MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊

Popunjava nastavnik

Ime i prezime: RJEŠENJE 2 **Broj indeksa:**

Vrijeme početka: **Vrijeme završetka:**

	Broj ↓ bodova
1. Riješiti integrale:	
(a) $\int_0^1 x e^{x^2}\,dx = \left\{ \begin{matrix} u = x^2 \\ du = 2x\,dx \end{matrix} \right\} = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u = \left[\frac{1}{2}e^{x^2}\right]_0^1 = \frac{1}{2}e - \frac{1}{2}$	10
(b) $\int_0^1 \frac{x^2-1}{x^2+2}\,dx = \left\{ \begin{matrix} \text{DIJELJENJE} \\ \text{POLINOMA} \end{matrix} \right\} = \int_0^1 1\,dx - 3\int_0^1 \frac{dx}{x^2+2} = 1 - \frac{3}{\sqrt{2}}\arctan\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}}\arctan\frac{0}{\sqrt{2}} = 1 - \frac{3}{\sqrt{2}}\arctan\frac{1}{\sqrt{2}}$	15
2. Da li je integral $\int_{-1}^{1} \frac{x^2-1}{x^2+2}\,dx$ nepravi i zašto? SEGMENT $[-1,1]$ JE U DOMENI FUNKCIJE $1-\frac{3}{x^2+2}$. ZATO INTEGRAL NIJE NEPRAVI.	5
3. Izračunati površinu lika omeđenog omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$.	20
4. Riješiti diferencijalnu jednadžbu: $y' + y + 3 = x^2 + 2x$. Može li se zadovoljiti početni uvjet $y(0) = 1$? Uvrstiti izračunato rješenje u jednadžbu i provjeriti da li je zadovoljena.	14+2+4
5. Istražiti ekstreme funkcije $f(x,y) = y^3 - 3xy + x^2$.	20
6. Razviti funkciju $f(x) = \ln(2x)$ u Taylorov red po potencijama od $x - 1$. Izračunati barem prva 4 člana.	10
	Ukupno:

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\|\cosh x\|$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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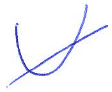
f	$\frac{df}{dx}$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

(3) $\left.\begin{matrix} y = 2x^2+9 \\ y = 9x \end{matrix}\right\}$ SJECIŠTA: $9x = 2x^2+9$, $2x^2-9x+9=0$

$x_{1,2} = \frac{9 \pm \sqrt{81-72}}{4} = \frac{9 \pm 3}{4}$

$x_1 = \frac{3}{2}$, $x_2 = 3$

SKICA:



$P = \int_{\frac{3}{2}}^{3} 9x-(2x^2+9)\,dx = \left[\frac{9}{2}x^2 - \frac{2}{3}x^3 - 9x\right]_{\frac{3}{2}}^{3} = \ldots = \frac{9}{8}$

---

(4) $y' + y + 3 = x^2+2x$

VARIJACIJA KONSTANTI: SEPARIRANA JEDNADŽBA $y'+y=0$

RJEŠENJE: $y = ce^{-x}$

UVRSTIM U JEDNADŽBU: $y(x) = C(x)e^{-x}$

$\Rightarrow C'(x) = (x^2+2x-3)e^x \Rightarrow C(x) = \int (x^2+2x-3)e^x = \left\{\begin{matrix} u = x^2+2x-3 & du = 2x+2 \\ dv = e^x dx & v = e^x \end{matrix}\right\} = (x^2+2x-3)e^x - 2\int (x+1)e^x dx =$

$= \left\{\begin{matrix} u = x+1 & dv = e^x dx \\ du = dx & v = e^x \end{matrix}\right\} = (x^2+2x-3)e^x - 2(x+1)e^x + 2e^x + C$

$= (x^2-3)e^x + C$

$\Rightarrow y(x) = e^{-x}\left[(x^2-3)e^x + C\right] = x^2 - 3 + Ce^{-x}$

$1 = y(0) = -3 + C \Rightarrow C = 4 \quad \Rightarrow y(x) = x^2 - 3 + 4e^{-x}$ ZADOVOLJAVA $y(0)=1$

$y'(x) = 2x - 4e^{-x}$

UVRŠTAVANJE U ODJ:

$2x \cancel{-4e^{-x}} + x^2 \cancel{-3} \cancel{+4e^{-x}} \cancel{+3} = 2x + x^2$

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(5) $f = y^3 - 3xy + x^2$

$\partial_x f = \ldots$

VIDI SKENIRANO RJEŠENJE 1

$T(0,0)$ STD.

$T(\frac{9}{4}, \frac{3}{2})$ MIN.

