 + 5 = \frac{5}{i}$$

$$z^3 = \frac{5}{i} - 5$$

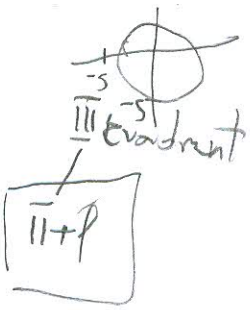
$$z^3 = \frac{5 - 5i}{i} \cdot \frac{-i}{-i}$$

$$z^3 = \frac{-5i + 5i^2}{1}$$

$$z^3 = -5 - 5i$$

x y

$$\tan\varphi = -\frac{5}{5} = 1$$



$$\varphi' = 0{,}79$$

$$\varphi = \Pi + \varphi'$$

$$\varphi = 3{,}14 + 0{,}79$$

$$\varphi = 3{,}93$$

$$r = \sqrt{x^2+y^2} = \sqrt{25+25} = \sqrt{50} = 7{,}07$$

$$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\varphi+2k\Pi}{n} + i\sin\frac{\varphi+2k\Pi}{n}\right)$$

$k = 0, 1, 2$

$\underline{k=0}$

$$z_1 = \sqrt[3]{7{,}07}\left(\cos\frac{3{,}93}{3} + i\sin\frac{3{,}93}{3}\right) = 1{,}92\cdot(\cos 1{,}31 + i\sin 1{,}31)$$

$$z_1 = 0{,}49 + 1{,}85i$$

$\underline{k=1}$

$$z_2 = 1{,}92\left(\cos\frac{3{,}93+2\Pi}{3} + i\sin\frac{3{,}93+2\Pi}{3}\right) = z_2 = -1{,}85 - 0{,}49i$$

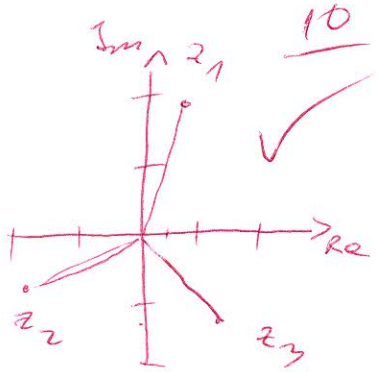
3,4 3,4

$\underline{k=2}$

$$z_3 = 1{,}92\left(\cos\frac{3{,}93+4\Pi}{3} + i\sin\frac{3{,}93+4\Pi}{3}\right)$$

16,5 16,5

$$z_3 = -1{,}35 - 1{,}36i$$



10

IME I PREZIME: Luka Stipić

BROJ INDEKSA:

$$\left[\begin{array}{cccc|c} 2 & 3 & -3 & -1 & -5 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array}\right] \sim \left[\begin{array}{cccc|c} -1 & 2 & 0 & -1 & -1 \\ 0 & -2 & 3 & 0 & 5 \\ 2 & 3 & -3 & -1 & -5 \\ -4 & 0 & 3 & 1 & 9 \end{array}\right] /\cdot(+1)$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 1 \\ 0 & -2 & 3 & 0 & 5 \\ 2 & 3 & -3 & -1 & -5 \\ -4 & 0 & 3 & 1 & 9 \end{array}\right] \begin{array}{l} \\ \\ /-1R\cdot 2 \\ /+1R\cdot 4 \end{array} \quad \left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 1 \\ 0 & -2 & 3 & 0 & 5 \\ 0 & 7 & -3 & -3 & -7 \\ 0 & -8 & 3 & 5 & 13 \end{array}\right] /\cdot(-\tfrac{1}{2})$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 1 \\ 0 & 1 & -3/2 & 0 & -5/2 \\ 0 & 7 & -3 & -3 & -7 \\ 0 & -8 & 3 & 5 & 13 \end{array}\right] \begin{array}{l} /+2R\cdot 2 \\ \\ /-2R\cdot 7 \\ /+2R\cdot 8 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -4 \\ 0 & 1 & -3/2 & 0 & -5/2 \\ 0 & 0 & 1/2 & -3 & 2\,1/2 \\ 0 & 0 & -9 & 5 & -7 \end{array}\right] /\cdot\tfrac{2}{15}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -4 \\ 0 & 1 & -3/2 & 0 & -5/2 \\ 0 & 0 & 1 & -3/5 & 7/5 \\ 0 & 0 & -9 & 5 & -7 \end{array}\right] \begin{array}{l} /+3R\cdot 3 \\ /+3R\cdot 3/2 \\ \\ /+3R\cdot 9 \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/5 & 1/5 \\ 0 & 1 & 0 & -3/5 & -2/5 \\ 0 & 0 & 1 & -2/5 & 7/5 \\ 0 & 0 & 0 & 7/5 & 28/5 \end{array}\right] /\cdot\tfrac{5}{7}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/5 & 1/5 \\ 0 & 1 & 0 & -3/5 & -2/5 \\ 0 & 0 & 1 & -2/5 & 7/5 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \begin{array}{l} /+4R\cdot\frac{1}{5} \\ /+4R\cdot 3/5 \\ /+4R\cdot 2/5 \\ \end{array} \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$2\cdot 1 + 3\cdot 2 - 3\cdot 3 - 4 = -5$

