MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: LOUZE MACOLA **Broj indeksa:** 56197 - 2008

Vrijeme: od 08:20 **do** 09:50 ♣4 **Broj bodova:** 35

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. ~~(15)~~ Integriraj

$$\int \frac{1+\sin(3x)}{\cos^2(3x)}dx$$

2. ~~(20)~~ ~~15~~ 10 Integriraj

$$\int_{-1}^{1} \frac{x}{(x+2)(x^2+1)}dx$$

3. ~~(20)~~ 15 Odredi površinu koju zatvaraju krivulja $y^2 = 2x + 1$ i pravac $y = x + 1$.

4. ~~(10+10)~~

a) Ispitaj ekstreme funkcije

$$f(x, y) = xy + 4x^2 - 3y^2$$

b) Odredi domenu funkcije:

$$f(x, y) = \ln(x^2 + y^2)$$

5. (20+15) Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' - 5y = x^4$$

b)

$$y'' + 6y' + 9y = 2\cos x$$

VIDI RJEŠENJE 2

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME!

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2/ $\int_{-1}^{1} \frac{x}{(x+2)(x^2+1)} dx = \int \frac{-\frac{2}{5}}{x+2} dx + \int \frac{\frac{2}{5}x+\frac{1}{5}}{(x^2+1)} dx =$

$= -\frac{2}{5}\int \frac{dx}{x+2} + \frac{2}{5}\int \frac{x}{x^2+1} dx + \frac{1}{5}\int \frac{dx}{(x^2+1)} = -\frac{2}{5}\ln|x+2| + \frac{2}{5}\int \frac{\frac{1}{2}dt}{t}$

$+ \frac{1}{5}\operatorname{arctan} x^2+1 + C = -\frac{2}{5}\ln|x+2| + \frac{2}{10}\ln|x^2+1| + \frac{1}{5}\operatorname{arctan} x^2+1 + C$

DOBRO $\underline{10}$

$= -\frac{2}{5}\ln|3| + \frac{2}{5}\ln|1| + \frac{2}{10}\ln|2| + \frac{2}{10}\ln|2| + \frac{1}{5}\operatorname{arctan} 2 + C + \frac{1}{5}\operatorname{arctan} 2 + C$

ALI NIJE DOBRO RIJEŠEN ODREĐENI INTEGRAL! X

$\frac{x}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{(x^2+1)} \Big/ \cdot (x+2)(x^2+1)$

$x = A(x^2+1) + Bx + C(x+2)$

$x = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$

$x = (A+B)x^2 + (2B+C)x + A + 2C$

$A+B=0 \qquad B=-A$

$2B+C=1 \qquad \boxed{B=\frac{2}{5}}$

$A+2C=0$

$\left[\begin{array}{l} x^2+1=t \\ 2x\,dx=dt \\ x\,dx=\frac{1}{2}dt \end{array}\right]$

$-2A+C=1 \,/\cdot(-2)$

$A+2C=0$

$4A-\cancel{2C}=-2$

$A+\cancel{2C}=0$

$5A=-2$

$\boxed{A=-\frac{2}{5}}$

$2\cdot\frac{2}{5}+C=1$

$\frac{4}{5}+C=1$

$C=1-\frac{4}{5}$

$C=\frac{5-4}{5}$

$\boxed{C=\frac{1}{5}}$

$\left[\begin{array}{l} x^2+1=t \\ 2x\,dx=dt \end{array}\right]$

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4/a) Ispitaj ekstreme funkcije

$f(x,y) = xy + 4x^2 - 3y^2$

$\partial_x f = y + 8x$ $\quad\quad \partial_x f = 0$

$\partial_{xx} f = 8$ $\quad\quad \partial_y f = 0$

$\partial_{xy} f = 1$

$\partial_y f = x - 6y$ $\quad\quad y + 8x = 0$

$\partial_{yy} f = -6$ $\quad\quad x - 6y = 0$ $\quad\quad \boxed{x = 6y}$

$\partial_{yx} f = 1$ $\quad\quad x = 6 \cdot 0$ $\quad\quad \boxed{x = 0}$

$y + 8 \cdot 6y = 0$

$y + 48y = 0$

$A = \partial_{xx} f$ $\quad\quad T(0,0)$ $\quad\quad 49y = 0 \,/ : 49$

$A = 8$ $\quad\quad \boxed{y = 0}$

$$\Delta = \begin{vmatrix} \partial_{xx} f & \partial_{xy} f \\ \partial_{yx} f & \partial_{yy} f \end{vmatrix} = \begin{vmatrix} 8 & 1 \\ 1 & -6 \end{vmatrix} = -48 - 1 = -49$$

