

MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: Antonio-Đorđić Galešić Broj indeksa: 17-1-0018-2010

Vrijeme: od 8:30 do 10:18 ♣5

Broj bodova: $\frac{20}{80} = 25\%$

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. ~~(15)~~ Integriraj

~~Integriraj~~ $\int_0^1 x \tan(x^2+1) dx$

2. ~~(20)~~ Integriraj

$$\int \frac{x^2+1}{(x+1)^2(x-1)} dx$$

3. (20) Odredi površinu koju zatvaraju krivulje $y = 1 - x^2$, $y = 3 + 2x - x^6$ i os apscisa. **125 BACENO**

4. (10+10)

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 + y^2 + xy - 3x - 6y$$

b) Odredi domenu funkcije:

$$f(x, y) = x - \sqrt{x+y}$$

5. ~~(15+15)~~ Riješi sljedeće diferencijalne jednačbe:

a) ~~ny~~ $y' = -\frac{y}{x}$

b)

$$y'' + y' + \frac{1}{4}y = 2$$

VIDI RJEŠENJE 7.

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NA PISATI IME I PREZIME!

$$\begin{aligned}
 1) \int_0^1 x \tan(x^2+1) dx &= \begin{cases} x^2+1 = t \\ 2x dx = dt \\ x dx = \frac{dt}{2} \end{cases} \\
 &= \int_0^1 \tan t \frac{dt}{2} \\
 &= \frac{1}{2} \int_0^1 \tan t dt \\
 &= \frac{1}{2} (-\ln|\cos x|) \Big|_0^1 \\
 &= \frac{1}{2} (-\ln|\cos(1-0)|) \\
 &= \frac{1}{2} (-\ln|\cos 1|) \\
 &= -\frac{1}{2} \ln|\cos 1|
 \end{aligned}$$

POGREŠKA U GRANICAMA!

$$\begin{aligned}
 &= -\frac{1}{2} \ln(1) \\
 &= -\frac{1}{2} \ln(1) \\
 &= 0^+
 \end{aligned}$$

$$2) \int \frac{x^2+1}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{(x+1)^2}$$

$$= \int \frac{\frac{1}{2}}{x-1} dx + \int \frac{\frac{1}{2}x - \frac{1}{2}}{(x+1)^2} dx$$

$$= \frac{1}{2} \int \frac{1}{x-1} dx + \int \frac{\frac{1}{2}x}{(x+1)^2} dx - \int \frac{\frac{1}{2}}{(x+1)^2} dx$$

$$= \left(\frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| \right) + \frac{1}{-2(x+1)} + \frac{13}{24} + C$$

IME I PREZIME: Antonio-Borđić Galešić

BROJ INDEKSA:

$$\frac{x^2+1}{(x+1)^2(x-1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x+1)^2} \quad / \cdot (x-1)(x+1)^2$$

$$x^2+1 = A(x+1)^2 + (Bx+C)(x-1)$$

$$x^2+1 = A(x^2+2x+1) + (Bx+C)(x-1)$$

$$x^2+1 = Ax^2 + 2Ax + A + Bx^2 - Bx + Cx - C$$

$$x^2+1 = x^2(A+B) + x(2A-B+C) + (A-C)$$

$$A+B=1 \rightarrow A=1-B \quad 2(1-B)-B+C=0$$

$$2A-B+C=0 \quad 2-2B-B+C=0$$

$$A-C=1 \quad (1-B)-(3B-2)=1 \quad 2-3B+C=0$$

$$1-B-3B+2=1 \quad C=3B-2$$

$$-4B=1-2-1$$

$$-4B=-2 \quad /: (-4)$$

$$\underline{\underline{B = \frac{1}{2}}}$$

$$A + \frac{1}{2} = 1$$

$$A = 1 - \frac{1}{2}$$

$$\underline{\underline{A = \frac{1}{2}}}$$

$$\frac{1}{2} - C = 1$$

$$-C = 1 - \frac{1}{2}$$

$$-C = \frac{1}{2} \quad / \cdot (-1)$$

$$\underline{\underline{C = -\frac{1}{2}}}$$

$$1^{\circ} \int \frac{1}{2} \frac{dx}{x-1} = \frac{1}{2} \left(\frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| \right) + C$$

$$= \frac{1}{4} \ln \left| \frac{x+1}{x-1} \right| + C$$

$$2^{\circ} \int \frac{\frac{1}{2}}{(x+1)^2} dx = \frac{1}{2} \int \frac{1}{(x+1)^2} dx = \left[\begin{array}{l} x+1=t \\ dx=dt \end{array} \right]$$

$$= \frac{1}{2} \int \frac{dt}{t^2} = \frac{1}{2} \int t^{-2} dt = -C$$

$$= \frac{1}{2} \cdot \frac{t^{-1}}{-1} = \frac{1}{2} \frac{t^{-1}}{-2}$$

$$= \frac{(x+1)^{-1}}{-2}$$

$$3^{\circ} \int \frac{\frac{1}{2}x}{(x+1)^2} = \frac{1}{2} \int \frac{x}{(x+1)^2} = \left[\begin{array}{l} u=x, du=dx \\ dv=(x+1)^2, v=\frac{(x+1)^3}{3} = \frac{x^3+3x^2+3x+1}{3} \end{array} \right]$$

