MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: Ivan Mandičić **Broj indeksa:** 56458- 2008

Vrijeme: od 08 : 15 **do** ________ ♣5 **Broj bodova:** Ø

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. ~~*(15)*~~ Integriraj

[redacted] $$\int_0^1 x \tan(x^2+1)\,dx$$

2. ~~*(20)*~~ Integriraj

$$\int \frac{x^2+1}{(x+1)^2(x-1)}dx$$

3. *(20)* Odredi površinu koju zatvaraju krivulje $y = 1 - x^2$, $y = 3 + 2x - x^6$ i os apscisa. IZBAČENO

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 + y^2 + xy - 3x - 6y$$

b) Odredi domenu funkcije:

$$f(x, y) = x - \sqrt{x+y}$$

5. *(10+15)* Riješi sljedeće diferencijalne jednadžbe:

a) [redacted] $y' = -\dfrac{y}{\text{[redacted]}}$

b)

$$y'' + y' + \frac{1}{4}y = 2$$

VIDI RJEŠENJE 1

PISATI JEDNOSTRANO !

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME!

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(2)

$$\int \frac{x^2+1}{(x+1)^2(x-1)}\,dx = \int \frac{x(x+1)-x+1}{(x+1)^2(x-1)} = \int \frac{x(x+1)}{(x+1)} - \int \frac{x}{(x+1)(x-1)}$$

$$= \int x\,dx - \int \frac{x}{(x+1)(x-1)} = \frac{x^2}{2} - \int \frac{x-1+1}{(x+1)(x-1)} = \frac{x^2}{2} - \left(\int \frac{x-1}{x-1}\right.$$

$$\left. + \int \frac{1}{x+1}\,dx\right) = \frac{x^2}{2} - \left(x + \int \frac{dx}{x+1}\right) = \frac{x^2}{2} - \left(x + \int \frac{dx}{x+1}\right) \left\{ \begin{array}{l} x+1=t \;/' \\ dx=dt \end{array} \right.$$

$$= \frac{x^2}{2} - \left(x + \int \frac{dt}{t}\right) = \frac{x^2}{2} - \left(x + \ln|x+1|\right) = \frac{x^2}{2} - x - \ln|x+1| + C$$

~~0~~ V

(1)

$$\int_0^1 x\tan(x^2+1)\,dx \left\{ \begin{array}{l} x^2+1=t \;/' \\ 2x\,dx = dt \\ x\,dx = \frac{dt}{2} \end{array} \right\} =$$

$$= \int_0^1 \frac{\tan(t)\,dt}{2} = \frac{1}{2}\int_0^1 \tan t\,dt =$$

$$= \frac{1}{2}\Big|_0^1 -\ln|\cos t| + C = \frac{1}{2}\Big|_0^1 -\ln|\cos(x^2+1)| + C$$

$$= \frac{1}{2}\left(-\ln|\cos(1^2+1-0^2+1)| + C = \frac{1}{2}\left(-\ln|\cos(1)|\right) + C\right.$$

$$= -\frac{\ln|\cos(1)|}{2} + C$$

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ZBOG OVOG JE NULA BODOVA.

