

Popunite odmah!

PISATI JEDNOSTRANO!

IME I PREZIME:

RJESENJE 1

BROJ INDEKSA:

DATUM: 26.6.2012. VRIJEME: OD DO

MATEMATIKA 1: Trajanje 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Broj ↓
bodova

1. Zadan je skup linearnih jednadžbi:

$$\begin{aligned} 2x + 3y - 3z - w &= -5 \\ 3z - 2y &= 5 \\ 2y - x - w &= -1 \\ w - 4x + 3z &= 9 \end{aligned}$$

- (a) zapisati dani sustav matrično, 2
 (b) riješiti matrični sustav Gaussovom metodom 10
 (c) provjeriti izračunato rješenje matričnim množenjem 3

2. Riješiti u kompleksnim brojevima sljedeće jednadžbe:

- (a) $z^3 + |3 - 4i| = \frac{5}{i}$. 10
 (b) $-z + |z - 4i| = 3 + 4i$. 10

3. Za funkciju $f(x) = x - \sqrt{x^2 - x}$:

- (a) odrediti asimptote i 10
 (b) odrediti prvu derivaciju 10

4. Za funkciju $g(x) = \frac{e^{2x}}{x^2}$:

- (a) uz pomoć L'Hopitalovog pravila odrediti: $\lim_{x \rightarrow +\infty} g(x)$ 10
 (b) uz pomoć zaključka iz (a) diskutirati konvergenciju reda: $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^2}$ 10

5. Zadana je funkcija: $h(x) = \frac{x^2 - 3}{x^2 + 3}$. Na temelju ispitivanja toka funkcije:

- (a) diskutirati da li je funkcija globalno ograničena ili ne, 5
 (b) navesti sve lokalne ekstreme, 5
 (c) navesti sve točke infleksije i 5
 (d) napraviti skicu grafa funkcije. 10

$$2.(b) \quad z = x + iy \Rightarrow -(x + iy) + |x + iy - 4i| = 3 - 4i$$

$$\left. \begin{aligned} \text{za } x > 0, \sqrt{x^2} &= x \\ \text{za } x < 0, \sqrt{x^2} &= -x \end{aligned} \right\}$$

$$\left. \begin{aligned} \operatorname{Re}(x + iy - 4i) &= x \\ \operatorname{Im}(x + iy - 4i) &= y - 4 \\ |x + iy - 4i| &= \sqrt{x^2 + (y - 4)^2} \end{aligned} \right\} \Rightarrow \begin{aligned} -x - yi + \sqrt{x^2 + (y - 4)^2} &= 3 - 4i \\ \text{Realni dio: } -x + \sqrt{x^2 + (y - 4)^2} &= 3 \\ \text{Imag. dio: } -y &= -4 \end{aligned}$$

$$\Rightarrow y = 4$$

$$\Rightarrow -x + \sqrt{x^2 + (4 - 4)^2} = 3$$

$$\begin{aligned} -x + \sqrt{x^2} = 3 &\Rightarrow \left. \begin{aligned} \text{za } x > 0 & -x + x = 3 \Rightarrow 0 = 3 \\ \text{za } x < 0 & -x - x = 3 \\ & -2x = 3 \\ & x = -\frac{3}{2} \end{aligned} \right\} \end{aligned}$$

$$\Rightarrow z = -\frac{3}{2} + 4i$$

$$z^3 + |3 - 4i| = \frac{5}{i}$$

$$z^3 + \sqrt{3^2 + 4^2} = \frac{5}{i}$$

$$z^3 + \sqrt{9 + 16} = \frac{5}{i}$$

$$z^3 + 5 = \frac{5}{i}$$

$$z^3 = \frac{5}{i} - 5$$

$$z^3 = \frac{5 - 5i}{i} \cdot \frac{-i}{-i}$$

$$z^3 = \frac{-5i + 5i^2}{1}$$

$$z^3 = -5 - 5i$$

\downarrow \downarrow
 x y

$$\frac{k=0}{z_1} = \sqrt[3]{7,07} \left(\cos \frac{3,93}{3} + i \sin \frac{3,93}{3} \right) = 1,92 \cdot \left(\cos 1,31 + i \sin 1,31 \right)$$

$$z_1 = 0,49 + 1,85i$$

$$\frac{k=1}{z_2} = 1,92 \left(\cos \frac{3,93 + 2\pi}{3} + i \sin \frac{3,93 + 2\pi}{3} \right) = z_2 = -1,85 - 0,49i$$

$$\frac{k=2}{z_3} = 1,92 \left(\cos \frac{3,93 + 4\pi}{3} + i \sin \frac{3,93 + 4\pi}{3} \right)$$

