MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE JEDNOSTRANO!**

POPUNJAVA NASTAVNIK
Broj ↓
bodova

IME I PREZIME: RJEŠENJE 1

BROJ INDEKSA:

1. Izračunati volumen područja između plašta stošca $x^2+y^2=z^2$ i plašta paraboloida $x^2+y^2=5z$.
2. Izračunati $\iint\limits_{\partial K} \mathbf{F}\cdot d\mathbf{S}$ gdje je $\mathbf{F}=\begin{pmatrix} 3x+z^{77} \\ y^2-\sin(x^2z) \\ xz+ye^{x^5}\end{pmatrix}$ i ∂K rub kvadra $K=\{(x,y,z)\,:\,0\le x\le 1,\,0\le y\le 3,\,0\le z\le 2\}$ koji je orijentiran vanjskom normalom.
3. Riješiti $x'''(t)+3x'(t)=t, \quad ,x'(0)=x''(0)=0,\ x(0)=1.$
4. Izračunati krivuljni integral skalarnog polja $f(x,y,z)=x+z$ po luku krivulje C zadane sa $x=2t$, $y=t^2$ i $z=\frac{1}{3}t^3$ ako je $0\le t\le 10$.
5. Zadan je X krug radijusa 3 oko točke $T(1,0)$ i $f(x,y)=xy$. Izračunati $\iint\limits_X f$.

Ukupno:

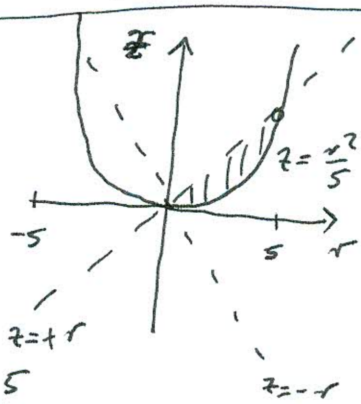
1)

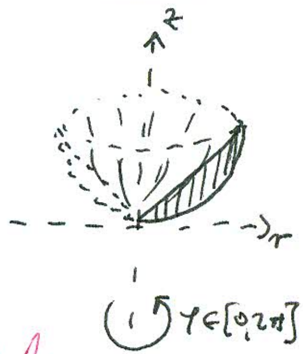
$r^2=x^2+y^2$

$r^2=z^2 \quad (+) \quad r^2=5z$

$r=\pm z$

$z=\pm r \qquad z=\frac{r^2}{5}$





15 bodova

$$V=\int_0^{2\pi}\int_0^5\int_{\frac{r^2}{5}}^{r} r\,dz\,dr\,d\varphi = 2\pi\int_0^5 r\left(r-\frac{r^2}{5}\right)dr=\frac{2\pi}{12}\cdot 5^3 \approx$$

5 bodova

2) TEOREM O DIVERGENCIJI: $\iint\limits_{\partial K} F\cdot dS = \iiint\limits_K \operatorname{div} F$

$\operatorname{div} F = 3+2y+x$

$$\Rightarrow \iint\limits_{\partial K} F\,dS = \int_0^1\int_0^3\int_0^2 (3+2y+x)\,dz\,dy\,dx = 2\int_0^1\int_0^3 (3+2y+x)\,dy\,dx =$$

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$$=2\int_0^1\left[3y+\frac{2y^2}{2}+xy\right]_{y=0}^{3}dx = 2\int_0^1 (9+9+3x)\,dx = 2\left[18x+3\frac{x^2}{2}\right]_{x=0}^{1}$$

$$=2\left(18+\frac{3}{2}\right)=39$$

5 bodova

4) $f(x,y,z) = x + z$

C ... parametrizacija $t \in [0,10]$
$$\begin{cases} x = 2t, & x' = 2 \\ y = t^2, & y' = 2t \\ z = \frac{1}{3}t^3, & z' = t^2 \end{cases}$$

$$\int_C f\,ds = \int_0^{10} \left(2t + \frac{1}{3}t^3\right) \cdot \sqrt{2^2 + (2t)^2 + (t^2)^2}\,dt$$

$$(*) = \int_0^{10} \left(2t + \frac{1}{3}t^3\right) \cdot \underbrace{\sqrt{4 + 4t^2 + t^4}}_{(\blacksquare)}\,dt$$

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AKO NE ZNAM EGZAKTNO INTEGRIRATI, KORISTIM TRAPEZNU FORMULU (MOŽE I SIMPSONOVA)

$f(t) = \left(2t + \frac{t^3}{3}\right) \cdot \sqrt{4 + 4t^2 + t^4}$, $n = 10$, $\Delta t = 1$, $t_0 = 0$, $t_{10} = 10$

t	0	1	2	3	4	5	6	7	8	9	10
$f(t)$	0	7	40	165	528	1395	3192	6545	12320	21663	36040

PO TRAPEZNOJ FORMULI

$$(*) \approx \left(\frac{0}{2} + 7 + 40 + 165 + 528 + 1395 + 3192 + 6545 + 12320 + 21663 + \frac{36040}{2}\right) \cdot 1$$

$$(*) \approx 63875$$

OVDJE 5 bodova

AKO PRIMIJETIM DA JE
$$4 + 4t^2 + t^4 = 4 + 2 \cdot 2 t^2 + (t^2)^2 = (2 + t^2)^2$$
$$\Rightarrow (\blacksquare) = 2 + t^2$$
$$\Rightarrow (*) = \int_0^{10} \left(2t + \frac{1}{3}t^3\right) \cdot (2 + t^2)\,dt$$

$$(*) = \int_0^{10} \left(4t + \frac{2}{3}t^3 + 2t^3 + \frac{1}{3}t^5\right) dt = \int_0^{10} 4t + \frac{8}{3}t^3 + \frac{1}{3}t^5\,dt = \left[4\frac{t^2}{2} + \frac{8}{3} \cdot \frac{t^4}{4} + \frac{1}{3}\frac{t^6}{6}\right]_0^{10}$$

$$= \underbrace{2 \cdot 100}_{200} + \underbrace{\frac{2}{3} \cdot 10000}_{\approx 6666} + \underbrace{\frac{1}{18} \cdot 1000000}_{\approx 55555} \approx 62422$$

