MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: Antonio-Đorđe Galešić **Broj indeksa:** 17-1-0018-2010

Vrijeme: od 8:50 **do** 10:18 ♣5 **Broj bodova:** $\frac{20}{80} = 25\%$

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. ~~(15)~~ Integriraj

$$\int_0^1 x \tan(x^2+1)\,dx$$

2. ~~(20)~~ Integriraj

$$\int \frac{x^2+1}{(x+1)^2(x-1)}dx$$

3. (20) Odredi površinu koju zatvaraju krivulje $y = 1 - x^2$, $y = 3 + 2x - x^6$ i os apscisa. IZBAČENO

4. (10+10)

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 + y^2 + xy - 3x - 6y$$

b) Odredi domenu funkcije:

$$f(x, y) = x - \sqrt{x+y}$$

5. ~~(10+15)~~ Riješi sljedeće diferencijalne jednadžbe:

a)

$$y' = -\frac{y}{x}$$

b)

$$y'' + y' + \frac{1}{4}y = 2$$

VIDI RJEŠENJE 1.

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME!

IME I PREZIME: Antonio-Djordi Galešić BROJ INDEKSA: 17-1-0018-2010

1) $\int_0^1 x\tan(x^2+1)\,dx = \left[\begin{array}{l} x^2+1=t \,/' \\ 2x\,dx = dt \\ x\,dx = \frac{dt}{2} \end{array}\right.$

$= \int_0^1 \tan t \frac{dt}{2}$

$= \frac{1}{2}\int_0^1 \tan t \, dt$

$= \frac{1}{2}\left(-\ln|\cos x| \Big|_0^1\right)$

$= \frac{1}{2}\left(-\ln|\cos(1-0)|\right)$

$= \frac{1}{2}\left(-\ln|\cos 1|\right)$

$= -\frac{1}{2}\ln|\cos 1|$

$\rightarrow = -\frac{1}{2}\ln(1)$

$= -\frac{1}{2}\cdot 0$

$= 0^+$

POGREŠKA U GRANICAMA!

$\emptyset$

2) $\int \frac{x^2+1}{(x+1)^2(x-1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)^2}$

$= \int \frac{\frac{1}{2}}{x-1}\,dx + \int \frac{\frac{1}{2}x - \frac{1}{2}}{(x+1)^2}\,dx$

$= \frac{1}{2}\underbrace{\int \frac{1}{x-1}}_{1^\circ}dx + \underbrace{\int \frac{\frac{1}{2}x}{(x+1)^2}dx}_{2^\circ} - \underbrace{\int \frac{\frac{1}{2}}{(x+1)^2}dx}_{2^\circ}$

$= \left(\frac{1}{4}\ln\left|\frac{x+1}{x-1}\right|\right) + \frac{1}{-2(x+1)} + \frac{13}{24} + C$

$\emptyset$

IME I PREZIME: Antonio-Dvorchi Galešić BROJ INDEKSA:

$$\frac{x^2+1}{(x+1)^2(x-1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x+1)^2} \quad / \cdot (x-1)(x+1)^2$$

$$x^2+1 = A(x+1)^2 + (Bx+C)(x-1)$$

$$x^2+1 = A(x^2+2x+1) + (Bx+C)(x-1)$$

$$x^2+1 = Ax^2 + 2Ax + A + Bx^2 - Bx + Cx - C$$

$$x^2+1 = x^2(A+B) + x(2A-B+C) + (A-C)$$

$$A+B=1 \rightarrow A = 1-B$$

$$2A-B+C=0$$

$$A-C=1$$

$$2(1-B)-B+C=0$$

$$2-2B-B+C=0$$

$$2-3B+C=0$$

$$C=3B-2$$

$$(1-B)-(3B-2)=1$$

$$1-B-3B+2=1$$

$$-4B=1-2-1$$

$$-4B=-2 \quad /:(-4)$$

$$\underline{\underline{B=\frac{1}{2}}}$$

$$A+\frac{1}{2}=1$$

$$A=1-\frac{1}{2}$$

$$\underline{\underline{A=\frac{1}{2}}}$$

$$\frac{1}{2}-C=1$$

$$-C=1-\frac{1}{2}$$

$$-C=\frac{1}{2} \quad /\cdot(-1)$$

$$\underline{\underline{C=-\frac{1}{2}}}$$

IME I PREZIME: Antonio-Đorđi Galešić BROJ INDEKSA:

$$1^\circ \quad \frac{1}{2}\int \frac{dx}{x-1} = \frac{1}{2}\left(\frac{1}{2}\ln\left|\frac{x+1}{x-1}\right|\right) + C$$