$3\cdot 3 - 2\cdot 2 = 5$

$2\cdot 2 - 1 - 4 = -1$

$4 - 4\cdot 1 + 3\cdot 3 = 9$

$$\left[\begin{array}{cccc} 2 & 3 & -3 & -1 \\ 0 & -2 & 3 & 0 \\ -1 & 2 & 0 & -1 \\ -4 & 0 & 3 & 1 \end{array}\right] \cdot \left[\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}\right] \checkmark$$

$$\left[\begin{array}{c} 2\cdot 1 + 3\cdot 2 + (-3)\cdot 3 + (-1)\cdot 4 \\ 0\cdot 1 + (-2)\cdot 2 + 3\cdot (3) + 0\cdot 4 \\ (-1)\cdot 1 + 2\cdot 2 + 0\cdot 3 + (-1)(4) \\ (-4)\cdot 1 + 0\cdot 2 + 3\cdot 3 + 1\cdot 4 \end{array}\right] \Rightarrow \left[\begin{array}{c} -5 \\ 5 \\ -1 \\ 9 \end{array}\right]$$

15

$f(x) = x - \sqrt{x^2 - x}$ $\quad \mathcal{D}(f) = \{x \in \mathbb{R} : x \in \langle -\infty, 0] \cup [1, +\infty \rangle$

$= \langle -\infty, 0] \cup [1, +\infty \rangle$

ESTIRAM V.A. $\lim_{x \to 0} f(x) = f(0) = 0$, $\lim_{x \to 1} f(x) = f(1) = 1$ $\Big\} \Rightarrow$ NEMA V.A.

ESTIRAM D.H.A. $\lim_{x \to +\infty} x - \sqrt{x^2 - x} = [+\infty - \infty] = \lim_{x \to +\infty} \frac{x - \sqrt{x^2 - x}}{1} \cdot \frac{x + \sqrt{x^2 - x}}{x + \sqrt{x^2 - x}} =$

$= \lim_{x \to +\infty} \frac{x^2 - (x^2 - x)}{x + \sqrt{x^2 - x}} = \lim_{x \to +\infty} \frac{x : x}{x + \sqrt{x^2 - x} : x} = \lim_{x \to +\infty} \frac{1}{1 + \sqrt{1 - \frac{1}{x}}} = \frac{1}{2}$

$\Rightarrow$ D.H.A. $y = \frac{1}{2}$ 5 badova

ESTIRAM L.H.A. $\lim_{x \to -\infty} x - \sqrt{x^2 - x} = -\infty - \infty = -\infty \Rightarrow$ NEMA L.H.A.

ESTIRAM L.K.A. $\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{x - \sqrt{x^2 - x}}{x} = \left\{ \begin{matrix} x \mapsto -x \\ -\infty \mapsto +\infty \end{matrix} \right\} =$

$= \lim_{x \to +\infty} \frac{-x - \sqrt{x^2 + x}}{-x} = \lim_{x \to +\infty} \frac{+x}{+x} + \frac{\sqrt{x^2 + x}}{x} = \lim_{x \to +\infty} 1 + \sqrt{1 + \frac{1}{x}} = 2$

$\Rightarrow \boxed{k = 2}$

$\lim_{x \to -\infty} f(x) - kx = \lim_{x \to -\infty} x - \sqrt{x^2 - x} - 2x = \lim_{x \to -\infty} -x - \sqrt{x^2 - x} = \left\{ \begin{matrix} x \mapsto -x \\ -\infty \mapsto +\infty \end{matrix} \right\}$

$= \lim_{x \to +\infty} x - \sqrt{x^2 + x} = [\infty - \infty] = \lim_{x \to +\infty} \frac{x - \sqrt{x^2 + x}}{1} \cdot \frac{x + \sqrt{x^2 + x}}{x + \sqrt{x^2 + x}} =$

$= \lim_{x \to +\infty} \frac{x^2 - (x^2 + x)}{x + \sqrt{x^2 + x}} = \lim_{x \to +\infty} \frac{-x : x}{x + \sqrt{x^2 + x} : x} = \lim_{x \to +\infty} \frac{-1}{1 + \sqrt{1 + \frac{1}{x}}} = -\frac{1}{2}$