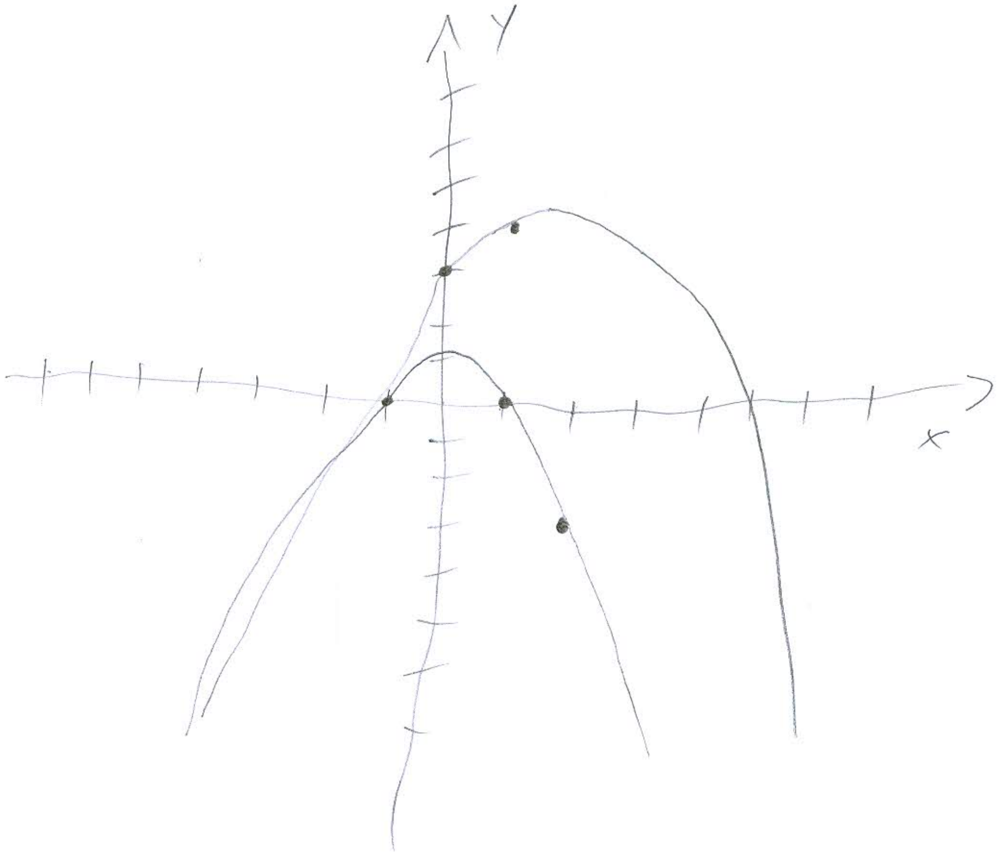
ODREĐENI INTEGRAL JE MJERA POVRŠINE (BROJ) A TU NEMA MJESTA NEODREĐENOJ KONSTANTI!

IME I PREZIME: Ivan Mandičić

BROJ INDEKSA: 56458-2008

(3) $y = 1 - x^2$

$y = 3 + 2x - x^6$



$x = 1 \quad y = 0$

$x = 2 \quad y = -3$

$x = -1 \quad y = 0$

$x = -1 \quad y = 0$

$x = 0 \quad y = 3$

$x = 1 \quad y = 4$

$1 - x^2 = 0$

$-x^2 = -1$

$x^2 = 1$

$x_1 = \sqrt{1}$

$x_2 = -\sqrt{1}$

$3 + 2x - x^6 = 0$

$-x^6 + 2x = -3 \; / \cdot (-1)$

$x^6 - 2x = 3$

$x(x^5 - 2) = 3$

$\cancel{x = 3}$

$\cancel{x^5 - 2 = 3}$

$\cancel{x^5 = 5}$

$\cancel{x = \sqrt[5]{5}}$

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3 ← OVA GRANICA NEMA VEZE!

(3) $$\int_{-1}^{1} 1-x^2\,dx + \int_{-1}^{3} 3+2x-x^6\,dx =$$

$$= -\int_{-1}^{1} x^2\,dx + \int_{-1}^{1} dx + \int_{-1}^{3} 3\,dx + \int_{-1}^{3} 2x\,dx - \int_{-1}^{3} x^6\,dx$$

$$= -\Big|_{-1}^{1} \frac{x^3}{3} + \Big|_{-1}^{1} x + 3\Big|_{-1}^{3} x + 2\Big|_{-1}^{3} \frac{x^2}{2} - \Big|_{-1}^{3} \frac{x^7}{7}$$

$$= -\left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{3}}\right) + \cancel{1} - \cancel{1} + 3\left(3-1\right) + 2\left(\frac{9}{2} - \frac{1}{2}\right) - \left(\frac{3^7}{7} - \frac{(-1)^7}{7}\right) = 6 + 8 - \left(\frac{3^7}{7} - \frac{-1}{7}\right) = 14 - \frac{3^7}{7} - \frac{1}{7} =$$

$$= \frac{98 - 2187 - 1}{7} = -\frac{2090}{7} = 298{,}57$$