---

(6) $f(x) = \ln(2x)$ $\quad f(1) = \ln 2$

$f'(x) = \frac{1}{2x}\cdot 2 = \frac{1}{x}$ $\quad f'(1) = 1$

$f''(x) = -\frac{1}{x^2}$ $\quad f''(1) = -1$

$f'''(x) = \frac{2}{x^3}$ $\quad f'''(1) = 2$

TAYLOROV RAZVOJ:

$f(x) \approx \ln 2 + 1\cdot(x-1) - \frac{1}{2}(x-1)^2 + \frac{2}{6}(x-1)^3$

$\frac{9}{2}\left(x^2\Big|_{\frac{3}{2}}^{3}\right) - \frac{2}{3}\left(x^3\Big|_{\frac{3}{2}}^{3}\right) - 9\left(x\Big|_{\frac{3}{2}}^{3}\right)$

$P = \int_{\frac{3}{2}}^{3}$

$= \frac{9}{2}\left(9 - \frac{9}{4}\right) - \frac{2}{3}\left(27 - \frac{27}{8}\right) - 9\left(3 - \frac{3}{2}\right)$

$= \frac{9}{2}\left(\frac{36-9}{4}\right) - \frac{2}{3}\left(\frac{216}{8}\right) - 9\left(\frac{6-3}{2}\right)$

$= \frac{9}{2}\left(\frac{27}{4}\right) - \frac{2}{3}\left(\frac{216}{8}\right) - 9\cdot\frac{3}{2}$

$= \frac{243}{8} - \frac{144}{8} - \frac{27}{2}$

$= \frac{99}{8} - \frac{27}{2} = \frac{99-108}{8} = —$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: DENI MILETIĆ **BROJ INDEKSA:** 57143

VRIJEME POČETKA: 08:25 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

20

1. Riješiti integrale:

   (a) $\int\limits_0^1 xe^{x^2}\,\mathrm{d}x$; 10

   (b) $\int\limits_0^1 \frac{x^2-1}{x^2+2}\,\mathrm{d}x.$ 15

2. Da li je integral $\int\limits_{-1}^{1} \frac{x^2-1}{x^2+2}\,\mathrm{d}x$ nepravi i zašto? 5

3. Izračunati površinu lika omeđenog omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. 20

4. Riješiti diferencijalnu jednadžbu: $y' + y + 3 = x^2 + 2x$. Može li se zadovoljiti početni uvjet $y(0) = 1$? Uvrstiti izračunato rješenje u jednadžbu i provjeriti da li je zadovoljena. 14+2+4

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Ukupno:

Tablica integrala

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$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
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1. a) $\int_0^1 x e^{x^2} dx = \left\{ \begin{array}{l} t = x^2;\ dt = 2x\,dx \\ x\,dx = \frac{dt}{2} \end{array} \right\} = \frac{1}{2}\int_0^1 e^t = \frac{1}{2}\left[e^{x^2}\right]_0^1 = \frac{1}{2} e^2 - \frac{1}{2} e^0 = \frac{e^2}{2} - \frac{1}{2}$

10

b) $\int_0^1 \frac{x^2-1}{x^2+2} = \underbrace{\int_0^1 dx}_{I} - 3\underbrace{\int_0^1 \frac{dx}{x^2+2}}_{II} =$

$\begin{array}{l} x^2 - 1 : x^2+2 = 1 - \frac{3}{x^2+2} \\ \underline{-x^2 - 2} \\ -3 \end{array}$

$I = \int_0^1 dx = [x]_0^1 = 1 - 0 = 1$

$II\ 3\int_0^1 \frac{dx}{x^2+2} = 3\int_0^1 \frac{dx}{x^2+(\sqrt{2})^2} = 3\cdot\left[\frac{1}{2}\arctan\frac{x}{2}\right]_0^1 = 3\cdot\left[\frac{1}{2}\arctan\frac{1}{2} - \frac{1}{2}\cancel{\arctan 0}\right]$

$= 1 - 3\cdot\frac{1}{\sqrt{2}}\arctan\frac{1}{\sqrt{2}}$

3. $\left.\begin{array}{l} y = 2x^2+9 \\ y = 9x \end{array}\right\} 2x^2 + 9 = 9x \Rightarrow 2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{9 \pm \sqrt{81-72}}{4} = \frac{9\pm\sqrt{9}}{4} = \frac{9\pm 3}{4}$

$x_1 = \frac{3}{2}$ $\quad x_2 = 3$

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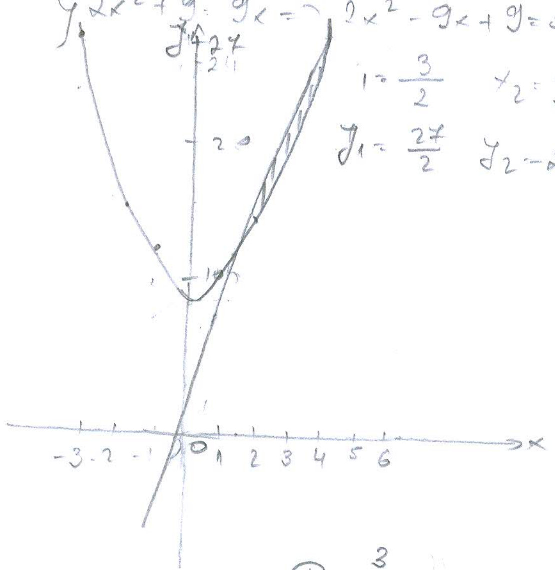

