MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE JEDNOSTRANO!**

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POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Izračunati volumen područja između plašta stošca $x^2+y^2=z^2$ i plašta paraboloida $x^2+y^2=5z$. 20
2. Izračunati $\iint\limits_{\partial K} \mathbf{F}\cdot d\mathbf{S}$ gdje je $\mathbf{F}=\begin{pmatrix} 3x+z^{77} \\ y^2-\sin(x^2z) \\ xz+ye^{x^5}\end{pmatrix}$ i ∂K rub kvadra $K=\{(x,y,z)\,:\,0\le x\le 1,\,0\le y\le 3,\,0\le z\le 2\}$ koji je orijentiran vanjskom normalom. $\varnothing$
3. Riješiti $x'''(t)+3x'(t)=t, \quad ,x'(0)=x''(0)=0,\, x(0)=1$. $\varnothing$
4. Izračunati krivuljni integral skalarnog polja $f(x,y,z)=x+z$ po luku krivulje C zadane sa $x=2t$, $y=t^2$ i $z=\frac{1}{3}t^3$ ako je $0\le t\le 10$. $\varnothing$
5. Zadan je X krug radijusa 3 oko točke $T(1,0)$ i $f(x,y)=xy$. Izračunati $\iint\limits_X f$. 15

Ukupno: 35

(5) $f(x,y)=xy$ $\quad r=3 \quad T(1,0)$

$r\in[0,3]$

$\varphi\in[0,2\pi]$

$x=r\cos\varphi$

$y=r\sin\varphi$

$x_1=r\cos\varphi+1$

$y_1=r\sin\varphi$

$$\iint xy\, r\, dr\, d\varphi=$$

$$\int_0^3\int_0^{2\pi}(r\cos\varphi+1)\,r\sin\varphi\, r\, dr\, d\varphi=$$ ✓

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$$\int_0^3\int_0^{2\pi} r^2(r\cos\varphi\sin\varphi+\sin\varphi)\,dr\,d\varphi=$$ ✓

$$\left|\begin{matrix} u=r\cos\varphi & dv=\sin\varphi \\ du=-\sin\varphi & v=-\cos\varphi\end{matrix}\right|$$

VIDI RJEŠENJE!

$$u\cdot v-\int v\,du = r\cos\varphi\cdot(-\cos\varphi)-\int\cos\varphi\sin\varphi$$ ✗

$$\int_0^3 r^2(-r-r)\,dr=-2\int_0^3 r^3\,dr=-2\,\frac{r^4}{4}\Big|_0^3$$

$$=-2\left(\frac{81}{4}-\frac{0}{4}\right)=-\frac{162}{4}=-\frac{81}{2}$$

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1. STOŽAC $x^2+y^2=z^2$

PARABOLOID $x^2+y^2=5z$

$\varphi\in[0,2\pi]$

$r\in[z,\sqrt{5z}]$

$z\,[0,5]$

$r^2=z^2=5z$

$z^2=5z$

$z(z-5)=0$

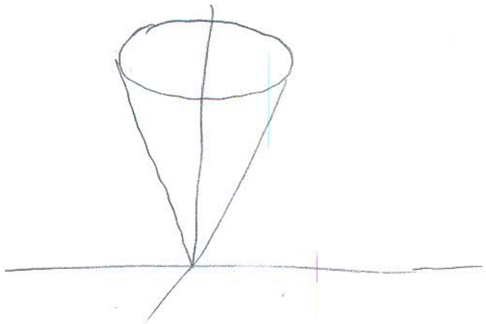
$z_1=0 \qquad z_2=5$

$x=r\cos\varphi$

$y=r\sin\varphi$

$r^2=z^2$

$r=\pm z$



$$\int_0^{2\pi} d\varphi \int_0^5 dz \int_z^{\sqrt{5z}} r\,dr = \int_0^{2\pi} d\varphi \int_0^5 \left(\frac{r^2}{2}\Big|_z^{\sqrt{5z}}\right) dr$$

$$= \int_0^{2\pi} d\varphi \int_0^5 \left(\frac{(\sqrt{5z})^2}{2} - \frac{z^2}{2}\right) dr = \frac{1}{2}\int_0^{2\pi} d\varphi \int_0^5 5z\,dr - \int_0^5 z^2\,dr$$

$$= \frac{1}{2}\int_0^{2\pi} d\varphi \left(5\frac{z^2}{2}\Big|_0^5 - \frac{z^3}{3}\Big|_0^5\right) = \frac{1}{2}\int_0^{2\pi}\left(\frac{125}{2}-\frac{125}{3}\right)d\varphi$$

