

MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: LUKA BRONIC' Broj indeksa: 57826-2009

Vrijeme: od 08:15 do 9:55 ♣5 Broj bodova:  $\frac{5}{80} = 6.25\%$

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. ~~(15)~~ Integriraj

~~Integriraj~~  $\int_0^1 x \tan(x^2+1) dx$

2. ~~(20)~~ Integriraj

$$\int \frac{x^2+1}{(x+1)^2(x-1)} dx$$

3. ~~(20)~~ Odredi površinu koju zatvaraju krivulje  $y = 1 - x^2$ ,  $y = 3 + 2x - x^6$  i os apscisa. **IZBAČENO!**

4. ~~(10+10)~~

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 + y^2 + xy - 3x - 6y$$

b) Odredi domenu funkcije:

$$f(x, y) = x - \sqrt{x+y}$$

5. ~~(10+15)~~ Riješi sljedeće diferencijalne jednadžbe:

a)  ~~$y' = -\frac{y}{x}$~~

b)

$$y'' + y' + \frac{1}{4}y = 2$$

VIDI RJEŠENJE 1

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPIŠATI IME I PREZIME!

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BROJ INDEKSA:

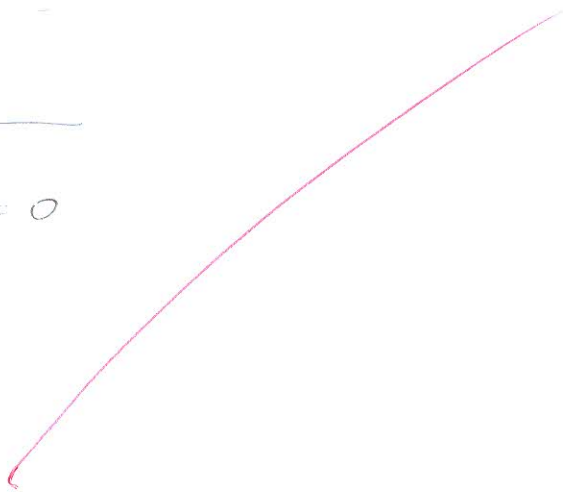
$$3. \quad y = 1 - x^2$$

$$y = 3 + 2x - x^6$$

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$$1 - x^2 - 3 + 2x + x^6 = 0$$

$$x^6 - 2x - 2 = 0$$



$$1. \int_0^1 x \tan(x^2+1) dx = \left. \begin{array}{l} x^2+1 = t \\ 2x dx = dt \\ x dx = \frac{dt}{2} \end{array} \right| \begin{array}{c|c|c} x & 0 & 1 \\ \hline t & 1 & 2 \end{array}$$

$$\int_1^2 \tan t \frac{dt}{2} = \frac{1}{2} \int_1^2 \tan t dt = \frac{1}{2} \left( -\ln |\cos x| \right) \Big|_1^2 =$$

$$= \frac{1}{2} - \ln |\cos 2| - \ln |\cos 1|$$

$$= -\frac{1}{2} \ln |\cos 1|$$

NJE DOBRO UVRSTENO U  
ODREĐENI INTEGRAL



$$2. \int \frac{x^2+1}{(x+1)^2(x-1)} dx = \frac{A}{x+1} + \frac{Bx+C}{(x+1)^2} + \frac{D}{x-1} \quad \text{K1A2}$$

$$x^2+1 = (Ax+A)(x-1) + (Bx+C)(x-1) + D(x+1)^2$$

$$x^2+1 = Ax^2 - Ax + Ax + A + Bx^2 - Bx + Cx - C + D(x^2 + 2x + 1)$$

$$x^2+1 = \underline{Ax^2} + \underline{A} + \underline{Bx^2} - \underline{Bx} + \underline{Cx} - \underline{C} + \underline{Dx^2} + \underline{2Dx} + \underline{D}$$

$$(A+B+D) = 1$$

$$A = 1 - B - C$$

$$(-B+C+2D) = 0$$

$$D = 1 - A + C$$

$$(A - C + D) = 1$$

$$B = 2D - C$$



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BROJ INDEKSA:

