MATEMATIKA 2
14. lipnja 2012.

Ime i prezime: ANTONIO VUJATOVIĆ **Broj indeksa:** 17-1-0011-2010

Vrijeme: od 8:30 **do** 10:20 ♣4 **Broj bodova:** ~~25~~ 20

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(15)* Integriraj

$$\int \frac{1+\sin(3x)}{\cos^2(3x)}dx$$

~~15~~ 10

2. *(~~20~~)* Integriraj

$$\int_{-1}^{1} \frac{x}{(x+2)(x^2+1)}dx$$

3. *(~~20~~ 15)* Odredi površinu koju zatvaraju krivulja $y^2 = 2x + 1$ i pravac $y = x + 1$.

4. *(10+10)* 0

a) Ispitaj ekstreme funkcije

$$f(x,y) = xy + 4x^2 - 3y^2$$

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x^2 + y^2)$$

5. *(~~20+15~~)* 2 Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' - 5y = x^4$$

b)

$$y'' + 6y' + 9y = 2\cos x$$

VIDI RJEŠENJE 2

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME!

IME I PREZIME: ANTONIO VUJATOVIĆ BROJ INDEKSA:

4

b) $f(x,y) = \ln(x^2+y^2)$

$x^2+y^2 > 0$

$D: f = R$

a) $f(x,y) = xy + 4x^2 - 3y^2$

$\frac{df}{dx} = y + 8x$

$\frac{df}{dy} = x - 6y$

$A = \frac{d^2f}{dx^2} = 8$

$B = \frac{d^2f}{dy^2} = -6$

$C = \frac{d^2f}{dxdy} = 1$

$\Delta = \begin{vmatrix} 8 & 1 \\ 1 & -6 \end{vmatrix} = -48 - 1 = -49 < 0$

nema ekstrem

10

1. $y^2 = 2x+1$

$y = \pm\sqrt{2x+1}$

$y = x+1$

0

Ime i prezime: Antonio Vujatović **Broj indeksa:**

2.

$$\int_{-1}^{1} \frac{x}{(x+2)(x^2+1)}dx = \frac{A}{x+2} + \frac{Bx+C}{x^2+1} \Big/ \cdot naz$$

$$x = A(x^2+1) + (Bx+c)(x+2)$$

$$x = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$$

$0 = A + B \qquad B = -A$

$1 = 2B + C \Rightarrow 1 = 2B + \frac{1}{5}$

$0 = A + 2C \qquad \frac{4}{5} = 2B$

$$\boxed{B = \frac{2}{5}}$$

$$\boxed{A = -\frac{2}{5}}$$

$1 = 2\cdot(-A) + C$

$0 = A + 2C$

$1 = -2A + C$

$0 = A + 2C \ / \cdot 2$

$1 = -2A + C$

$0 = 2A + 4C$

$1 = 5C$

$$\boxed{C = \frac{1}{5}}$$

$$= \int_{-1}^{1} \frac{-\frac{2}{5}dx}{x+2} + \frac{\frac{2}{5}x + \frac{1}{5}}{(x^2+1)}dx$$

$$= -\frac{2}{5}\int_{-1}^{1}\frac{dx}{x+2} + \int_{-1}^{1}\frac{\frac{2}{5}x}{(x^2+1)}dx + \int_{-1}^{1}\frac{\frac{1}{5}}{x^2+1}dx$$

$$= -\frac{2}{5}\int_{-1}^{1}\frac{dx}{x+2} + \frac{2}{5}\int_{-1}^{1}\frac{x\,dx}{(x^2+1)} + \frac{1}{5}\int\frac{dx}{x^2+1}$$

Ime i prezime: ANTONIO VUJATOVIĆ Broj indeksa:

$$\frac{x\,dx}{x^2+1} = \left| \begin{array}{l} x^2+1=t \\ 2x\,dx=dt \,/:2 \\ x\,dx=\frac{1}{2}dt \end{array} \right|$$

$$= -\frac{2}{5}\int_{-1}^{1}\frac{dx}{x+2} + \frac{2}{10}\int_{-1}^{1}\frac{dt}{t} + \frac{1}{5}\int\frac{dx}{x^2+1}$$

$$= -\frac{2}{5}\ln|x+2|\Big|_{-1}^{1} + \frac{1}{5}\ln|x^2+1|\Big|_{-1}^{1} + \frac{1}{5}\arctan 1 + C$$

$$= -\frac{2}{5}\ln|1+2| + \frac{2}{5}\ln|-1+2| + \frac{1}{5}\ln|1^2+1| \overline{+}\frac{1}{5}\ln|(-1)^2+1| + \frac{1}{5}\arctan 1 + \ldots \overline{+}\frac{1}{5}\arctan(-1)$$

$$= \left(-\frac{2}{5}\cdot 1{,}098 + \frac{2}{5}\cdot 0\right) + \left(\frac{1}{5}\cdot 0{,}693\right) + \frac{1}{5}\cdot 0{,}693 + 9 + 9 + C$$

$= 17,83$

~~15~~ 10 Mat

Kosor

=

5.

a) $xy' - 5y = x^4$

$$xy' - 5y = 0$$

$$x\frac{dy}{dx} = 5y \;/\cdot dx$$

$$x\,dy = 5y\,dx \;/\cdot\frac{1}{xy}$$

$$\frac{dy}{y} = \frac{5dx}{x}$$

$$\int\frac{dy}{y} = \int\frac{5\,dx}{x}$$

$$\ln|y| = 5\ln|x| + \ln|c|$$

$$y = C\cdot x^5$$

0

OVO JE RJEŠENJE HOMOGENE ODJ.
KOJE JE RJEŠENJE POLAZNE ODJ?