$y = 9x$

x	1	2
y	9	18

$y = 2x^2 + 9$

$y' = 4x \Rightarrow x = 0$

$y = 9$

x	1	2	-1	-2
y	11	15	11	15

$P = \int_{\frac{3}{2}}^{3} 9x - (2x^2+9) = \int_{\frac{3}{2}}^{3} 9x - 2x^2 + 9 = \left[9\frac{x^2}{2} - \cancel{2}\frac{x^2}{\cancel{2}} + x\right]_{\frac{3}{2}}^{3}$

$= \left[9\frac{3^2}{2} - 3^2 + 3\right] - \left[9\cdot\frac{(\frac{3}{2})^2}{2} - (\frac{3}{2})^2 + \frac{3}{2}\right] = \left[\frac{72}{2} - 9 + 3\right] - \left[9\cdot\frac{\frac{9}{4}}{2} - \frac{9}{4} + \frac{3}{2}\right] = \left[\frac{72-12}{2}\right] - \left[\frac{72}{6} - \frac{3}{4}\right]$

$= \frac{\cancel{60}\,30}{\cancel{2}} - \left[\frac{144-9}{12}\right] = 30 - \frac{\cancel{135}\,45}{\cancel{12}\,4} = 30 - \frac{45}{4} = \frac{120-45}{4} = \frac{75}{4}$

IME I PREZIME: DENI MILETIĆ **BROJ INDEKSA:** 57143

4. $y' + y + 3 = x^2 + 2x$

$y_0 = y' + y + 3$

$y_0 = r^2 + r + 3$

$r_{1,2} = \frac{-1 \pm \sqrt{1-12}}{2} = \frac{-1 \pm \sqrt{-11}}{2}$

$r_1 = -\frac{1}{2} - i\frac{\sqrt{-11}}{2} \quad r_2 = -\frac{1}{2} + i\frac{\sqrt{-11}}{2}$

$y_0 = C_1 \cos x\, e^{-\frac{1}{2} - i\frac{\sqrt{-11}}{2}} - C_2 \sin x\, e^{-\frac{1}{2} + i\frac{\sqrt{-11}}{2}} + 1$

$y = y_0 + \eta \Rightarrow 0 + 2$

$\eta = 1$

$\eta = a_2 x^2 + a_1 x + a_0$

$\eta' = 2a_2 x + a_1$

$2a_2 x + a_1 + a_2 x^2 + a_1 x + a_0$

$a_2 = 1$

$2a_2 + a_1 = 2 \Rightarrow 2 + a_1 = 2 \Rightarrow a_1 = 0$

$a_1 + a_0 = 0 \Rightarrow a_0 = 0$

5. $f(x,y) = y^3 - 3xy + x^2$

$\partial_x f = -3y + 2x \Rightarrow -3x + 2x = 0$

$-x = 0$

$x = 0$

$\partial_y f = 3y - 3x \Rightarrow 3x = 3y$

$x = y$

$T(0,0)$

SEDLASTA TOČKA

10

$\partial_{xx} f = 2 > 0$

$\partial_{yy} f = 3$

$\partial_{xy} f = -3$

$\Delta = \begin{bmatrix} 2 & -3 \\ -3 & 3 \end{bmatrix} = 6 - 9 = -3 < 0$

6. $f(x) = \ln(2x) \Rightarrow f(0) = 0$

$f'(x) = \frac{1}{2x} \cdot 2 = \frac{1}{x} \qquad f(1) = 1$

$f''(x) = -\frac{1}{x^2} \qquad f(2) = -\frac{1}{4}$

$f'''(x) = \frac{2}{x^3} \qquad f(3) = -\frac{2}{9}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ **IME I PREZIME:** AUGUSTIN PTIČAR **BROJ INDEKSA:** 17-1-0055-2011 **VRIJEME POČETKA:** 08²⁵ **VRIJEME ZAVRŠETKA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova 15

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$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
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$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

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$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

IME I PREZIME: AUGUSTIN PTIČAR **BROJ INDEKSA:** 17-1-0055-2011

1. a) $\int_0^1 x e^{x^2} dx = \left| \begin{array}{l} x^2 = t \,/' \\ 2x \cdot dx = dt \\ x dx = \frac{dt}{2} \end{array} \right.$

$\int_0^1 \frac{1}{2} e^t dt = \frac{1}{2} \int_0^1 e^t dt = \frac{1}{2} \cdot \left(e^{x^2} \Big|_0^1 \right) = \frac{1}{2} \cdot \left(e^1 - e^0 \right) = \frac{1}{2} \cdot (2.7 - 1)$

$= \frac{1}{2} \cdot 1.7 = \frac{1}{2} \cdot \frac{17}{10} = \frac{17}{20}$ ✓ 1)

b) $\int_0^1 \frac{x^2 - 1}{x^2 + 2} dx$

$(x^2 + 2) (x^2 - 1) = 1 + \frac{3}{x^2 - 1}$

$\underline{-x^2 \mp 1}$

3

$\int_0^1 \left(1 + \frac{3}{x^2 - 1}\right) dx = \int_0^1 1 \cdot dx + 3 \int_0^1 \frac{dx}{x^2 - 1} =$

$= x \Big|_0^1 + 3 \cdot \int_0^1 \frac{dx}{x^2 - 1^2} = 1 + 3 \cdot \left(\frac{1}{2} \ln \left| \frac{1 + x}{1 - x} \right| \right) \Big|_0^1$

$= 1 + 3 \left(-\frac{1}{2} \ln \frac{1 + 0}{1 - 0} \right) = 1 + 3 \cdot \left(-\frac{1}{2} \ln 1 \right) = 1 + 3 \cdot 0 = 1$

3. $y = 2x^2 + 9$

$\underline{y = 9x}$

$9x = 2x^2 + 9$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{9 \pm \sqrt{81 - 72}}{4}$

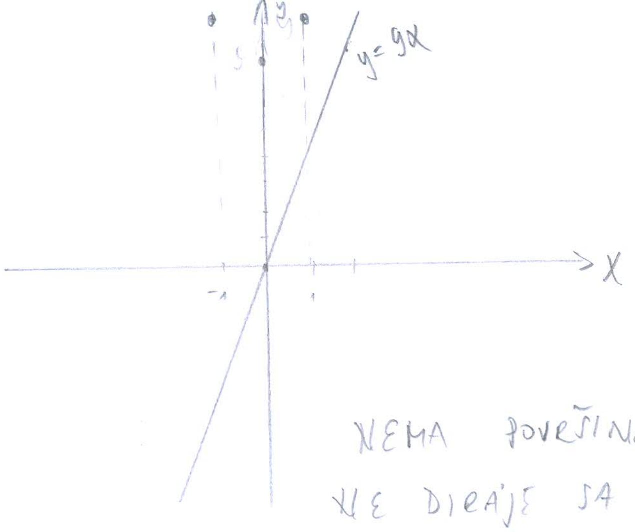
$x_{1,2} = \frac{9 \pm \sqrt{9}}{4}$

$x_{1,2} = \frac{9 \pm 3}{4}$

$x_1 = 3$

$x_2 = \frac{6}{4} = \frac{3}{2}$

→



$x = 0, 1, 2, \ldots$

$y_1 = 0 ; (0,0)$

$y_2 = 1 \cdot 9 = 9 \; (1,9)$

$x = 0, 1, -1, 2, -2$

$y = 9 \; T(0,9)$

$y_1 = 11 \; (1,11)$

$y_2 = 11 \; (-1,11)$

$y_2 = 17 \; (2,17)$

$y_3 = 17 \; (-2,17)$

NEMA POVRŠINE

NE DIRAJE SA PRAVAC

I PARABOLA

5. $f(x,y) = y^3 - 3xy + x^2$

$\frac{df}{dx} = -3y + 2x$

$\frac{df}{dy} = 3y^2 - 3x$

$\frac{d^2f}{dx^2} = 2$

$\frac{d^2f}{dy^2} = 6y$

$\frac{d^2f}{dydx} = -3$

$\frac{d^2f}{dx\,dy} = -3$

$$\begin{vmatrix} 2 & -3 \\ -3 & 6y \end{vmatrix}$$

$-3y + 2x = 0 \;/ \cdot (+3)$

$3y^2 - 3x = 0 \;/ \cdot (2)$

$-9y + \cancel{6x} = 0$

$6y^2 - \cancel{6x} = 0$

$-9y + 6y^2 = 0$

$y(6y - 9) = 0$

$y = 0$

$y = \frac{9}{6} = \frac{3}{2}$

?

$$\begin{vmatrix} 2 & -3 \\ -3 & 0 \end{vmatrix} \quad \text{ili} \quad \begin{vmatrix} 2 & -3 \\ -3 & 9 \end{vmatrix}$$

$\downarrow$

$0 - (-3 \cdot -3)$

$= -9 ; \det A_1 = -9$

$\det A_2 = 18 - (9)$

$\det A_2 = 9$

IME I PREZIME: AUGUSTIN PTIČAR **BROJ INDEKSA:** 17-1-0055-2011

~~4. $y'+y+3=x^2+2x$~~

2. $\int_{-1}^{1} \frac{x^2-1}{x^2+2}\,dx =$ | $(x^2+2):(x^2-1) = 1+\frac{3}{x^2-1}$

$$\int_{-1}^{1}\left(1+\frac{3}{x^2-1}\right)dx = \int_{-1}^{1} dx + 3\int_{-1}^{1}\frac{dx}{x^2-1} = x\Big|_{-1}^{1} + 3\left(\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|\right)\Big|_{-1}^{1}$$