$$4. a) f(x, y) = x^2 + y^2 + xy - 3x - 6y$$

$$\frac{\partial f}{\partial x} = 2x + y - 3$$

$$\frac{\partial f}{\partial y} = 2y + x - 6$$

$$\frac{\partial^2 f}{\partial x^2} = 2$$

$$\frac{\partial^2 f}{\partial y^2} = 2$$

$$\frac{\partial^2 f}{\partial x \partial y} = 1$$

$$\Delta = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3 > 0$$

$$\Delta = 3 > 0$$

$$\frac{\partial^2 f}{\partial x^2} = 2 > 0$$

MINIMUM FUNKCIJE

$$2y + x - 6 = 0 \quad / \cdot \frac{1}{2}$$

$$y + \frac{1}{2}x - 3 = 0$$

$$y = -3 - \frac{1}{2}x$$

$$2 - 3 - \frac{1}{2}x + y - 6 = 0$$

$$-1 - \frac{1}{2}2x - 6 = 0$$

$$-1 - x - 6 = 0$$

$$-x - 7 = 0$$

$$x = -7$$

$$y = -3 - \frac{1}{2}(-7)$$

$$y = -3 + \frac{7}{2}$$

$$y = \frac{-6+7}{2} = \frac{1}{2}$$

$$T(-7, \frac{1}{2}) \quad \times$$



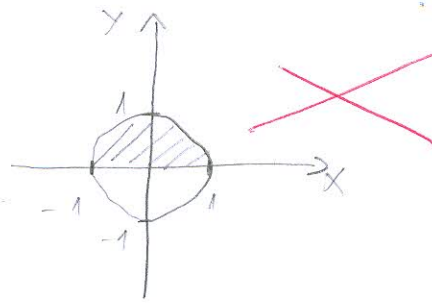
4. IME I PREZIME: LUKA BROJIC

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b)  $f(x,y) = x - \sqrt{x+y}$

$$x+y \geq 0$$

$$D_f = \mathbb{R} \left\{ \begin{array}{l} \{x < 1, -1\} \\ \{y < 1, -1\} \end{array} \right\} \times$$



5. a)  $y' = -\frac{y}{x} / :$

$$\frac{dy}{dx} = -\frac{y}{x} / : \frac{dx}{x}$$

$$dy \cdot x = -y dx / : \frac{y}{y}$$

$$\frac{dy}{y} = -\frac{dx}{x} / \int$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\ln|y| = -\ln|x| + \ln|C|$$

$$\ln|y| = \ln|x^{-1}| + \ln|C|$$

$$\ln|y| = \ln\left|\frac{1}{x}\right| + \ln|C| \checkmark$$

$$\ln|y| = \ln\left|\frac{C}{x}\right|$$

$$y = \frac{C}{x} \quad \underline{5}$$

$$y' = \frac{C(x) \cdot x + C(x)'}{x^2}$$

$$y' = \frac{C(x) \cdot x + C(x)'}{x^2}$$

$$2^o \frac{C(x) \cdot x + C(x)'}{x^2} = -\frac{C(x)}{x}$$

$$\frac{C(x) \cdot x + C(x)'}{x^2} = -\frac{C(x) \cdot x}{x^2}$$

$$\frac{C(x)'}{x^2} = x$$

$$\frac{C(x)' \cdot x + C(x)'}{x^2} = -\frac{C(x)}{x^2}$$

$$\frac{C(x)' \cdot x + C(x)'}{x^2} = \frac{C(x) \cdot x^2}{C(x) \cdot x}$$

$$C(x)' \cdot x = C$$

NETOČNO

RJEŠENJE TOČNO 1

DALJE NESTO POGREŠNO

=> POLA BODOVA

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BROJ INDEKSA:

$$5. b) y'' + y' + \frac{1}{4}y = 2$$

$$y'' + y' + \frac{1}{4}y = 0$$

$$r^2 + r + \frac{1}{4} = 0$$

$$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot \frac{1}{4}}}{2} = \frac{-1 \pm \sqrt{1-1}}{2} = \frac{-1 \pm 0}{2}$$

$$r_1 = \frac{-1-0}{2} = -\frac{1}{2}$$

$$r_2 = \frac{-1+0}{2} = -\frac{1}{2}$$

$$y = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

$$y = C_1 e^{-x/2} + C_2 e^{-x/2}$$

$$y = C_1 e^{-x/2} + C_2 e^{-x/2} \quad \text{⊘}$$

KOJE JE PJEŠENJE  
NEHOMOGENE JEDNAČBE: