

odgovornosti studenata. **PIŠITE JEDNOSTRANO!**

IME I PREZIME:

RJEŠENJE 1

BROJ INDEKSA:

- Izračunati volumen područja između plašta stošca $x^2 + y^2 = z^2$ i plašta paraboloida $x^2 + y^2 = 5z$.
- Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} 3x + z^{77} \\ y^2 - \sin(x^2 z) \\ xz + ye^{x^5} \end{pmatrix}$ i ∂K rub kvadra $K = \{(x, y, z) : 0 \leq x \leq 1, 0 \leq y \leq 3, 0 \leq z \leq 2\}$ koji je orijentiran vanjskom normalom.
- Riješiti $x'''(t) + 3x'(t) = t$, $x'(0) = x''(0) = 0$, $x(0) = 1$.
- Izračunati krivuljni integral skalarnog polja $f(x, y, z) = x + z$ po luku krivulje C zadane sa $x = 2t$, $y = t^2$ i $z = \frac{1}{3}t^3$ ako je $0 \leq t \leq 10$.
- Zadan je X krug radijusa 3 oko točke $T(1, 0)$ i $f(x, y) = xy$. Izračunati $\iint_X f$.

1)

$$r^2 = x^2 + y^2$$

$$r^2 = z^2 \quad (+) \quad r^2 = 5z$$

$$r = \pm z$$

$$z = \pm r$$

Ukupno:

$$V = \int_0^{2\pi} \int_0^5 \int_{\frac{r^2}{5}}^r r \, dz \, dr \, d\varphi = 2\pi \int_0^5 r(r - \frac{r^2}{5}) \, dr = \frac{2\pi}{12} \cdot 5^3 \approx 5 \text{ bodova}$$

2) TEOREM O DIVERGENCIJI: $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S} = \iiint_K \text{div } \mathbf{F}$

$$\text{div } \mathbf{F} = 3 + 2y + x$$

$$\begin{aligned} \Rightarrow \iint_{\partial K} \mathbf{F} \cdot d\mathbf{S} &= \iiint_K (3 + 2y + x) \, dz \, dy \, dx = 2 \int_0^1 \int_0^3 (3 + 2y + x) \, dy \, dx = \\ &= 2 \int_0^1 \left[3y + \frac{2y^2}{2} + xy \right]_{y=0}^3 \, dx = 2 \int_0^1 (9 + 9 + 3x) \, dx = 2 \left[18x + \frac{3x^2}{2} \right]_{x=0}^1 \\ &= 2 \left(18 + \frac{3}{2} \right) = 39 \end{aligned}$$

5 bodova

$$4) f(x, y, z) = x + z$$

$$C \dots \text{parametrizacija} \begin{cases} x = 2t & x' = 2 \\ y = t^2 & y' = 2t \\ z = \frac{1}{3}t^3 & z' = t^2 \end{cases} \\ t \in [0, 10]$$

$$\int_C f ds = \int_0^{10} \left(2t + \frac{1}{3}t^3\right) \cdot \sqrt{2^2 + (2t)^2 + (t^2)^2} dt$$

$$(*) = \int_0^{10} \left(2t + \frac{1}{3}t^3\right) \cdot \underbrace{\sqrt{4 + 4t^2 + t^4}}_{(1)} dt \quad \underline{15 \text{ bodova}}$$

AKO NE ZNAM EGZAKTNO INTEGRIRATI, KORISTIM TRAPEZNU FORMULU (MOŽE I SIMPSONOVA)

$$f(t) = \left(2t + \frac{t^3}{3}\right) \cdot \sqrt{4 + 4t^2 + t^4}, \quad n = 10, \quad \Delta t = 1, \quad t_0 = 0, \quad t_{10} = 10$$

t	0	1	2	3	4	5	6	7	8	9	10
f(t)	0	7	40	165	528	1395	3192	6545	12320	21663	36040

PO TRAPEZNOJ
FORMULI

$$(*) \approx \left(\frac{0}{2} + 7 + 40 + 165 + 528 + 1395 + 3192 + 6545 + 12320 + 21663 + \frac{36040}{2} \right) \cdot 1$$

$$(*) \approx 63875 \quad \underline{\text{OVDJE 5 bodova}}$$

AKO PRIMIJETIM DA JE $4 + 4t^2 + t^4 = 4 + 2 \cdot 2t^2 + (t^2)^2 = (2 + t^2)^2$

$$\Rightarrow (1) = \sqrt{2 + t^2}$$

$$\Rightarrow (*) = \int_0^{10} \left(2t + \frac{1}{3}t^3\right) \cdot (2 + t^2) dt$$