$$= \frac{1}{4}\ln\left|\frac{x+1}{x-1}\right| + C$$

$$2^\circ \quad \int \frac{\frac{1}{2}}{(x+1)^2}dx = \frac{1}{2}\int \frac{1}{(x+1)^2}dx = \left[\begin{array}{l} x+1=t \\ dx = dt \end{array}\right.$$

$$= \frac{1}{2}\int \frac{dt}{t^2} = \frac{1}{2}\int t^{-2}\,dt \quad C$$

$$= \frac{1}{2}\cdot\frac{t^{-1}}{-1} = \frac{t^{-1}}{-2}$$

$$= \frac{(x+1)^{-1}}{-2}$$

$$3^\circ \quad \int \frac{\frac{1}{2}x}{(x+1)^2} = \frac{1}{2}\int \frac{x}{(x+1)^2} = \left[\begin{array}{l} u = x, \quad du = dx \\ dv = (x+1)^2 \quad v = \frac{(x+1)^3}{3} = \frac{x^3+3x^2+3x+1}{3} \end{array}\right.$$

$$\frac{1}{2}\int u\,dv = uv - \int v\,du$$

$$\int (x^2+2x+1)dx = \int x^2dx + 2\int x\,dx + \int dx = \frac{x^3}{3} + \frac{x^2}{2} + x$$

$$= x\cdot\left(\frac{x^3}{3} + \frac{x^2}{2} + x\right) - \int\left(\frac{x^3}{3} + \frac{x^2}{2} + x\right)dx$$

$$= \frac{x^4}{3} + \frac{x^3}{2} + x^2 - \left(\frac{x^4}{4} + \frac{x^3}{6} + \frac{x^2}{2}\right) =$$

$$= \frac{x^4}{3} + \frac{x^3}{2} + x^2 - \frac{x^4}{4} - \frac{x^3}{6} + \frac{x^2}{2}$$

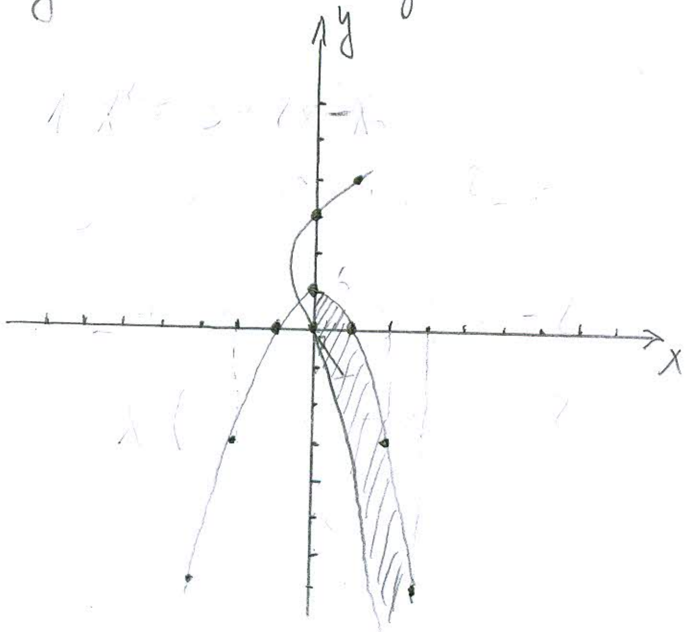
$$= \frac{1}{2} + \frac{1}{12} + \frac{1}{2} = \frac{6+1+6}{12} = \frac{13}{12}\cdot\frac{1}{2} = \frac{13}{24}$$

$$\frac{1}{3}\int \frac{x^4}{4} \qquad \frac{1}{2}\,\frac{x^3}{3} \qquad \frac{x^2}{2}$$

$$\frac{x^3}{2} - \frac{x^3}{6} = \frac{3x^3 - x^3}{6} = \frac{3}{6} = \frac{1}{2} \qquad \frac{x^4}{3} - \frac{x^4}{4} = \frac{4x^4 - 3x^4}{12} = \frac{1}{12}$$

Ime i prezime: Antonio-Djordji Galešić Broj indeksa:

3) $y = 1 - x^2$ $y = 3 + 2x - x^6$



$y = 1 - x^2$

x	y
0	1
1	0
2	-3
3	-7
-1	0
-2	-3

$y = 3 + 2x - x^6$

x	y
0	3
1	4
-1	0

OVO NEMA VEZE

$f(x) = 4$

$$\int_0^1 (1 - x^2 - (3 + 2x - x^6))\,dx = \int_0^1 (1 - x^2 - 3 - 2x + x^6)\,dx$$

$$= \int_0^1 (-2 - x - x^2)\,dx$$

$$= -2\int_0^1 dx - \int_0^1 x\,dx - \int_0^1 x^2\,dx$$

$$= -2x \Big|_0^1 - \frac{x^2}{2}\Big|_0^1 - \frac{x^3}{3}\Big|_0^1$$

$$= -2(1-0) - \frac{1}{2}(1^2 - 0) - \frac{1}{3}(1^3 - 0)$$

$$= -2 - \frac{1}{2} - \frac{1}{3} = \frac{-12 - 3 - 2}{6} = -\frac{17}{6}$$

IME I PREZIME: Antonio-Đorđi Galešić BROJ INDEKSA:

