MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE JEDNOSTRANO!**

IME I PREZIME: JOSIP MIJALIĆ **BROJ INDEKSA:** 57154

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati volumen područja između plašta stošca $x^2 + y^2 = z^2$ i plašta paraboloida $x^2 + y^2 = 5z$. 20

2. Izračunati $\iint\limits_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} 3x + z^{77} \\ y^2 - \sin(x^2 z) \\ xz + ye^{x^5} \end{pmatrix}$ i ∂K rub kvadra $K = \{(x,y,z) : 0 \leq x \leq 1, 0 \leq y \leq 3, 0 \leq z \leq 2\}$ koji je orijentiran vanjskom normalom.

3. Riješiti $x'''(t) + 3x'(t) = t, \quad , x'(0) = x''(0) = 0, x(0) = 1.$ 20

4. Izračunati krivuljni integral skalarnog polja $f(x,y,z) = x + z$ po luku krivulje C zadane sa $x = 2t$, $y = t^2$ i $z = \frac{1}{3}t^3$ ako je $0 \leq t \leq 10$. 20

5. Zadan je X krug radijusa 3 oko točke $T(1,0)$ i $f(x,y) = xy$. Izračunati $\iint\limits_X f$. 20

Ukupno: 80

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(4) $\int_C f\, ds = \int_a^b (f \circ r) \|r'\| \, t \, dt$

$f(x,y,z) = x + z$

$x = 2t$

$y = t^2$

$z = \frac{1}{3} t^3$

$r(t) = 2t\, i \; t^2 j + \frac{1}{3} t^3 k$

$\| r'(t) \| = \sqrt{[(2t)']^2 + [(t^2)']^2 + [(\frac{1}{3} t^3)']^2}$

$\| r'(t) \| = \sqrt{2^2 + (2t)^2 + (\frac{1}{3} 3t)^2} = \sqrt{4 + 4t^2 + t^4}$ ✓

$\| r'(t) \| = \sqrt{(2+t)^2} = 2 + t^2$ ✓

$f \circ r(t) = 2t + \frac{1}{3} t^3$ ✓

$\int_C f\, ds = \int_0^{10} (2t + \frac{1}{3} t^3)(2 + t^2)\, dt$ ✓ $= \int_0^{10} (4t + \frac{2}{3} t^3 + 2t^3 + \frac{1}{3} t^5)\, dt$

$= \int_0^{10} \left(4t + t^3 \left(\frac{2}{3} + 2\right) + \frac{t^6}{3}\right) dt = \int_0^{10} \left(4t + \frac{8}{3} + \frac{t^6}{3}\right) dt =$

$= \left(\cancel{4}^{2} \frac{t^2}{\cancel{2}} + \frac{\cancel{8}^{2}}{3} \cdot \frac{t^4}{\cancel{4}} + \frac{1}{3} \cdot \frac{t^6}{6} \right) \Bigg|_0^{10} = \left(2t^2 + \frac{2}{3} t^4 + \frac{t^6}{18} \right) \Bigg|_0^{10} =$

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$$= 2\cdot(10)^2 + \frac{2}{3}(10)^4 + \frac{(10)^6}{18} = 2\cdot 100 + \frac{2}{3}\cdot 10000 + \frac{1000000}{18}$$

$$= 62422{,}22$$

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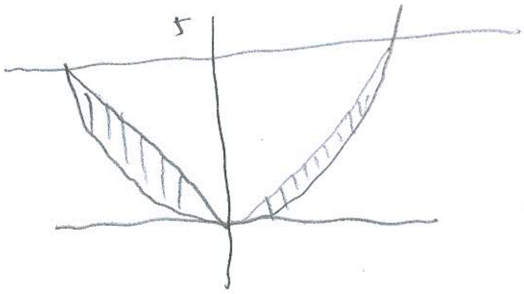

(1)

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$$V = \int_0^{2\pi} d\varphi \int_0^5 dz \int_2^{\sqrt{5z}} r\,dr = \int_0^{2\pi} d\varphi \int_0^5 dz \left(\frac{r^2}{2}\right)\Big|_2^{\sqrt{5z}} =$$

$$= \int_0^{2\pi} d\varphi \int_0^5 dz \left(\frac{5z^2}{2} - \frac{z^2}{2}\right) = \int_0^{2\pi} d\varphi \left(\frac{5}{2}\cdot\frac{z^2}{2} - \frac{1}{2}\frac{z^3}{3}\right)\Big|_0^5$$

$$= \int_0^{2\pi} d\varphi \left(\frac{5z^2}{4} - \frac{z^3}{6}\right)\Big|_0^5 = \int_0^{2\pi} d\varphi \left(\frac{5\cdot 5^2}{4} - \frac{5^3}{6}\right) =$$

$$= \int_0^{2\pi} d\varphi \left(\frac{5^3}{4} - \frac{5^3}{6}\right) = \longrightarrow = \int_0^{2\pi} d\varphi \frac{5^3}{12} =$$

$$= \frac{5^3}{12}\varphi\Big|_0^{2\pi} = \frac{5^3}{\cancel{12}}\cdot \cancel{2}\pi = \frac{5^3}{6}\pi = \frac{125}{6}\pi$$