ILI OVDJE 5 bodova

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3) $x'''(t) + 3x'(t) = t \qquad x'(0) = x''(0) = 0 \quad x(0) = 1$

$$x'''(t) \Rightarrow s^3 X(s) - s^2 X(0) - s X'(0) - X''(0)$$

$$\Rightarrow s^3 X(s) - s^2 //$$

$$x'(t) \Rightarrow s X(s) - 1 //$$

$$t \Rightarrow \frac{1}{s^2}$$

$$s^3 X(s) - s^2 + 3 s X(s) - 3 = \frac{1}{s^2}$$

$$X(s)(s^3 + 3s) = \frac{1}{s^2} + s^2 + 3$$

$$X(s)(s^3 + 3s) = \frac{1 + s^4 + 3s^2}{s^2}$$

$$X(s) = \frac{1 + s^4 + 3s^2}{s^3(s^2+3)}$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+3} \quad \cdot s^3(s^2+3)$$

$$= A(s^2)(s^2+3) + B(s)(s^2+3) + C(s^2+3)$$

$$+ Ds+E(s^3)$$

$$= A(s^4 + 3s^2) + B(s^3+3) + C(s^2+3) + Ds+E(s^3)$$

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$$A s^4 + 3As^2 + Bs^3 + 3B + Cs^2 + 3C + Ds^4 + Es^3$$

$s^4(A+D)$

$s^3(B+E)$

$s^2(3A+C)$

$\boxed{D=\frac{1}{9}}$ $\boxed{B=0}$ $\boxed{C=\frac{1}{3}}$

$\boxed{E=0}$

$3A+\frac{1}{3}=3$

$\boxed{A=\frac{8}{9}}$

$A+D=1$

$B+E=0$

$3A+C=3$

$3B=0$

$3C=1$

$$X(s)=\frac{8}{9}\cdot\frac{1}{s}+\frac{1}{3}\cdot\frac{1}{s^3}+\frac{1}{9}\frac{s}{s^2+3}$$

$$x(t)=\frac{8}{9}+\frac{1}{3}\cdot\frac{1}{2}t^2+\frac{1}{9}\cos(\sqrt{3}\,t)$$

✓ 20

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5

$$\iint_X f = \iint_X xy\,dx\,dy$$

$$(x-1)^2 + y^2 = 3^2$$

$x\,2 = x - 1$

$x = x\,2 + 1$

$x = r\cos\varphi + 1$

$y = r\sin\varphi$

$$\int_0^{2\pi}\int_0^{3}\left[(r\cos\varphi+1)\right]\cdot r\,dr\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^{3} \underbrace{(r\cos\varphi+1)}_{=x}\underbrace{(r^2\sin\varphi)}_{y = r\sin\varphi}\,dr\,d\varphi =$$

$x \cdot y \cdot r\,dr\,d\varphi$

$$= \int_0^{2\pi}\int_0^{3}\left(r^3\cos\varphi\sin\varphi + r^2\sin\varphi\right)dr\,d\varphi =$$

$$= \int_0^{2\pi}\left(\frac{r^4}{4}\sin\frac{2\varphi}{2} + \frac{r^3}{3}\sin\varphi\right)\Big|_0^3\,d\varphi =$$

$$= \int_0^{2\pi}\left(\frac{3^4}{4}\sin\frac{2\varphi}{2} + \frac{3^3}{3}\sin\varphi\right)d\varphi = \int_0^{2\pi}\left(\frac{81}{42}\sin 2\varphi + 9\sin\varphi\right)d\varphi$$

NASTAVAK

→

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$$\int \sin 2\varphi \, d\varphi = \left[\begin{matrix} 2\varphi = t \\ 2d\varphi = dt \end{matrix} \Rightarrow d\varphi = \frac{dt}{2} \right] = \int \sin \frac{dt}{2}$$

$$= \frac{1}{2} \int \sin t\,dt = \frac{1}{2} (-\cos t)\,dt = -\frac{1}{2} \cos t\,dt =$$

$$= -\frac{1}{2} \cos 2\varphi + C$$

nastavak 5.

$$= \left[\frac{81}{8} \left(-\frac{1}{2} \cos 2\varphi\right) + 9(-\cos\varphi \right]_0^{2\pi} =$$

$$= \left(-\frac{81}{16} \cos 2\varphi - 9 \cos\varphi\right)_0^{2\pi} =$$

$$= -\frac{81}{16} \cancel{\cos 4\pi} + \frac{81}{16} \cancel{\cos 0} - 9 \cancel{\cos 2\pi} + 9 \cancel{\cos 0} =$$

$$= \cancel{-\frac{81}{16}} + \cancel{\frac{81}{16}} \cancel{-9} \cancel{+9} = 0$$

20