MATEMATIKA 2
14. lipnja 2012.

Ime i prezime: Toma Medić **Broj indeksa:** 17-2-0052

Vrijeme: od 8:25 **do** 8:35 ♣4 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(15)* Integriraj

$$\int \frac{1+\sin(3x)}{\cos^2(3x)}dx$$

2. *(~~20~~ 15)* Integriraj

$$\int_{-1}^{1} \frac{x}{(x+2)(x^2+1)}dx$$

~~VIDI VUJATOVIĆ~~

3. *(~~20~~ 15)* Odredi površinu koju zatvaraju krivulja $y^2 = 2x + 1$ i pravac $y = x + 1$.

VIDI MACOLA

4. *(10+10)*

a) Ispitaj ekstreme funkcije

$$f(x,y) = xy + 4x^2 - 3y^2$$ VIDI MACOLA

b) Odredi domenu funkcije:

$$f(x,y) = \ln(x^2 + y^2)$$

$x^2+y^2>0$ za sve x,y osim $x=0$; $y=0$

$\Rightarrow D(f)=\mathbb{R}^2 \setminus \{(0,0)\}$

5. *(~~20+15~~)* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' - 5y = x^4$$

b)

$$y'' + 6y' + 9y = 2\cos x$$

VIDI RJEŠENJA NA SLJEDEĆIM STRANICAMA

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME!

NASTAVNI KOLA RJEŠENJA

1) $\displaystyle\int \frac{1+\sin(3x)}{\cos^2(3x)}dx = \left\{ \begin{matrix} 3x = t \\ 3dx = dt \end{matrix} \right\} = \int \frac{1+\sin t}{\cos^2 t} \frac{dt}{3} =$

$\displaystyle = \frac{1}{3}\int \frac{1+\sin t}{\cos^2 t}dt = (*)$

$\displaystyle \left\{ \begin{matrix} z = \cos t \\ dz = -\sin t\,dt \end{matrix} \right\} = -\int \frac{dz}{z^2} = \frac{1}{z} = \frac{1}{\cos t} + C$

PRVI NAČIN:

$\displaystyle (*) = \frac{1}{3}\int \frac{1}{\cos^2 t}dt + \frac{1}{3}\cdot\int \frac{\sin t}{\cos^2 t}dt$

$\displaystyle = \frac{1}{3}\tan t + \frac{1}{3}\cdot\frac{1}{\cos t} = \frac{1}{3}\left(\tan(3x) + \frac{1}{\cos(3x)}\right) + C$

DRUGI NAČIN:

UZ POMOĆ TRIGONOMETRIJSKIH SUPSTITUCIJA ...

5a) PRVI NAČIN: VIDI BILJEŠKU UZ KURIĆ

DRUGI NAČIN: VARIJACIJA KONSTANTE

1° $xy' - 5y = 0 \Rightarrow xy' = 5y \Rightarrow \frac{y'}{y} = \frac{5}{x} \;/\int \Rightarrow \ln|y| = 5\ln|x| + c$

$\Rightarrow y = Cx^5$

2° UVRSTITI $y = c(x)\cdot x^5$ U POLAZNU ODJ. $xy' - 5y = x^4$

$$\left. \begin{aligned} x\left(c'(x)\cdot x^5 + \cancel{c(x)\cdot 5x^4}\right) \cancel{- 5\cdot c(x)\cdot x^5} &= x^4 \\ c'(x)\cdot x^6 &= x^4 \\ c'(x) &= x^{-2} \\ c(x) &= -\frac{1}{x} + C \end{aligned} \right\} \Rightarrow y = \left(-\frac{1}{x} + C\right)\cdot x^5$$

