MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE JEDNOSTRANO!**

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: Tomislav Jurić

BROJ INDEKSA:

1. Izračunati volumen područja između plašta stošca $x^2 + y^2 = z^2$ i plašta paraboloida $x^2 + y^2 = 5z$. 5
2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} 3x + z^{77} \\ y^2 - \sin(x^2 z) \\ xz + ye^{x^5} \end{pmatrix}$ i ∂K rub kvadra $K = \{(x, y, z) : 0 \le x \le 1, 0 \le y \le 3, 0 \le z \le 2\}$ koji je orijentiran vanjskom normalom. 19
3. Riješiti $x'''(t) + 3x'(t) = t, \quad , x'(0) = x''(0) = 0, x(0) = 1.$ 20
4. Izračunati krivuljni integral skalarnog polja $f(x, y, z) = x + z$ po luku krivulje C zadane sa $x = 2t$, $y = t^2$ i $z = \frac{1}{3}t^3$ ako je $0 \le t \le 10$. ~~0~~
5. Zadan je X krug radijusa 3 oko točke $T(1, 0)$ i $f(x, y) = xy$. Izračunati $\iint_X f$. ~~0~~

Ukupno: 44

(4)

$x = 2t$
$y = t^2$
$z = \frac{1}{3}t^3$

$t \in [0, 20\pi]$

$r'(t) = \begin{bmatrix} 2 \\ 2t \\ t^2 \end{bmatrix}$

$$\|r(t)\| = \sqrt{2^2 + 2t^2 + (t^2)^2}$$
$$= \sqrt{4 + 4t^2 + t^4}$$
$$= \sqrt{4 + t^2(2t + t^2)} \quad \text{X}$$
$$= 2 + t\sqrt{2t + t^2} \quad \text{X}$$

~~0~~

X

$$L = \int_0^{20\pi} 2 + t\sqrt{2t + t^2}\, dt = \frac{1}{2}\int \sqrt{u}\, du = \frac{1}{2}\int u^{\frac{1}{2}}\, du = \frac{1}{2}\frac{2}{3}u^{\frac{3}{2}}$$

X X

$2t + t^2 = u$
$2 + 2t\, dt = du$
$2 + t\, dt = \frac{1}{2}du$

$$= \frac{1}{3}\sqrt{u^3}$$
$$= \frac{1}{3}(2 + t)\sqrt{(2t + t)^3}\Big|_0^{20\pi}$$
$$= \frac{1}{3}(2 + 20\pi)\sqrt{(40\pi + 20\pi)^3}$$
$$= \frac{2}{3} + \frac{20}{3}\pi\sqrt{(60\pi)^3}$$

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$X'(0) = X''(0) = 0 \qquad X(0) = 1$

$\ldots + 3\left(sX(s) - X(0)\right) = \frac{1}{s^2}$

$\ldots(0) - \cancel{X''(0)} + 3\left(sX(s) \ldots\right)$

$\ldots\left(sX(s) - 1\right) = \frac{1}{s^2}$

$\ldots sX(s) - 3 = \frac{1}{s^2}$

$\ldots s) = \frac{1}{s^2} + s^2 + 3$

$\ldots + 3s) = \frac{1 + s^4 + 3s^2}{s^2} \quad \Big| \cdot \frac{1}{s^3 + 3s}$

$s_{1,2,3} = 0$

$s^2 \neq -3$

$\ldots \frac{s^4 + 3s^2 + 1}{s^3(s^2+3)}$

$\ldots \frac{\ldots + 1}{\ldots + 3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds + E}{s^2 + 3} \quad \Big| \, s^3(s^2+3)$

$\ldots 3s^2 + 1 = As^2(s^2+3) + Bs(s^2+3) + C(s^2+3) + (Ds+E)s^3$

$1 = 3C \qquad \boxed{C = \frac{1}{3}}$

$s_{1,2,3} = 0$

$\ldots + 3s^2 + 1 = As^4 + 3As^2 + Bs^3 + 3Bs + Cs^2 + 3C + Ds^4 + Es^3$

$D = -A + 1 = -\frac{8}{9} + 1 = \frac{1}{9} \qquad \boxed{D = \frac{1}{9}}$

$3A = -\frac{1}{3} + 3 = \frac{8}{3} \cdot \frac{1}{3}$

$A = \frac{8}{9}$

$\ldots s^4 \Rightarrow \quad 1 = A + D$

$\ldots s^3 \Rightarrow \quad 0 = B + E \qquad \boxed{E = 0}$

$\ldots s^2 \Rightarrow \quad 3 = 3A + C \qquad 3A = -C + 3 \Rightarrow$

$\ldots s \Rightarrow \quad 0 = 3B \qquad \boxed{B = 0}$

$X(s) = \frac{8}{9} \cdot \frac{1}{s} + \frac{1}{3} \cdot \frac{1}{s^3} + \frac{1}{9} \frac{s}{s^2+3}$

$X(t) = \frac{8}{9} + \frac{1}{3} \cdot \frac{t^2}{2} + \frac{1}{9} \cos\sqrt{3}t = \frac{8}{9} + \frac{1}{6}t^2 + \frac{1}{9}\cos\sqrt{3}t$