20

$$= \frac{125}{12}\int_0^{2\pi} d\varphi = \frac{125}{12}\varphi\Big|_0^{2\pi} = \frac{125}{12}2\pi = \frac{250}{12}\pi$$

$$= \frac{125}{6}\pi$$

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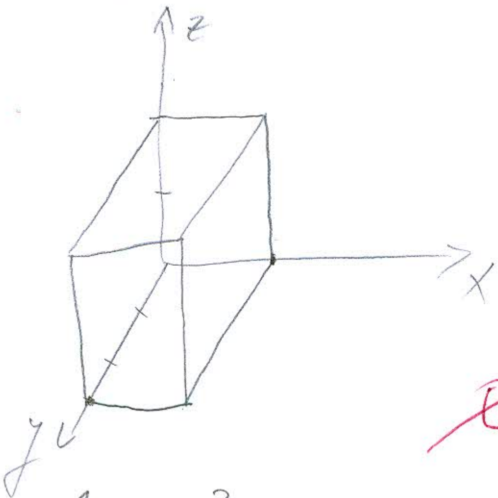
(2)

$$F = \begin{bmatrix} 3x + z^{77} \\ y^2 - \sin(x^2 z) \\ xz + y e^{x^5} \end{bmatrix} = \operatorname{div} = 3 + 2y + x$$



$x \in [0,1]$

$y \in [0,3]$

$z \in [0,2]$

$$3\int_0^2 dz \int_0^1 x\,dx \int_0^3 2y\,dy = 3\cdot 2\int_0^2 dz \int_0^1 x\,dx \int_0^3 y\,dy$$

$$= 6\int_0^2 dz \int_0^1 x\,dx \left(\frac{y^2}{2}\right)\Big|_0^3 = 6\int_0^2 dz \int_0^1 \frac{9}{2}x\,dx$$

$$= 6\,\frac{9}{2}\int_0^2 \left(\frac{x^2}{2}\right)\Big|_0^1 dz = \frac{54}{2}\int_0^2 \frac{1}{2}\,dz$$

$$= \frac{54}{4}(2-0) = \frac{108}{4} = \underline{\underline{27}}$$

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(3) $x'''(t) + 3x'(t) = t$ $\quad x'(0) = x''(0) = 0$

$x(0) = 1$

$s^3X(s) - s^2x(0) - sx'(0) - x''(0) + 3(sX(s) - x(0)) = \frac{1}{s^2}$

$s^3X(s) - s^2x(0) - \cancel{sx'(0)}^{0} - \cancel{x''(0)}^{0} + 3sX(s) - 3x(0) = \frac{1}{s^2}$

$s^3X(s) + 3sX(s) - s^2 - 3 = \frac{1}{s^2}$

$X(s)(s^3 + 3s) = \frac{1}{s^2} + s^2 + 3$

$X(s)\, s(s^2+3) = \frac{1 + s^4 + 3s^2}{s^2} \quad /: s(s^2+3)$

$X(s) = \frac{s^4 + 3s^2 + 1}{s^3(s^2+3)}$

$\frac{s^4 + 3s^2 + 1}{s^3(s^2+3)} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{Ds + E}{s^2+3}$

$s^4 + 3s^2 + 1 = A(s^3(s^2+3)) + B(s^4(s^2+3)) + C(s^5(s^2+3))$
$+ Ds(s^6) + E(s^6)$

$= As^5 + 3As^3 + Bs^6 + 3Bs^4 + Cs^7 + 3Cs^5$
$+ Ds^7 + Es^6$

$0 = D + C$
$0 = E + B \rightarrow \boxed{E = -\frac{1}{3}}$
$0 = A + 3C$
$1 = 3B \Rightarrow \boxed{B = \frac{1}{3}}$
$0 = 3A \Rightarrow A = \frac{0}{3} = \phi$

$F(s) = \frac{1}{3}\frac{1}{s^2} - \frac{1}{3}\frac{1}{\sqrt{3}}\frac{1}{s^2+3}\cdot\sqrt{3}$

$x(t) = \frac{1}{3}t - \frac{1}{3\sqrt{3}}\sin\sqrt{3}t$

$\emptyset$

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(4)

$$r'(t) = \begin{bmatrix} 2 \\ 2t \\ t^2 \end{bmatrix} \qquad t \in [0, 10]$$

$$\|r'(t)\| = \sqrt{2^2 + (2t)^2 + (t^2)^2}$$

$$= \sqrt{4 + 4t^2 + t^4}$$

$$\int_0^{10} \sqrt{4+4t^2+t^4}\, x+z \, dt = \sqrt{4+4t^2+t4} \int_0^{10} x+z \, dt$$

NE VADITI IZ INTEGRALA

$\varnothing$