$$\neq \frac{1}{2} \int u dv = u dv - \int v du$$

$$= x \cdot \left(\frac{x^3}{3} + \frac{x^2}{2} + x \right) - \int \left(\frac{x^3}{3} + \frac{x^2}{2} + x \right) dx$$

$$= \frac{x^4}{3} + \frac{x^3}{2} + x^2 - \left(\frac{x^4}{4} + \frac{x^3}{6} + \frac{x^2}{2} \right)$$

$$= \frac{x^4}{3} + \frac{x^3}{2} + x^2 - \frac{x^4}{4} - \frac{x^3}{6} + \frac{x^2}{2}$$

$$= \frac{1}{2} + \frac{1}{12} + \frac{1}{2} = \frac{6+1+6}{12} = \frac{13}{12} \cdot \frac{1}{2} = \frac{13}{24}$$

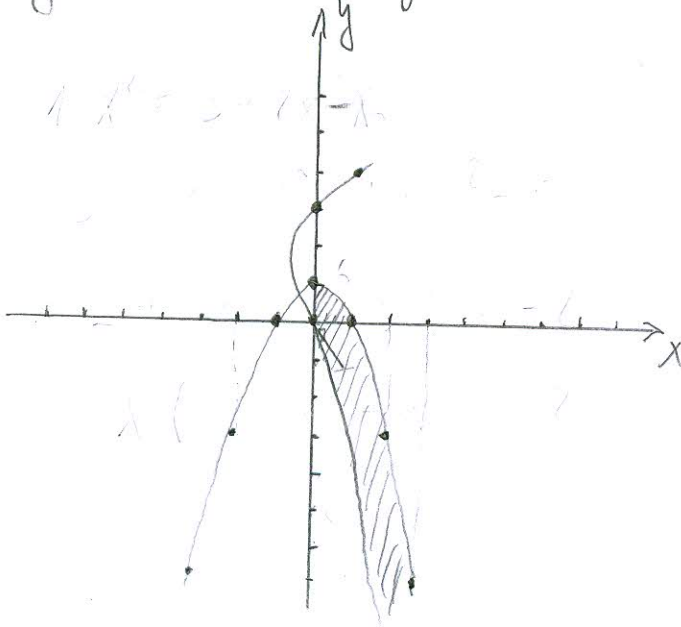
$$\frac{1}{3} \int \frac{x^4}{4} \quad \frac{1}{2} \int \frac{x^3}{3} \quad \frac{x^2}{2}$$

$$\frac{x^3}{2} - \frac{x^2}{6} = \frac{3x^3 - x^2}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{x^4}{3} - \frac{x^4}{4} = \frac{4x^4 - 3x^4}{12} = \frac{1}{12}$$

3) $y = 1 - x^2$

$y = 3 + 2x - x^2$



$y = 1 - x^2$

x	y
0	1
1	0
2	-3
3	-7
-1	0
-2	-3

$y = 3 + 2x - x^2$

x	y
0	3
1	4
-1	0

DVO MEMA VEZE

$f(x) = 4$

$\int_0^1 (1 - x^2 - (3 + 2x - x^2)) dx = \int_0^1 (1 - x^2 - 3 - 2x + x^2) dx$

$= \int_0^1 (-2 - x - x^2) dx$

$= -2 \int_0^1 dx - \int_0^1 x dx - \int_0^1 x^2 dx$

$= -2x \Big|_0^1 - \frac{x^2}{2} \Big|_0^1 - \frac{x^3}{3} \Big|_0^1$

$= -2(1-0) - \frac{1}{2}(1^2-0) - \frac{1}{3}(1^3-0)$

$= -2 - \frac{1}{2} - \frac{1}{3} = \frac{-12 - 3 - 2}{6} = -\frac{17}{6}$

4) a) $f(x, y) = x^2 + y^2 + xy - 3x - 6y$

$\frac{\partial f}{\partial x} = 2x + y - 3$ \ STAC TOČK.

$\frac{\partial f}{\partial y} = 2y + x - 6$

$2x + y - 3 = 0$

$y = 3 - 2x$

$2(3 - 2x) + x - 6 = 0$

$6 - 4x + x - 6 = 0$

$4x + x = 0$

$5x = 0 \quad /: 5$

$x = 0$

$2y + x - 6 = 0$

$2y + 0 - 6 = 0$

$2y = 6 \quad /: 2$

$y = 3$

$T(0, 3)$ ✓

$\frac{\partial^2 f}{\partial x^2} = 2$

$\frac{\partial^2 f}{\partial y^2} = 2$

$\frac{\partial^2 f}{\partial x \partial y} = 1$

$D = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3 > 0$ ima ekstrem

10

$\frac{\partial^2 f}{\partial x^2} = 2 > 0$

točka $T(0, 3)$ je minimum ✓

funkcije $f(x, y) = x^2 + y^2 + xy - 3x - 6y$

