MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: Marin Mavoš **Broj indeksa:** 57 651

Vrijeme: od 9:00 **do** 10:21 ♣5 **Broj bodova:** $\frac{10}{80} = 12.5\%$

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. ~~(15)~~ Integriraj

$$\int_0^1 x \tan(x^2+1)\,dx$$

2. ~~(20)~~ Integriraj

$$\int \frac{x^2+1}{(x+1)^2(x-1)}dx$$

3. (20) Odredi površinu koju zatvaraju krivulje $y = 1 - x^2$, $y = 3 + 2x - x^6$ i os apscisa. IZBAČENO!

4. (10+10)

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 + y^2 + xy - 3x - 6y$$

b) Odredi domenu funkcije:

$$f(x, y) = x - \sqrt{x+y}$$

5. ~~(10+15)~~ Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' = -\frac{y}{x}$$

b)

$$y'' + y' + \frac{1}{4}y = 2$$

VIDI RJEŠENJE 1

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME!

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1) $\int_0^1 x \tan(x^2+1)\,dx = \left| \begin{array}{l} x^2+1=t \\ 2x=dt \;\Big|\cdot\frac{1}{2} \\ x=\frac{dt}{2} \end{array} \right|$

$\int_0^1 \tan t \frac{dt}{2} = \frac{1}{2}\int_0^1 \tan t\, dt = \frac{1}{2}\left(-\ln|\cos t|+C\right)\Big|_0^1$

$= \frac{1}{2} - \ln|\cos x^2+1| + C \Big|_0^1$

POGREŠNO:

OVDJE NEDOSTAJE ZAGRADA. $\frac{1}{2}$ SE NE ODUZIMA VEĆ MNOŽI.

$= \frac{1}{2} - \ln|\cos 1^2+1| - \frac{1}{2} - \ln|\cos 0^2+1|$

I INAČE JE POGREŠNO UVRŠTENO!

$\varnothing$

$= -1,3861$

2) $\int \frac{x^2+1}{(x+1)^2\cdot(x-1)}\,dx = \frac{A}{(x+1)^2} + \frac{B}{(x+1)} + \frac{C}{(x-1)}$

$x^2+1 = A(x-1) + B(x+1)(x-1) + C(x+1)^2$

$x^2+1 = Ax - A + Bx^2 + \cancel{Bx} - \cancel{Bx} - B + C(x^2+2x+1)$

$x^2+1 = Ax - A + Bx^2 - B + Cx^2 + 2Cx + C$

$1 = B + C$

$0 = A + 2C$

$1 = -A - B + C$

$-C = B \qquad B = -C$

$-A = 2C$

$-A - 2C$

$1 = 2C - B + C$

$1 = 3C - B$

$0 = A + 2C$

$1 = -A + C + C$

X

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$0 = A + 2C$

$1 = -A + 2C$

$1 = 2C$

$2C = 1 \quad / \cdot \frac{1}{2}$

$\boxed{C = \frac{1}{2}}$

$\boxed{B = -\frac{1}{2}}$

$1 = -A + \frac{1}{2} + \frac{1}{2}$

$A = -1 + \frac{1}{2} + \frac{1}{2}$

$\boxed{A = 0}$

$\left| \begin{array}{l} x+1 = t \\ dx = dt \end{array} \right|$

$\int \frac{dt}{t^2} = t^{-2} dt$

$= t^{-1} dt$

$= \frac{1}{t} dt$

$= \frac{1}{x+1} dx$

$* \int \frac{dx}{(x+1)^2} - \frac{1}{2} \int \frac{dx}{x+1} + \frac{1}{2} \int \frac{dx}{x-1}$

$= \frac{1}{x+1} - \frac{1}{2} \ln|x+1| + \frac{1}{2} \ln|x-1|$

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4) $f(x,y) = x^2 + y^2 + xy - 3x - 6y$

$\frac{\partial f}{\partial x} = 2x + y - 3$

$\frac{\partial f}{\partial y} = 2y - x - 6$

$2x + y - 3 = 0$
$2y - x - 6 = 0$

$2x + y = 3$
$2y - x = 6 \,|\cdot 2$

$\cancel{2x} + y = 3$
$-\cancel{2x} + 4y = 12$

$5y = 15$
$y = 3$

$2x + 3 - 3 = 0$
$2x = 0 \,|\cdot \frac{1}{2}$
$x = 0$

$T(0, 3)$

A: $\frac{\partial f}{\partial^2 x} = 2 > 0$ min

B: $\frac{\partial f}{\partial x \partial y} = 1$

C: $\frac{\partial f}{\partial^2 y} = 2$

$f(min) = 0^2 + 3^2 + 0 \cdot 3 - 3 \cdot 0 - 6 \cdot 3$
$= -9$

$\Delta = A \cdot C - B^2$
$\Delta = 2 \cdot 2 - 1^2$
$\Delta = 4 - 1$
$\Delta = 3$ IMA!

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5. b) $y'' + y' + \frac{1}{4}y = 2$

$y = 1$

$y' = \lambda$

$y'' = \lambda^2$

$y'' + y' + \frac{1}{4}y = 0$

$\lambda^2 + \lambda + \frac{1}{4} = 0$

$\boxed{y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} +}$

$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\lambda_1 \neq \lambda_2$

$= \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot \frac{1}{4}}}{2}$

$y = A\cos x + B\sin x$ NE!

$y' = -A\sin x + B\cos x$

$= \frac{-1 \pm \sqrt{1-1}}{2}$

$y'' = -A\cos x - B\sin x$

$= \frac{-1 \pm 0}{2}$

$= -A\cos x - B\sin x - A\sin x + B\cos x + \frac{1}{4}(A\cos x + B\sin x)$

$\boxed{\lambda_1 = -\frac{1}{2}}$

$= -A\cos x - B\sin x - A\sin x + B\cos x + \frac{1}{4}A\cos x + \frac{1}{4}B\sin x$

$\boxed{\lambda_2 = \frac{1}{2}}$

$1 = -A + B + \frac{1}{4}A$

$0 = -B - A + \frac{1}{4}B$

0

$1 = -\frac{3}{4}A + B$

$0 = -A - \frac{3}{4}B \quad / \cdot \left(-\frac{3}{4}\right)$

$1 = -\cancel{\frac{3}{4}A} + B$

$0 = \cancel{\frac{3}{4}A} + \frac{9}{16}B$

$1 = \frac{9}{16}B$

$0 = \frac{3}{4}A + \frac{9}{16} \cdot \frac{16}{9}$

$\frac{9}{16}B = 1 \quad / \cdot \frac{16}{9}$

$0 = \frac{3}{4}A + 1$

$\boxed{B = \frac{16}{9}}$

$\frac{7}{4}A = 0$

$\boxed{A = 0}$