$\Rightarrow l = -\frac{1}{2}$

$\Rightarrow$ L.K.A. $y = 2x - \frac{1}{2}$ 5 badova

3. (b) $\left(x - \sqrt{x^2 - x} \right)' = 1 - \left(x^2 - x \right)^{\frac{1}{2}\prime} = 1 - \frac{1}{2} \left(x^2 - x \right)^{-\frac{1}{2}} \cdot (2x - 1)$

$f'(x) = 1 - \frac{2x - 1}{2\sqrt{x^2 - x}}$

④ IME I PREZIME: JURE MIČIĆ BROJ INDEKSA: 17-1-0083-11

a) $g(x)=\frac{e^{2x}}{x^2}$

$$\lim_{x\to\infty}\frac{e^{2x}}{x^2}=\lim_{x\to\infty}\frac{(e^{2x})'}{(x^2)'}=\lim_{x\to\infty}\frac{e^{2x}\cdot 2}{2x}=\lim_{x\to\infty}\frac{e^{2x}}{x}=\frac{e^{2x}\cdot 2}{1}=2e^{2x}=\infty$$ ✓

b) $\sum_{n=1}^{\infty}\frac{e^{2n}}{n^2}$ NE KONVERGIRA JER $\lim_{x\to\infty} a_n$ NIJE JEDNAK NULI ✓

⑤ $h(x)=\frac{x^2-3}{x^2+3}$ $x\in\langle-\infty,+\infty\rangle$

a) FUNKCIJA JE OGRANIČENA ✓
GLOBALNI SUPREMUM MAX JOJ JE 1
A GLOBALNI MIN JE −1 ✓

b) $Y=-1$ JE LOKALNI MINIMUM ✓

$x^2-3=0$

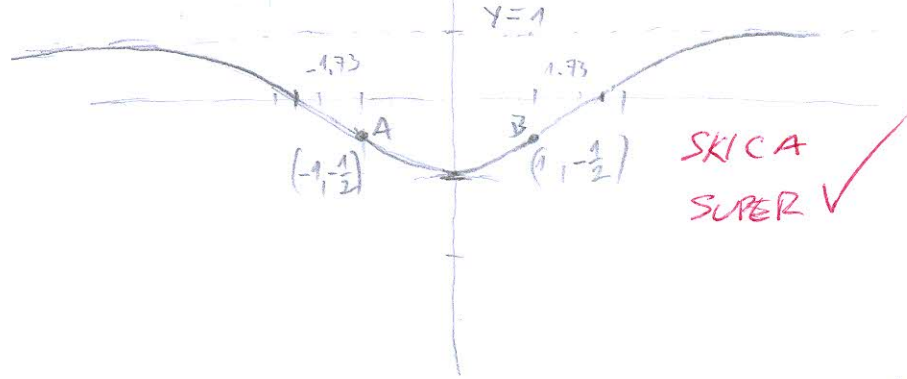
$x_1=\sqrt{3}=1.73$

$x_2=-\sqrt{3}=-1.73$

$\lim_{x\to+\infty}\frac{x^2-3}{x^2+3}=1$

$\lim_{x\to-\infty}\frac{x^2-3}{x^2+3}=1$

$Y=1$ – HORIZONTALNA ASIMTOTA ✓



SKICA SUPER ✓

$\lim_{x\to 0}\frac{x^2-3}{x^2+3}=1$

a) $h(x)\in[-1,1\rangle$

c) $A(-1,-\frac{1}{2})$, $B(1,-\frac{1}{2})$ TOČKE INFLEKSIJE ✓

$$h(x)'=\frac{2x\cdot(x^2+3)-(2x\cdot(x^2-3))}{(x^2+3)^2}$$

$$h(x)'=\frac{2x^3+6x-2x^3+6x}{(x^2+3)^2}$$

$$h(x)'=\frac{12x}{(x^2+3)^2}\qquad h(x)'=0 \text{ KAD JE } x=0$$

	$-\infty$		0		$+\infty$
$\frac{12x}{(x^2+3)^2}$		−	0	+	
		↘		↗	

	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
$-36\frac{x^2+1}{(x^2+3)^3}$	−	0	+	+	+	0	−
	↘		↗		↗		↘

$$h(x)''=\frac{12\cdot(x^2+3)^2-(12x\cdot 2(x^2+3)^1\cdot 2x)}{(x^2+3)^4}$$

$$h(x)''=\frac{12(x^2+3)^2-48x^2(x^2+3)}{(x^2+3)^4}$$

$$h(x)''=\frac{12x^2+36-48x^2}{(x^2+3)^3}=\frac{-36x^2+36}{(x^2+3)^3}$$