$$z_3 = -1,35 - 1,36i$$

$$\tan \rho = \frac{-5}{5} = -1$$

$$\rho' = 0,79$$

$$\rho = \pi + \rho'$$

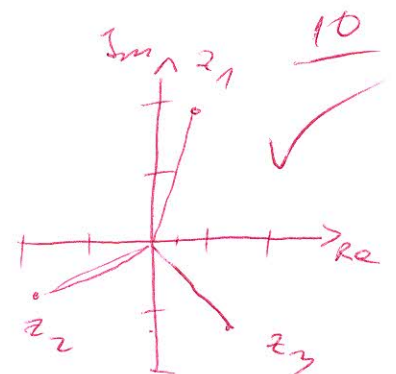
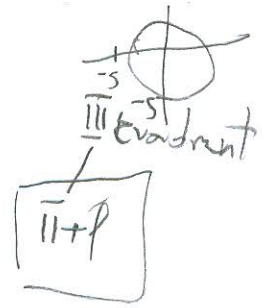
$$\rho = 3,14 + 0,79$$

$$\rho = 3,93$$

$$r = \sqrt{x^2 + y^2} = \sqrt{25 + 25} = \sqrt{50} = 7,07$$

$$\sqrt[n]{z} = \sqrt[n]{r} \left(\cos \frac{\rho + 2k\pi}{n} + i \sin \frac{\rho + 2k\pi}{n} \right)$$

$$k = 0, 1, 2$$



IME I PREZIME:

Luka STIPE

BROJ INDEKSA:

$$\left[\begin{array}{cccc|c} 2 & 3 & -3 & -1 & -5 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array} \right] \sim \left[\begin{array}{cccc|c} -12 & 0 & -1 & -1 & -1 \\ 0 & -2 & 3 & 0 & 5 \\ 2 & 3 & -3 & -1 & -5 \\ -4 & 0 & 3 & 1 & 9 \end{array} \right] \cdot (-1)$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 1 \\ 0 & -2 & 3 & 0 & 5 \\ 2 & 3 & -3 & -1 & -5 \\ -4 & 0 & 3 & 1 & 9 \end{array} \right] \begin{array}{l} | -1R \cdot 2 \\ | +1R \cdot 4 \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 1 \\ 0 & -2 & 3 & 0 & 5 \\ 0 & 7 & -3 & -3 & -7 \\ 0 & -8 & 3 & 5 & 13 \end{array} \right] \cdot (-\frac{1}{2})$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & 1 \\ 0 & 1 & -\frac{3}{2} & 0 & -\frac{5}{2} \\ 0 & 7 & -3 & -3 & -7 \\ 0 & -8 & 3 & 5 & 13 \end{array} \right] \begin{array}{l} | +2R \cdot 2 \\ | -2R \cdot 7 \\ | +2R \cdot 8 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -4 \\ 0 & 1 & -\frac{3}{2} & 0 & -\frac{5}{2} \\ 0 & 0 & \frac{1}{2} & -3 & \frac{21}{2} \\ 0 & 0 & -9 & 5 & -7 \end{array} \right] \cdot \frac{2}{15}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -4 \\ 0 & 1 & -\frac{3}{2} & 0 & -\frac{5}{2} \\ 0 & 0 & \frac{1}{2} & -3 & \frac{21}{2} \\ 0 & 0 & -9 & 5 & -7 \end{array} \right] \begin{array}{l} | +3R \cdot 3 \\ | +3R \cdot \frac{3}{2} \\ | +3R \cdot 9 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ 0 & 1 & 0 & -\frac{7}{5} & -\frac{2}{5} \\ 0 & 0 & 1 & -\frac{2}{5} & \frac{7}{5} \\ 0 & 0 & 0 & \frac{7}{5} & \frac{28}{5} \end{array} \right] \cdot \frac{5}{7}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} \\ 0 & 1 & 0 & -\frac{3}{5} & -\frac{2}{5} \\ 0 & 0 & 1 & -\frac{2}{5} & \frac{7}{5} \\ 0 & 0 & 0 & 1 & 4 \end{array} \right] \begin{array}{l} | +4R \cdot \frac{1}{5} \\ | +4R \cdot \frac{3}{5} \\ | +4R \cdot \frac{2}{5} \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{9}{5} \\ 0 & 1 & 0 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & 0 & \frac{3}{5} \\ 0 & 0 & 0 & 1 & 4 \end{array} \right]$$

$$2 \cdot 1 + 3 \cdot 2 - 3 \cdot 3 - 4 = -5$$

$$3 \cdot 3 - 2 \cdot 2 = 5$$

$$2 \cdot 2 - 1 - 4 = -1$$

$$4 - 4 \cdot 1 + 3 \cdot 3 = 9$$

$$\left[\begin{array}{cccc|c} 2 & 3 & -3 & -1 & -5 \\ 0 & -2 & 3 & 0 & 5 \\ -1 & 2 & 0 & -1 & -1 \\ -4 & 0 & 3 & 1 & 9 \end{array} \right] \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \checkmark$$