4) a) $f(x,y) = x^2 + y^2 + xy - 3x - 6y$

$$\frac{\partial f}{\partial x} = 2x + y - 3$$

STAC TOČK.

$$\frac{\partial f}{\partial y} = 2y + x - 6$$

$2x + y - 3 = 0$

$y = 3 - 2x$

$2(3-2x) + x - 6 = 0$

$6 - 4x + x - 6 = 0$

$4x + x = 0$

$5x = 0 \,/ : 5$

$x = 0$

$2y + x - 6 = 0$

$2y + 0 - 6 = 0$

$2y = 6 \,/ : 2$

$y = 3$

$T(0,3)$ ✓

$$\frac{\partial^2 f}{\partial x^2} = 2$$

$$\frac{\partial^2 f}{\partial y^2} = 2$$

$$\frac{\partial^2 f}{\partial x \partial y} = 1$$

10

$$D = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3 > 0$$ ima ekstrem

$$\frac{\partial^2 f}{\partial x} = 2 > 0$$ točka $T(0,3)$ je minimum ✓

funkcije $f(x,y) = x^2 + y^2 + xy - 3x - 6y$

Ime i prezime: Antonio-Djordi Galetić   Broj indeksa:

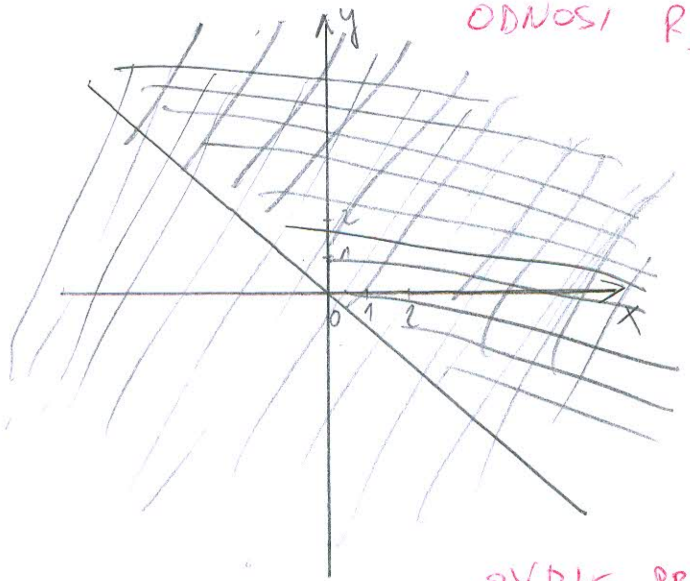
4) b) $f(x,y) = x - \sqrt{x+y}$

$x = \mathbb{R}$

$x + y \geq 0$

$y \geq -x$

TREBA JASNIJE OZNAČITI NA KOJI DIO SE ODNOSI RJEŠENJE



OVDJE PRIZNATO

10

5) a) $y' = -\frac{y}{x}$

$y' \overset{+}{-} \frac{y}{x} = 0$

$f(x) = \frac{1}{x}$

$g(x) = 0$

$y(x) = e^{-\int f(x)} \left[C + \int \underbrace{g(x)}_{=0} e^{\int f(x)} dx \right]$

$y(x) = e^{-\ln|x|} \left[C + \int \underbrace{0 \cdot e^{\ln|x|}}_{=0} dx \right]$

$y(x) = x^{-1} \left[C + \frac{x^2}{2} \right]$

$-\int \frac{1}{x} \cancel{dx} - \ln|x| + C$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int 0 dx = C$

PREVIŠE GREŠAKA! ∅

$\int e^{\ln|x|} dx = \int x dx = \frac{x^2}{2}$

5) IME I PREZIME: Antonio-Đuorđi Galešić

BROJ INDEKSA:

$\lambda^2 + 1 + \frac{1}{4} = 0$

b) $p^2 + 4q = D$

$1^2 - 4q = 0$

$\boxed{D = 0}$

$\lambda_{1,2} = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot \frac{1}{4}}}{2}$

$\lambda_{1,2} = \frac{-1}{2}$

Ø

$y(x) = C_1 e^{\lambda_1 x} + C_2 x e^{\lambda_1 x}$

$y(x) = C_1 e^{-\frac{1}{2}x} + C_2 (-\frac{1}{2}) e^{-\frac{1}{2}x} + 0$

ZADANA JE NEHOMOGENA

ZODJ 2. REDA.