MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE JEDNOSTRANO!**

IME I PREZIME: MARINO PREHODA

BROJ INDEKSA: 57659

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati volumen područja između plašta stošca $x^2 + y^2 = z^2$ i plašta paraboloida $x^2 + y^2 = 5z$. ∅

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} 3x + z^{77} \\ y^2 - \sin(x^2 z) \\ xz + ye^{x^5} \end{pmatrix}$ i ∂K rub kvadra $K = \{(x, y, z) : 0 \leq x \leq 1, 0 \leq y \leq 3, 0 \leq z \leq 2\}$ koji je orijentiran vanjskom normalom.

3. Riješiti $x'''(t) + 3x'(t) = t, \quad , x'(0) = x''(0) = 0, x(0) = 1.$ ∅

4. Izračunati krivuljni integral skalarnog polja $f(x, y, z) = x + z$ po luku krivulje C zadane sa $x = 2t$, $y = t^2$ i $z = \frac{1}{3}t^3$ ako je $0 \leq t \leq 10$.

5. Zadan je X krug radijusa 3 oko točke $T(1, 0)$ i $f(x, y) = xy$. Izračunati $\iint_X f$. 15

Ukupno: 15

5. $r = 3$ $T(1,0)$ $f(x,y) = xy$

$r \in [0, 3]$ $\quad x = r\cos\varphi + 1$ ✓

$\varphi \in [0, 2\pi]$ $\quad y = r\sin\varphi$ ✓ ✓

15

$$\int_0^{2\pi} d\varphi \int_0^3 (r\cos\varphi + 1)(r\sin\varphi)\, r\, dr = \int_0^{2\pi} d\varphi \int_0^3 (r^3 \cos\varphi \sin\varphi + r^2 \sin\varphi)\, dr =$$

$$= \int_0^{2\pi} d\varphi \int_0^3 r^3 \cos\varphi \sin\varphi\, dr + \int_0^{2\pi} d\varphi \int_0^3 r^2 \sin\varphi\, dr = \int_0^{2\pi} d\varphi \cos\varphi \sin\varphi \left(\frac{r^4}{4}\right)\Big|_0^3 + \int_0^{2\pi} d\varphi \sin\varphi \left(\frac{r^3}{3}\right)\Big|_0^3$$

$$= \frac{81}{4} \int_0^{2\pi} \cos\varphi \sin\varphi\, d\varphi + 9 \int_0^{2\pi} \sin\varphi\, d\varphi = \frac{81}{9} \int_0^{2\pi} \cos\varphi \sin\varphi\, d\varphi - 9\cos\varphi \Big|_0^{2\pi}$$

$$\left| \begin{array}{ll} \cos\varphi = u & du = dv \\ \sin\varphi = v & dv = du \end{array} \right| = 1$$

VIDI RJEŠENJE 1

IME I PREZIME: MARINO PRENDŽA BROJ INDEKSA: 57659

3. $x'''(t) + 3x'(t) = t \qquad x'(0) = x''(0) = 0 \quad x(0) = 1$

$$s^3X(s) - s^2x(0) - sx'(0) - x''(0) + 3[sX(s) - x(0)] = \frac{1}{s^2}$$

$$s^3X(s) - s^2 + 3sX(s) - 3 = \frac{1}{s^2}$$

$$X(s)(s^3 + 3s) = \frac{1}{s^2} + s^2 + 3$$

$$X(s)(s^3+3s) = \frac{1 + s^4 + 3s^2}{s^2} \;/\; (s^3+3s)$$

$$X(s) = \frac{s^4+3s^2+1}{s^2(s^3+3s)}$$

$$X(s) = \frac{s^4+3s^2+1}{s^2(s^2+3)s}$$

$$\frac{s^4+3s^2+1}{s^2(s^2+3)s} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs+D}{s^2+3} + \frac{E}{s}$$

$$s^4+3s^2+1 = A(s^3+3s) + Bs(s^3+3s) + (Cs+D)s^3 + E(s^2(s^2+3))$$

$$s^4+3s^2+1 = As^3 + 3As + Bs^4 + 3Bs^2 + Cs^4 + Ds^3 + Es^4 + 3Es^2$$

$$s^4+3s^2+1 = s^4(B+C+E) + s^3(A+D) + s^2(3B+3E) + 3As$$

$B + C + E = 1$

$A + D = 0 \rightarrow \boxed{D = 0}$

$3B + 3E = 3 \longrightarrow -3B - 3E = 3 \;/\; (-3)$

$-3A = 0 \;/\; (-3)$

$\boxed{A = 0}$

$B + E = -1$

$B = -1 - E$

$-1 - E + C + E = 1$

$\boxed{C = 2}$

$$X(s) = \frac{2s}{s^2+3} = 2 \cdot \frac{s}{s^2+3}$$

$$X(t) = 2 \cdot \frac{s}{s^2+3} = 2\cos(3t)$$

IME I PREZIME: MARINO PREHDŽA BROJ INDEKSA: 57659

1. $x^2+y^2=z^2$ $x^2+y^2=5z$ $r \in [z, \sqrt{5}z]$

$r^2=z^2/\sqrt{\ }$ $r^2=5z/\sqrt{\ }$ $\varphi \in [0, 2\pi]$

$r=z$ $r=\sqrt{5}z$ $z \in [$