$= 2 + 3\left(-\frac{1}{2}\ln 0\right) \Rightarrow$ NEPRAVI INTEGRAL

$\Rightarrow$ jer domena funkcije ln je $x>0$, a u ovom slučaju nam ispada 0 pa stoga ne možemo dobiti određeno rješenje

5

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ **IME I PREZIME:** Mateja Pečarić **BROJ INDEKSA:** 17-0032-2010 **VRIJEME POČETKA:** 8:00 **VRIJEME ZAVRŠETKA:** 8:55	POPUNJAVA NASTAVNIK Broj ↓ bodova 0

1. Riješiti integrale:

   (a) $\int\limits_0^1 x\mathrm{e}^{x^2}\,\mathrm{d}x$; — 10

   (b) $\int\limits_0^1 \frac{x^2-1}{x^2+2}\,\mathrm{d}x.$ — 15

2. Da li je integral $\int\limits_{-1}^1 \frac{x^2-1}{x^2+2}\,\mathrm{d}x$ nepravi i zašto? — 5

3. Izračunati površinu lika omeđenog omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. — 20

4. Riješiti diferencijalnu jednadžbu: $y' + y + 3 = x^2 + 2x$. Može li se zadovoljiti početni uvjet $y(0) = 1$? Uvrstiti izračunato rješenje u jednadžbu i provjeriti da li je zadovoljena. — 14+2+4