$A = 8 > 0$

$\Delta = -49 < 0$ } Funkcija nema ekstrema

b) $f(x,y) = \ln|x^2 + y^2|$ $\quad\quad \boxed{x^2 + y^2 \leq r^2}$

$\ln|x^2 + y^2| \geq 0$

$Df(x,y) = \mathbb{R} \geq 0$

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3/ Odredi površinu koju zatvaraju krivulja i pravac:

$y^2 = 2x + 1$ krivulja

$y = x + 1$ pravac

$y^2 = 2x + 1$

$y = \pm\sqrt{2x+1}$

x	0	1	2
$y = \pm\sqrt{2x+1}$	± 1	$\pm\sqrt{3}$	$\pm\sqrt{5}$
		± 1.73	2.2

$(x+1)^2 = 2x + 1$

$x^2 + \cancel{2x} + \cancel{1} - \cancel{2x} - \cancel{1} = 0$

$x^2 = 0$

$x = 0$

$y = x + 1$

$y = 0 + 1$

$T(0, 1)$

DIRALIŠTE

$y = x + 1$

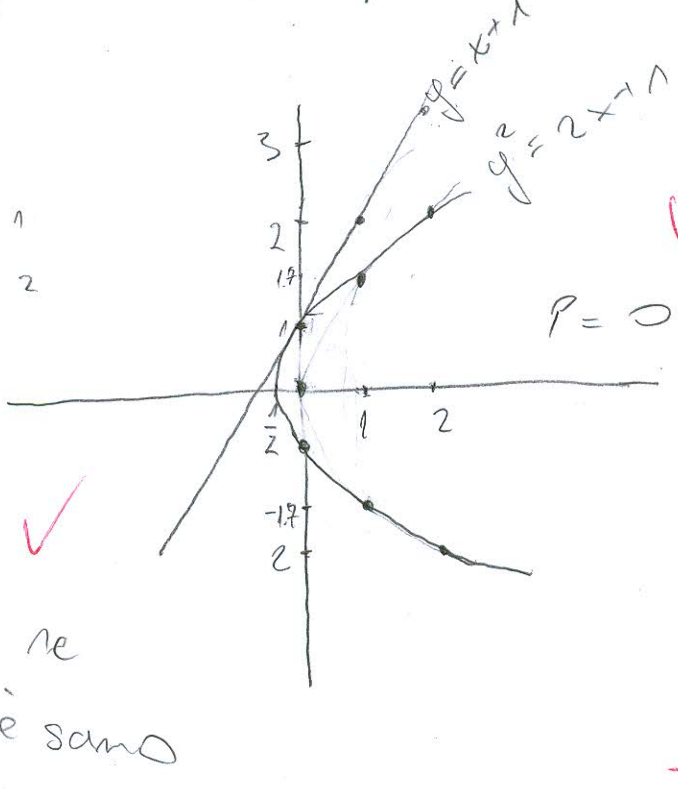
x	0	1	2
$y = x + 1$	1	2	3

$y^2 = 2x + 1$

$y^2 = 2 \cdot 0 + 1$

$y^2 = 1 \;/:2$

$y = \frac{1}{2}$



$P = 0$

Površina će biti jednaka 0 ✓

zbog toga što pravac ne ulazi u krivulju nego je samo dira! ~~u stacionarnoj točki~~

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BROJ INDEKSA:

1./ Integrivaj:

$$\int \frac{1+\sin(3x)}{\cos^2(3x)}\,dx = \left[\begin{array}{l} \cos^2(3x) = t \\ 1+\sin(3x) = dt \end{array}\right]$$

$$= \int \frac{dt}{t} = \ln|t| + c = \ln|\cos^2(3x)| + c$$