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3) $x'''(t) + 3x'(t) = t$ $\quad x'(0) = x''(0) = 0 \quad x(0) = 1$

$x'''(t) \Rightarrow s^3 X(s) - s^2 X(0) - s X'(0) - X''(0)$

$\Rightarrow s^3 X(s) - s^2 //$

$x'(t) \Rightarrow s X(s) - 1 //$

$t \Rightarrow \frac{1}{s^2}$

$s^3 X(s) - s^2 + 3 s X(s) - 3 = \frac{1}{s^2}$

$X(s)(s^3 + 3s) = \frac{1}{s^2} + s^2 + 3$

$X(s)(s^3 + 3s) = \frac{1 + s^4 + 3s^2}{s^2}$

$X(s) = \frac{1 + s^4 + 3s^2}{s^3(s^2+3)}$

$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+3} \quad \cdot s^3(s^2+3)$

$= A(s^2)(s^2+3) + B(s)(s^2+3) + C(s^2+3)$

$+ Ds+E(s^3)$

$= A(s^4 + 3s^2) + B(s^3+3) + C(s^2+3) + Ds+E(s^3)$

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$A s^4 + 3As^2 + Bs^3 + 3B + Cs^2 + 3C + Ds^4 + Es^3$

$s^4(A+D)$

$s^3(B+E)$

$s^2(3A+C)$

$\boxed{D=\frac{1}{9}}$ $\boxed{B=0}$ $\boxed{C=\frac{1}{3}}$

$\boxed{E=0}$

$3A+\frac{1}{3}=3$

$\boxed{A=\frac{8}{9}}$

$A+D=1$

$B+E=0$

$3A+C=3$

$3B=0$

$3C=1$

$\frac{8}{9}\cdot\frac{1}{s}+\frac{1}{3}\cdot\frac{1}{s^3}+\frac{1}{9}\frac{s}{s^2+3}$

$=\frac{8}{9}+\frac{1}{3}\cdot\frac{1}{2}t^2+\frac{1}{9}\cos(\sqrt{3}\,t)$ //

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(5)

$$\iint_X f = \iint_X xy \, dx \, dy$$

$$(x-1)^2 + y^2 = 3^2$$

$x^2 = x - 1$

$x = x^2 + 1$

$x = r \cos \varphi + 1$

$y = r \sin \varphi$

$$\int_0^{2\pi} \int_0^3 \left[(r \cos \varphi + 1) \right] \cdot r \, dr \, d\varphi = \times$$

$$= \int_0^{2\pi} \int_0^3 \underbrace{(r \cos \varphi + 1)}_{= x} \underbrace{(r^2 \sin \varphi)}_{y = r \sin \varphi} \, dr \, d\varphi = \checkmark$$

$x \cdot y \cdot r \, dr \, d\varphi$

$$= \int_0^{2\pi} \int_0^3 (r^3 \cos \varphi \sin \varphi + r^2 \sin \varphi) \, dr \, d\varphi = \checkmark$$

$$= \int_0^{2\pi} \left(\frac{r^4}{4} \sin \frac{2\varphi}{2} + \frac{r^3}{3} \sin \varphi \right) \Big|_0^3 \, d\varphi =$$

$$= \int_0^{2\pi} \left(\frac{3^4}{4} \sin \frac{2\varphi}{2} + \frac{3^3}{3} \sin \varphi \right) d\varphi = \int_0^{2\pi} \left(\frac{81}{42} \sin 2\varphi + 9 \sin \varphi \right) d\varphi$$

NASTAVAK →

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$$\int \sin 2\varphi \, d\varphi = \left[\begin{array}{l} 2\varphi = t \\ 2d\varphi = dt \end{array} \Rightarrow d\varphi = \frac{dt}{2} \right] = \int \sin \frac{dt}{2}$$

$$= \frac{1}{2} \int \sin t \, dt = \frac{1}{2} (-\cos t) \, dt = -\frac{1}{2} \cos t \, dt =$$

$$= -\frac{1}{2} \cos 2\varphi + C$$

nastavak 5.

$$= \left[\frac{81}{8} \left(-\frac{1}{2} \cos 2\varphi \right) + 9 (-\cos \varphi \right]_0^{2\pi} =$$

$$= \left(-\frac{81}{16} \cos 2\varphi - 9 \cos \varphi \right)_0^{2\pi} =$$

$$= -\frac{81}{16} \cancel{\cos 4\pi}^{1} + \frac{81}{16} \cancel{\cos 0}^{1} - 9 \cancel{\cos 2\pi}^{1} + 9 \cancel{\cos 0}^{1} =$$

$$= \cancel{-\frac{81}{16}} + \cancel{\frac{81}{16}} \cancel{- 9} \cancel{+ 9} = 0 \,//$$

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