5. Istražiti ekstreme funkcije $f(x, y) = y^3 - 3xy + x^2$. — 20

6. Razviti funkciju $f(x) = \ln(2x)$ u Taylorov red po potencijama od $x - 1$. Izračunati barem prva 4 člana. — 10

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

5. $f(x) = y^3 - 3xy + x^2$

$\frac{df}{dx} = 0 - 3(x' \cdot y) + (x \cdot y') + 2x$

$= -3y + 2x$

$= 2x - 3y$

$\frac{df}{dy} = 3y^2 - 3(x' \cdot y) + (x \cdot y') + 0$

$= 3y^2 + x$

$= x + 3y^2$

$2x - 3y = 0$

$x + 3y^2 = 0$

$2x = 3y$

$x = \frac{3}{2}y$

$\Longrightarrow \frac{3}{2}y + 3y^2 = 0$

$y\left(\frac{3}{2} + 3y\right) = 0$

$\boxed{y = 0}$

$\frac{3}{2} + 3y = 0$

$3y = -\frac{3}{2} \quad / \cdot 2$

$6y = -3$

$y = -\frac{3}{6}$

$\boxed{y = -\frac{1}{3}}$

$\rightarrow 2x - 3 \cdot 0 = 0$

$2x = 0$

$x = 0$

$S_1(0,0)$

$2x - 3 \cdot \left(-\frac{1}{3}\right) = 0$

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

$S_2\left(-\frac{1}{2}, -\frac{1}{3}\right)$

$2x - 3y$

$r = 2$

$s = -3$

$x + 3y^2$

$t = 1$

$S_1(0,0)$

$r \cdot t - s^2 = 2 - (-3)^2 =$

$= 2 - 9$

< 0

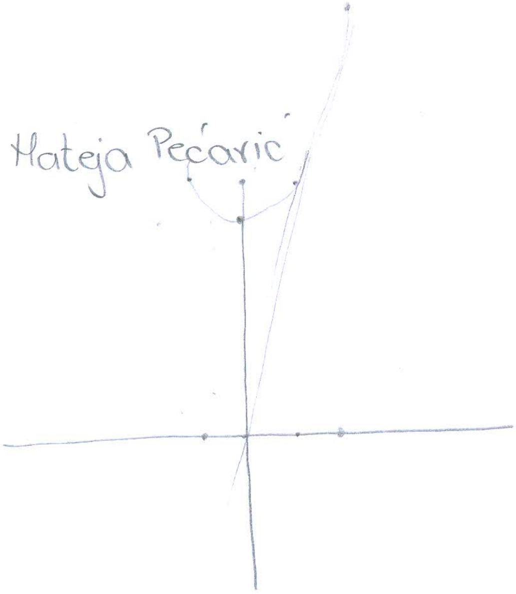
lokalni maximum

$S_2\left(-\frac{1}{2}, -\frac{1}{3}\right)$

$r t - s^2 =$

Ime i prezime: Mateja Pećarić **Broj indeksa:** 17-0032-2010

3. $y=2x^2+9$
$y=9x$



$x=-1$	$y=11$	$y=-9$
$x=0$	$y=9$	$y=0$
$x=1$	$y=11$	$y=9$
$x=2$	$y_1=17$	

$2x^2+9=9x$

$2x^2-9x+9=0$

$x_{1,2}=\frac{-9\pm\sqrt{81-4\cdot2\cdot9}}{4}$

$=\frac{-9\pm\sqrt{81-72}}{4}$

$x_1=\frac{-9+3}{4}$ $x_2=\frac{-9-3}{4}$

$x_1=-\frac{6}{4}$ $=-\frac{12}{4}$

$=-\frac{3}{2}$ $=-3$

$y=2\cdot(-1)^2+9$

$y=2+9=11$

$y=2\cdot0+9$

$y=9$

$P=\int_{-3}^{-\frac{3}{2}}(9x-(2x^2+9)dx$

$P=\int_{-3}^{-\frac{3}{2}}9x-2x^2-9$

$P=\left|9\frac{x^2}{2}-4\frac{x^3}{3}-9x\right|_{-3}^{-\frac{3}{2}}$

$P=\left|9\frac{\left(-\frac{3}{2}\right)^2}{2}-4\frac{\left(-\frac{3}{2}\right)^3}{3}-9\cdot\left(-\frac{3}{2}\right)\right)-\left(9\frac{(-3^2)}{2}-4\frac{(-3)^3}{3}-9\cdot(-3)\right.$

$=\left(9\frac{\frac{9}{4}}{2}-4\frac{\left(-\frac{27}{8}\right)}{3}+\frac{27}{2}\right)-\left(9\frac{9}{2}+4\frac{27}{3}+27\right)$

$=\left(9\frac{9}{8}+4\frac{27}{24}+\frac{27}{2}\right)-\left(9\frac{9}{2}+4\frac{27}{3}+27\right)$

$=\left(\frac{36+31+39}{24}\right)-\left(\frac{36+35+162}{6}\right)$

$=$

1.

a) $\int_0^1 x e^{x^2} dx = \left\{ \begin{matrix} x = u & e^{x^2} dx = du \\ dx = du & e^{x^2} = v \end{matrix} \right\}$

$u \cdot v - \int v du$

$x \cdot e^{x^2} - \int e^{x^2} \cdot dx = \left\{ \begin{matrix} x^2 = u & e dx = du \\ 2x dx = du & e = v \end{matrix} \right\}$

$= x^2 \cdot e - \int e \cdot 2x dx$

$= x^2 \cdot e - 2 \int e x dx$

4. $y' + y + 3 = x^2 + 2x \ / dx$

$\frac{y}{dx} + y dx + 3 dx = x^2 dx + 2x dx$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: MARKO TKALČEC **BROJ INDEKSA:** 56188-2008 0269024538

VRIJEME POČETKA: 08 20 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

0

1. Riješiti integrale:

   (a) $\displaystyle\int_0^1 xe^{x^2}\,\mathrm{d}x$; 10

   (b) $\displaystyle\int_0^1 \frac{x^2-1}{x^2+2}\,\mathrm{d}x.$ 15

2. Da li je integral $\displaystyle\int_{-1}^1 \frac{x^2-1}{x^2+2}\,\mathrm{d}x$ nepravi i zašto? 5

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4. Riješiti diferencijalnu jednadžbu: $y' + y + 3 = x^2 + 2x$. Može li se zadovoljiti početni uvjet $y(0) = 1$? Uvrstiti izračunato rješenje u jednadžbu i provjeriti da li je zadovoljena. 14+2+4

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$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ANTONIJA SEKULA **BROJ INDEKSA:** 56190	0
VRIJEME POČETKA: 8^{00} **VRIJEME ZAVRŠETKA:** 8^{05}	

1. Riješiti integrale:

   (a) $\displaystyle\int_0^1 xe^{x^2}\,\mathrm{d}x$; — 10

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$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

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$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
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MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: Kristina Dragić **BROJ INDEKSA:** 56179

VRIJEME POČETKA: 8:00 h **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

Ø

1. Riješiti integrale:

   (a) $\int\limits_0^1 x e^{x^2}\,\mathrm{d}x$; 10

   (b) $\int\limits_0^1 \frac{x^2-1}{x^2+2}\,\mathrm{d}x.$ 15

2. Da li je integral $\int\limits_{-1}^{1} \frac{x^2-1}{x^2+2}\,\mathrm{d}x$ nepravi i zašto? 5

3. Izračunati površinu lika omeđenog omeđenog parabolom $y = 2x^2+9$ i pravcem $y = 9x$. 20

4. Riješiti diferencijalnu jednadžbu: $y' + y + 3 = x^2 + 2x$. Može li se zadovoljiti početni uvjet $y(0) = 1$? Uvrstiti izračunato rješenje u jednadžbu i provjeriti da li je zadovoljena. 14+2+4

