

MATEMATIKA 2

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Vrijeme: od \_\_\_\_\_ do \_\_\_\_\_ ♣5

Broj bodova:  $\frac{10}{80} = 12.5\%$

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. ~~(15)~~ <sup>5</sup> Integriraj

~~Integriraj~~  $\int_0^1 x \tan(x^2+1) dx$

2. ~~(20)~~ Integriraj

$$\int \frac{x^2+1}{(x+1)^2(x-1)} dx$$

3. (20) Odredi površinu koju zatvaraju krivulje  $y = 1 - x^2$ ,  $y = 3 + 2x - x^6$  i os apscisa. **IZBAČENO**

4. ~~(10+10)~~

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^2 + y^2 + xy - 3x - 6y$$

b) Odredi domenu funkcije:

$$f(x, y) = x - \sqrt{x+y}$$

5. ~~(10+10)~~ <sup>5</sup> Riješi sljedeće diferencijalne jednačbe:

a)

~~y~~  $y' = -\frac{y}{x}$

b)

$$y'' + y' + \frac{1}{4}y = 2$$

VIDI RJEŠENJE 1

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPIŠATI IME I PREZIME!

3a)  $f(x,y) = x^2 + y^2 + xy - 3x - 6y$

$f'(x,y)_x = 2x + y - 3$

$f'(x,y)_y = 2y + x - 6$

$2x + y - 3 = 0$

$2y + x - 6 = 0$

$6y + y = 3$

$7y = 3$

$y = \frac{3}{7}$

$2y + x = 6$

$x = \frac{6}{2}$

$x = 3y$

$x = 3 \cdot \frac{3}{7} = \frac{9}{7}$

$A(\frac{9}{7}, \frac{3}{7})$

~~X~~ ~~0~~

$f''(x,y)_{xx} = 2$

$f''(x,y)_{xy} = 1$

$f''(x,y)_{yx} = 1$

$f''(x,y)_{yy} = 2$

$H_{\Delta} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 > 0$

ALLG MINIMUM

4)  $f(x,y) = x - \sqrt{x^2 + y^2}$   
 $= x - \sqrt{x^2 + y^2}$   
 $= x^2 + y^2$

$x + y \geq 0$

1)  $y' = -\frac{y}{x}$   
 $\frac{dy}{dx} = -\frac{y}{x} \quad | : y$   
 $\frac{dy}{y} = -\frac{1}{x} \quad | \cdot dx$   
 $\frac{dy}{y} = -\frac{dx}{x}$   
 $\int \frac{dy}{y} = -\int \frac{dx}{x}$   
 $\ln|y| = -\ln|x| + \ln|c|$   
 $\ln|y| = \ln|\frac{c}{x}|$

$y = \frac{c}{x}$  ✓  
 $y' = \frac{c' \cdot x - c \cdot 1}{x^2}$   
 $y' = \frac{c'x - c}{x^2}$

$\frac{c'x - c}{x^2} = -\frac{c}{x}$   
 $\frac{c'x}{x^2} - \frac{c}{x^2} = -\frac{c}{x^2}$   
 $\frac{c'x}{x^2} = \frac{c}{x} + \frac{c}{x^2}$   
 $\frac{c'x}{x^2} = 0$   
 $\frac{c'}{x} = 0 \quad | \cdot x$   
 $c' = 0$

$c = \int x dx$   
 $c = \frac{x^2}{2} + D$   
 $y = \frac{\frac{x^2}{2} + D}{x}$

~~X~~  
 OD PONUDENA 2  
 RESENJA JEDNO SE  
 TOČNO!

③  $y = 1 - x^2$ ,  $y = 3 + 2x - x^6$

$y = -x^2 + 1$

$-x^2 + 1 = 0$

$-x^2 = -1$

$x^2 = 1$

$x = \pm 1$

$y = -x^6 + 2x + 3$

$x = 1$

$-x^6 + 2x + 3 = 0$

$k = 2$

$-x^6 + 2x = -3$  /  $-4 = 3$

$x^6 - 2x = 3$

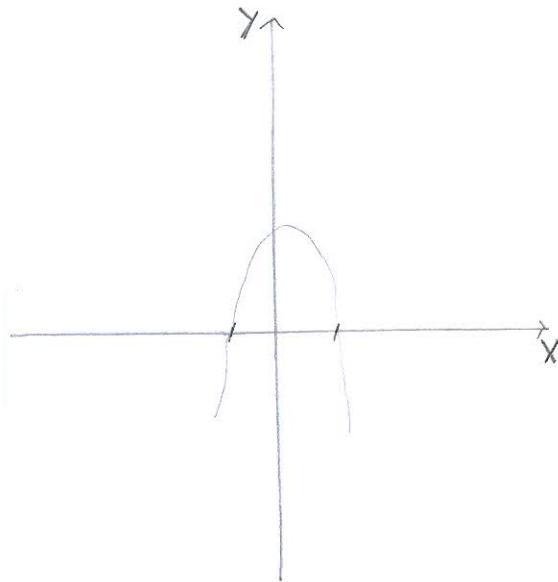
$x(x^5 - 2) = 3$

$x =$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 3}}{2}$

$x_{1,2} = \frac{-2 \pm \sqrt{4 - 12}}{2}$



5.  $y'' + y' + \frac{1}{4}y = 2$

$\lambda^2 + \lambda + \frac{1}{4} = 2$

$\lambda^2 + \lambda - \frac{7}{4} = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-\frac{7}{4})}}{2}$

$x_{1,2} = \frac{-1 \pm \sqrt{1 + \frac{28}{4}}}{2}$

$x_{1,2} = \frac{-1 \pm 2\sqrt{2}}{2}$

$x_0 =$



1.  $\int_0^1 x \tan(x^2+1) dx =$

$$\int_0^1 x \tan t \cdot \frac{dt}{2x} = \frac{1}{2} \int_0^1 \tan t dt = \frac{1}{2} \int$$

$$\frac{1}{2} - \ln |\cos t| \Big|_0^1 = -\frac{1}{2} \ln |\cos(x^2+1)| \Big|_0^1 = \frac{1}{2} \ln 2 - \frac{1}{2} \ln |\cos 2|$$

$$x^2+1 = t$$

$$2x dx = dt$$

$$dx = \frac{dt}{2x}$$

LOŠE UVRŠTENNO!

9

2.  $\int \frac{x^2+1}{(x+1)(x-1)} dx =$

$$\frac{x^2+1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{x-1} \quad | \quad (x+1)(x+1)(x-1)$$

$$x^2+1 = A(x+1)(x-1) + B(x+1)(x-1) + C(x+1)(x+1)$$

$$x^2+1 = A(x^2+x-x-1) + B(x^2+x-x-1) + C(x^2+x+x+1)$$

$$\underline{x^2+1} = \underline{Ax^2 - A} + \underline{Bx^2 - B} + \underline{Cx^2 + 2Cx + C}$$

$$1 = A+B+C$$

$$1 = -A-B+C$$

$$0 = 2Cx \Rightarrow C=0$$

$$1 = A+B$$

$$1 = -A-B$$

7