$$\left[\begin{array}{cccc|c} 2 \cdot 1 + 3 \cdot 2 + (-3) \cdot 3 + (-1) \cdot 4 & & & & \\ 0 \cdot 1 + (-2) \cdot 2 + 3 \cdot 3 + 0 \cdot 4 & & & & \\ (-1) \cdot 1 + 2 \cdot 2 + 0 \cdot 3 + (-1) \cdot 4 & & & & \\ (-4) \cdot 1 + 0 \cdot 2 + 3 \cdot 3 + 1 \cdot 4 & & & & \end{array} \right] \Rightarrow \begin{bmatrix} -5 \\ 5 \\ -1 \\ 9 \end{bmatrix} \checkmark$$

$$f(x) = x - \sqrt{x^2 - x}$$

$$D(f) = \{x \in \mathbb{R} \mid x \geq 0\}$$

$$= (-\infty, 0] \cup [1, +\infty)$$

$$x \in (-\infty, 0] \cup [1, +\infty)$$

STIRAM V.A. $\left. \begin{array}{l} \lim_{x \rightarrow 0} f(x) = f(0) = 0 \\ \lim_{x \rightarrow 1} f(x) = f(1) = 1 \end{array} \right\} \Rightarrow \text{NEMA V.A.}$

STIRAM D.H.A. $\lim_{x \rightarrow +\infty} x - \sqrt{x^2 - x} = [+\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{x - \sqrt{x^2 - x}}{1} \cdot \frac{x + \sqrt{x^2 - x}}{x + \sqrt{x^2 - x}} =$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 - (x^2 - x)}{x + \sqrt{x^2 - x}} = \lim_{x \rightarrow +\infty} \frac{x}{x + \sqrt{x^2 - x}} \cdot \frac{1}{x} = \lim_{x \rightarrow +\infty} \frac{1}{1 + \sqrt{1 - \frac{1}{x}}} = \frac{1}{2}$$

$$\Rightarrow \text{D.H.A. } y = \frac{1}{2} \quad \underline{\text{5 bodova}}$$

ESTIRAM L.H.A. $\lim_{x \rightarrow -\infty} x - \sqrt{x^2 - x} = -\infty - \infty = -\infty \Rightarrow \text{NEMA L.H.A.}$

ESTIRAM L.K.A. $\lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} \frac{x - \sqrt{x^2 - x}}{x} = \left\{ \begin{array}{l} x \rightarrow -x \\ -\infty \rightarrow +\infty \end{array} \right\} =$

$$= \lim_{x \rightarrow +\infty} \frac{-x - \sqrt{x^2 + x}}{-x} = \lim_{x \rightarrow +\infty} \frac{+x}{+x} + \frac{\sqrt{x^2 + x}}{x} = \lim_{x \rightarrow +\infty} 1 + \sqrt{1 + \frac{1}{x}} = 2 \Rightarrow \boxed{k=2}$$

$$\lim_{x \rightarrow -\infty} f(x) - kx = \lim_{x \rightarrow -\infty} x - \sqrt{x^2 - x} - 2x = \lim_{x \rightarrow -\infty} -x - \sqrt{x^2 - x} = \left\{ \begin{array}{l} x \rightarrow -x \\ -\infty \rightarrow +\infty \end{array} \right\}$$

$$= \lim_{x \rightarrow +\infty} x - \sqrt{x^2 + x} = [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{x - \sqrt{x^2 + x}}{1} \cdot \frac{x + \sqrt{x^2 + x}}{x + \sqrt{x^2 + x}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{x^2 - (x^2 + x)}{x + \sqrt{x^2 + x}} = \lim_{x \rightarrow +\infty} \frac{-x}{x + \sqrt{x^2 + x}} \cdot \frac{1}{x} = \lim_{x \rightarrow +\infty} \frac{-1}{1 + \sqrt{1 + \frac{1}{x}}} = -\frac{1}{2} \Rightarrow l = -\frac{1}{2}$$

$$\Rightarrow \text{L.K.A. } y = 2x - \frac{1}{2} \quad \underline{\text{5 bodova}}$$

$$3. (b) \left(x - \sqrt{x^2 - x} \right)' = 1 - (x^2 - x)^{\frac{1}{2}'} = 1 - \frac{1}{2} (x^2 - x)^{-\frac{1}{2}} \cdot (2x - 1)$$

$$f'(x) = 1 - \frac{2x - 1}{2\sqrt{x^2 - x}}$$