5. Istražiti ekstreme funkcije $f(x,y) = y^3 - 3xy + x^2$. 20

6. Razviti funkciju $f(x) = \ln(2x)$ u Taylorov red po potencijama od $x-1$. Izračunati barem prva 4 člana. 10

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

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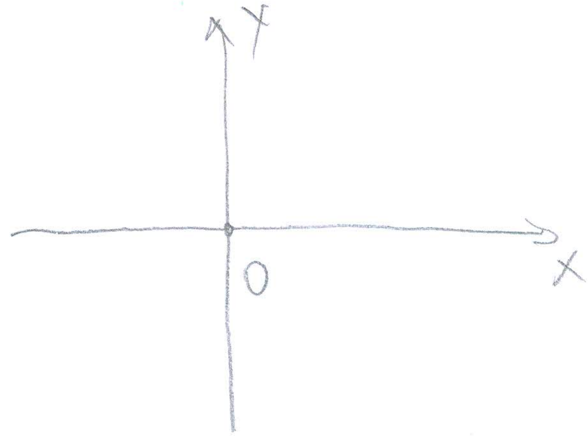
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

3. $y = 2x^2 + 3$

$y = 3x$

$2x^2 + 3 = 3x$

$2x^2 + 3 - 3x = 0$



MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: IVAN STOJANOV **BROJ INDEKSA:** 0269031670

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

$\emptyset$

1. Riješiti integrale:

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$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

$\int_0 x e^{x} dx$

$\int = \frac{...}{75} = \frac{76}{75}$

$\int x e^{x^2} dx = \left\{ \begin{matrix} u = x & dv = e^{x^2} \\ du = dx & v = e^{\frac{x^3}{3}} \end{matrix} \right\} = x \cdot e^{\frac{x^3}{3}} - \int e^{\frac{x^3}{3}} \cdot du$

~~$\int e^{x^2} dx = \left\{ \begin{matrix} u = e & dv = x^2 \\ du = e dx & v = \frac{x^3}{3} \end{matrix} \right\}$~~ $= x \cdot e^{\frac{x^3}{3}} - \frac{1}{3} \int e^{x^3} du$

$= x \cdot e^{\frac{x^3}{3}} - \frac{1}{3} \cdot e^{\frac{x^5}{5}} + C$

$\int_0^1 x e^{x^2} dx = \left(x \cdot e^{\frac{x^3}{3}} - \frac{1}{3} \cdot e^{\frac{x^5}{5}} \right) \Big|_0^1 = \left[1 \cdot e^{\frac{1^3}{3}} - \frac{1}{3} \cdot e^{\frac{1^5}{5}} \right] - \left[0 \cdot e^{\frac{0^3}{3}} - \frac{1}{3} \cdot e^{\frac{0^5}{5}} \right]$

$= \left[e^{\frac{1}{3}} - \frac{1}{3} \cdot e^{\frac{1}{5}} \right] - \left[-\frac{1}{3} \cdot e^0 \right]$

$= \left[\frac{1}{3} \cdot e - \frac{1}{3} \cdot \frac{1}{4} \cdot e \right] - \left[-\frac{1}{3} \cdot 1 \right]$

$= \left[\frac{1}{3} \cdot \frac{272}{100} - \frac{1}{12} \cdot \frac{272}{100} \right] + \frac{1}{3}$

$= \frac{272}{300} - \frac{272}{1200} + \frac{1}{3}$

$= \frac{68}{75} - \frac{68}{300} + \frac{1}{3} = \frac{68}{75} - \frac{17}{75} + \frac{1}{3} = \frac{51}{75} + \frac{1}{3} =$

b) $\int_0^1 \frac{x^2 - 1}{x^2 + 2} dx$

~~$x^2 + 2 = ($~~

$\int \frac{x^2 - 1}{x^2 + 2} dx = \cancel{\int x(x-}$

$x^2 + 2 = \cancel{(x-1)(x-2) = x^2 - 2x - x + 2}$

$\cancel{= (x+1)(x+2) = x^2 + 2x + x + 2}$

$\cancel{= (x+1)(x+1) = x^2 + x + x + 1}$

$\cancel{= (x+2)}$

$\cancel{= (x+1)^2 = x^2 + 2x + 1}$

$\cancel{= (x+2)(x-1) = x^2 - x + 2x - 2}$

$\neq$

$x^2 - 1 : (x^2 + 2) = x - \frac{1}{2}$

$\frac{-x^2 + 2}{-3}$

$\int \frac{x^2 - 1}{x^2 + 2} dx = \int (x - \frac{1}{2}) dx - \int \frac{3}{x^2 + 2} dx$

$= \frac{x^2}{2} - \frac{1}{2} x - 3 \int \frac{dx}{x^2 + 2}$

$\cancel{= \frac{x^2}{2} - \frac{1}{2} x - 3 \cdot \left(\frac{1}{\sqrt{2}} \arctan \right.}$

$\cancel{= \frac{x^2}{2} - \frac{1}{2} x - 3 \cdot \operatorname{arccot}(x+2) + C}$