$$(*) = \int_0^{10} \left(4t + \frac{2}{3}t^3 + 2t^3 + \frac{1}{3}t^5\right) dt = \int_0^{10} \left(4t + \frac{8}{3}t^3 + \frac{1}{3}t^5\right) dt = \left[4 \frac{t^2}{2} + \frac{8}{3} \cdot \frac{t^4}{4} + \frac{1}{3} \cdot \frac{t^6}{6} \right]_0^{10}$$

$$= \frac{2 \cdot 100}{200} + \frac{2}{3} \cdot 10000 + \frac{1}{18} \cdot 1000000 \approx 62422$$

41 OVDJE
5 bodova

$$3) \quad x'''(t) + 3x'(t) = t \quad x'(0) = x''(0) = 0 \quad x(0) = 1$$

$$x'''(t) \Rightarrow s^3 X(s) - s^2 X(0) - s X'(0) - X''(0)$$

$$\Rightarrow s^3 X(s) - s^2 //$$

$$x'(t) \Rightarrow s X(s) - 1 //$$

$$t \Rightarrow \frac{1}{s^2}$$

$$s^3 X(s) - s^2 + 3s X(s) - 3 = \frac{1}{s^2}$$

$$X(s) (s^3 + 3s) = \frac{1}{s^2} + s^2 + 3$$

$$X(s) (s^3 + 3s) = \frac{1 + s^4 + 3s^2}{s^2}$$

$$X(s) = \frac{1 + s^4 + 3s^2}{s^3 (s^2 + 3)}$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+3} \cdot s^3 (s^2+3)$$

$$= A (s^2)(s^2+3) + B (s)(s^2+3) + C (s^2+3)$$

$$+ Ds+E (s^3)$$

$$= A (s^4 + 3s^2) + B (s^3 + 3s) + C (s^2 + 3) + Ds + E (s^3)$$

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$$A s^4 + 3A s^2 + B s^3 + 3B + C s^2 + 3C + D s^4 + E s^3$$

$$s^4 (A + D)$$

$$s^3 (B + E)$$

$$s^2 (-3A + C)$$

s

$$D = \frac{1}{9}$$

$$B = 0$$

$$C = \frac{1}{3}$$

$$E = 0$$

$$3A + \frac{1}{3} = 3$$

$$A = \frac{8}{9}$$

$$A + D = 1$$

$$B + E = 0$$

$$3A + C = 3$$

$$3B = 0$$

$$3C = 1$$

$$X(s) = \frac{8}{9} \cdot \frac{1}{s} + \frac{1}{3} \cdot \frac{1}{s^3} + \frac{1}{9} \frac{s}{s^2+3}$$

$$X(t) = \frac{8}{9} + \frac{1}{3} \cdot \frac{1}{2} t^2 + \frac{1}{9} \cos(\sqrt{3} t)$$

✓

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⑤

$$\iint_X f = \iint_X xy \, dx \, dy$$

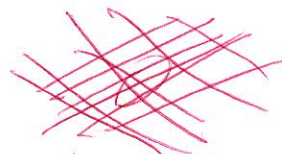
$$(x-1)^2 + y^2 = 3^2$$

$$x^2 = x - 1$$

$$\lambda = x^2 + 1$$

$$x = r \cos \varphi + 1$$

$$y = r \sin \varphi$$



$$\int_0^{2\pi} \int_0^3 [(r \cos \varphi + 1)] \cdot r \, dr \, d\varphi = \quad \times$$

$$= \int_0^{2\pi} \int_0^3 \underbrace{(r \cos \varphi + 1)}_{= x} \underbrace{(r^2 \sin \varphi)}_{y = r \sin \varphi} \, dr \, d\varphi = \quad \checkmark$$

$$= \int_0^{2\pi} \int_0^3 (r^3 \cos \varphi \sin \varphi + r^2 \sin \varphi) \, dr \, d\varphi = \quad \checkmark$$

$$= \int_0^{2\pi} \left(\frac{r^4}{4} \sin \frac{2\varphi}{2} + \frac{r^3}{3} \sin \varphi \right) \Big|_0^3 \, d\varphi =$$

$$= \int_0^{2\pi} \left(\frac{3^4}{4} \sin \frac{2\varphi}{2} + \frac{3^3}{3} \sin \varphi \right) \, d\varphi = \int_0^{2\pi} \left(\frac{81}{4} \sin 2\varphi + 9 \sin \varphi \right) \, d\varphi$$