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Broj indeksa:

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$$W = \begin{bmatrix} 3x + z^{77} \\ y^2 - \sin(x^2 z) \\ xz + y e^{x^5} \end{bmatrix}$$

$x \in [0,1]$

$y \in [0,3]$

$z \in [0,2]$

$$\operatorname{div} W = \frac{\partial 3x+z^{77}}{\partial x} + \frac{\partial y^2 - \sin(x^2 z)}{\partial y} + \frac{xz + ye^{x^5}}{\partial z}$$

$$\operatorname{div} W = 3 + 2y + x$$

$$\int_0^1\int_0^3\int_0^2 3+2y+x \, dz\, dy\, dx = \int_0^1\int_0^3 3z\Big|_0^2 + 2yz\Big|_0^2 + xz\Big|_0^2 \, dy\, dx$$

$$\int_0^1\int_0^3 (6+4y+2x)\, dy\, dx = \int_0^1 (6y + 2y^2 + 2xy)\Big|_0^3 dx = \int_0^1 (18 + 18 + 6x)\, dx$$

$$\int_0^1 (\cancel{38}\,36 + 6x)\, dx = \cancel{38}\,36\, x + 3x^2\Big|_0^1 = \cancel{38}\,36 + 3 = \cancel{41}\,39$$

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$$x^2+y^2=z^2$$

$$x^2+y^2=5z \Rightarrow r^2=5z \qquad r=\sqrt{5z} \qquad r=\sqrt{5z}$$

$$z=\frac{r^2}{5}$$

$$r^2=z^2$$

$$r=\pm z$$

$$z=\pm r$$

$$r\in[z,\sqrt{5z}]$$

$$z\in[0,5]$$

$$\varphi\in[0,2\pi]$$

SJECIŠTE

$$z=\frac{r^2}{5}$$

$$z^2=r^2$$

$$5z=z^2$$

$$z_1=0 \quad z_2=5$$

$$V=\int_0^{2\pi}\int_0^5\int_z^{\sqrt{5z}} r\,dr\,dz\,d\varphi$$

$$V=\int_0^{2\pi}\int_0^5 \frac{r^2}{2}\left(\sqrt{5z}-z\right)dz\,d\varphi=\int_0^{2\pi}\int_0^5\left(\frac{5z}{2}-\frac{z^2}{2}\right)dz\,d\varphi \qquad 5$$

$$V=\int_0^{2\pi}\frac{5}{4}z^2-\frac{z^3}{6}\Big|_0^5\,d\varphi=\int_0^{2\pi}\left(\frac{125}{4}-\frac{125}{6}\right)d\varphi=\int_0^{2\pi}\left(\frac{375-250}{12}\right)d\varphi$$

$$\int_0^{2\pi}\frac{125}{12}\,d\varphi=\frac{125}{12}\varphi\Big|_0^{2\pi}=\frac{125}{6}\pi$$

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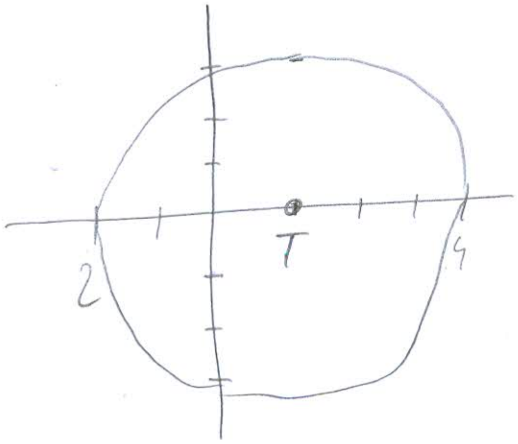
$r = 3$

$T(1,0)$

$\iint xy$



$x-1 = r\cos\varphi$

$x = r\cos\varphi + 1$

$y = r\sin\varphi$

$r \in [0,3]$

0

$$\int_0^{2\pi}\int_0^{3} r(r\cos\varphi+1)\,dr\,d\varphi + \int_0^{2\pi}\int_0^{3} r(r\sin\varphi)\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^{3} (r^2\cos\varphi + r)\,dr\,d\varphi + \int_0^{2\pi}\int_0^{3} (r^2\sin\varphi)\,dr\,d\varphi$$

$$\int_0^{2\pi} \frac{r^3}{3}\cos\varphi + \frac{r^2}{2}\Big|_0^3 d\varphi + \int_0^{2\pi} \frac{r^3}{3}\sin\varphi\Big|_0^3 d\varphi$$

$$\int_0^{2\pi} \left(9\cos\varphi + \frac{9}{2}\right) d\varphi + \int_0^{2\pi} (9\sin\varphi)\,d\varphi$$

$$9\sin\varphi + \frac{9}{2}\varphi\Big|_0^{2\pi} - 9\cos\varphi\Big|_0^{2\pi} = \cancel{9\sin 2\pi} + 9\pi - 9\cancel{\cos 2\pi} \cancel{- 9\sin 0} \cancel{+ 9\cos 0}$$

$$= 9\pi$$