$\int_0^1 \frac{x^2 - 1}{x^2 + 2} dx = \left(\frac{x^2}{2} - \frac{1}{2} x - 3 \cdot \operatorname{arccot}(x+2) \right) \Big|_0^1$

$= \left[\frac{1^2}{2} - \frac{1}{2} \cdot 1 - 3 \cdot \operatorname{arccot}(1+2) \right] -$

$\left[\frac{0^2}{2} - \frac{1}{2} \cdot 0 - 3 \cdot \operatorname{arccot}(0+2) \right.$

$= \left[\frac{1}{2} - \frac{1}{2} - 3 \cdot \operatorname{arccot}(3) \right]$

$- \left[-3 \cdot \operatorname{arccot}(2) \right]$

$\cancel{\int \frac{dx}{x^2 + 2} = \left\{ x + 2 = t \right.}$

$= -3 \cdot \operatorname{arccot}(3)$

$+ 3 \cdot \operatorname{arccot}(2)$

$= -3 \cdot 71{,}565 + 3 \cdot 63{,}435$

$=$

NASTAVAK NA DRUGOM PAPIRU

Ime i prezime: IVAN STOJANOV **Broj indeksa:** 0269031670

$$\int_0^1 \frac{x^2-1}{x^2+2} = \left(\frac{x^2}{2}-\frac{1}{2}x-3\cdot\frac{1}{\sqrt2}\arctan\frac{x}{\sqrt2}\right)\Bigg|_0^1$$

$$=\left(\frac{1^2}{2}-\frac{1}{2}\cdot1-3\cdot\frac{1}{\sqrt2}\arctan\frac{1}{\sqrt2}\right)-\left(\frac{0^2}{2}-\frac{1}{2}\cdot0-3\cdot\frac{1}{\sqrt2}\arctan\frac{0}{\sqrt2}\right)$$

$$=\left(\frac12-\frac12-3\cdot\frac{1}{\sqrt2}\arctan\frac{1}{\sqrt2}\right)-\left(-3\cdot\frac{1}{\sqrt2}\arctan0\right)$$

$$=-\frac{3}{\sqrt2}\arctan\frac{1}{\sqrt2}+3\cdot\frac{1}{\sqrt2}\arctan0$$

$$=2{,}122\arctan0{,}707+2{,}122\cdot0$$

$$=2{,}122\cdot0{,}88$$

$$=1{,}86736\approx1{,}88$$

~~2. DA~~

3. $y=2x^2+9$

$y=9x$

$\cancel{2x^2+9=9x}$

$\cancel{2x^2+9x-9=}$

$\cancel{2x^2-9x=-9}$

$\cancel{2x}$

$$2x^2+9=9x$$

$$2x^2-9x+9=0$$

$$x_{1,2}=\frac{9\pm\sqrt{81-4\cdot2\cdot9}}{2\cdot2}$$

$$x_{1,2}=\frac{9\pm\sqrt{81-36}}{4}$$

$$x_{1,2}=\frac{9\pm\sqrt{45}}{4}$$

$$x_1=\frac{9+\sqrt{45}}{4}\quad x_2=\frac{9-\sqrt{45}}{4}$$

$$P=\int_{\frac{9-\sqrt{45}}{4}}^{\frac{9+\sqrt{45}}{4}}\left((2x^2+9)-9x\right)dx=\int(2x^2-9x+9)\,dx=\frac{2x^3}{3}-\frac{9x^2}{2}+9x+C$$

$$P=\left(\frac{2x^3}{3}-\frac{9x^2}{2}+9x\right)\Bigg|_{\frac{9-\sqrt{45}}{4}}^{\frac{9+\sqrt{45}}{4}} \cancel{=\left[\frac{2\cdot3{,}9^2}{3}\right]=\frac{2}{3}\cdot(3{,}93)^3-9}$$

3.) NASTAVAK $P = \left[\frac{2}{3}\cdot(3.93)^3 - \frac{9}{2}(3.93)^2 + 9\cdot 3.93\right]$

$$- \left[\frac{2}{3}\cdot(0.57)^3 - \frac{9}{2}\cdot(0.57)^2 + 9\cdot 0.57\right]$$

$$P = \left[0.667\cdot 60.699 - 4.5\cdot 15.445 + 35.37\right]$$

$$- \left[0.667\cdot 0.185 - 4.5\cdot 0.325 + 5.13\right]$$

$$P = \left[40.486 - 69.503 + 35.37\right]$$

$$- \left[0.123 - 1.463 + 5.13\right]$$

$$P = 6.353 - 3.79 = 2.563$$

~~2.) DA JER DOBIJEMO ZA REZULTAT NULU.~~

4.) $y' + y + 3 = x^2 + 2x$

$$\lambda^2 + \lambda + 3 = 0$$

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1 - 4\cdot 1\cdot 3}}{2}$$

$$y_H = C_1$$