NASTAVNIKOVA RJEŠENJA

5b) $y'' + 6y' + 9y = 2\cos x$

HOMOGENA:

$$y'' + 6y' + 9y = 0$$

KARAKTERISTIČNA:

$$\lambda^2 + 6\lambda + 9 = 0$$

$$\lambda_{1,2} = \frac{-6 \pm \sqrt{36-36}}{2} = -3$$

RJEŠENJE HOMOGENE

$$y_0 = e^{-3x}(A+Bx)$$

POSEBNO RJEŠENJE TRAŽITI U OBLIKU

$$Y = A\cos x + B\sin x$$

UVRSTITI U POLAZNU JEDNADŽBU:

$$Y'' + 6Y' + 9Y = 2\cos x$$

$$(-A\cos x - B\sin x) + 6(-A\sin x + B\cos x) + 9(A\cos x + B\sin x) = 2\cos x$$

$$(-A + 6B + 9A)\cos x + (-B - 6A + 9B)\sin x = 2\cos x$$

$$\left.\begin{aligned} -A + 6B + 9A &= 2 \\ -B - 6A + 9B &= 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} 8A + 6B &= 2 \\ -6A + 8B &= 0 \end{aligned}\right\}$$

$$\Rightarrow \begin{cases} A = \frac{4}{25} \\ B = \frac{3}{25} \end{cases}$$

$$Y = \frac{4}{25}\cos x + \frac{3}{25}\sin x$$

$$y = y_0 + Y$$

OPĆE RJEŠENJE POLAZNE ODJ:

$$y = e^{-3x}(A+Bx) + \frac{4}{25}\cos x + \frac{3}{25}\sin x$$

NASTAVNIKOVA RJEŠENJA

2) $$\int_{-1}^{1} \frac{x}{(x+2)(x^2+1)}\,dx = \dots = (*)$$

$$\frac{x}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1} = \frac{Ax^2+A+Bx^2+2Bx+Cx+2C}{(x+2)(x^2+1)}$$

$$= \frac{(A+B)x^2+(2B+C)x+A+2C}{(x+2)(x^2+1)}$$

$$\left.\begin{aligned} A+B&=0\\ 2B+C&=1\\ A+2C&=0 \end{aligned}\right\} \Rightarrow \left\{ A \right.$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0\\ 0 & 2 & 1 & 1\\ 1 & 0 & 2 & 0 \end{array}\right]_{R3-R1} \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0\\ 0 & 2 & 1 & 1\\ 0 & -1 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0\\ 0 & 1 & -2 & 0\\ 0 & 2 & 1 & 1 \end{array}\right]_{R3-2R2} \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0\\ 0 & 1 & -2 & 0\\ 0 & 0 & 5 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2/5\\ 0 & 1 & 0 & 2/5\\ 0 & 0 & 1 & 1/5 \end{array}\right] \qquad \begin{aligned} A &= -\tfrac{2}{5}\\ B &= \tfrac{2}{5}\\ C &= \tfrac{1}{5} \end{aligned}$$

$$\Rightarrow (*) = \int_{-1}^{1} \frac{-\frac{2}{5}}{x+2}\,dx + \underbrace{\int_{-1}^{1} \frac{\frac{2}{5}x+\frac{1}{5}}{x^2+1}\,dx}_{= \frac{1}{5}\int_{-1}^{1} \frac{2x}{x^2+1}dx + \frac{1}{5}\int_{-1}^{1} \frac{dx}{x^2+1}}$$

$$= \left[-\frac{2}{5}\ln|x+2| + \frac{1}{5}\ln|x^2+1| + \frac{1}{5}\arctan(x)\right]_{-1}^{1}$$

$$= -\frac{2}{5}\ln 3 + \frac{1}{5}\ln 2 + \frac{1}{5}\arctan 1 - \left(-\frac{2}{5}\ln 1 + \frac{1}{5}\ln 2 + \frac{1}{5}\arctan(-1)\right)$$

$$= -\frac{2}{5}(\ln 3 - \underbrace{\ln 1}_{=0}) + \frac{1}{5}(\underbrace{\ln 2 - \ln 2}_{=0}) + \frac{1}{5}(\arctan 1 - \arctan(-1))$$

$$\approx -\frac{2}{5}\cdot 1.1 + \frac{1}{5}\cdot 1.57 \approx -0.12$$