MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊	POPUNJAVA NASTAVNIK
IME I PREZIME: RJEŠENJE 1 **BROJ INDEKSA:**	Broj ↓ bodova
VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**	

$\int u\,dv = uv - \int v\,du$

1. Riješiti integrale:

(a) $\displaystyle\int_0^{\pi} x^2 \sin(x)\,dx = \left\{ \begin{matrix} u = x^2 & dv = \sin x\,dx \\ du = 2x\,dx & v = -\cos x \end{matrix} \right\} = -x^2\cos x + 2\int x\cos x\,dx \left\{ \begin{matrix} u = x & dv = \cos x\,dx \\ du = dx & v = \sin x \end{matrix} \right\}$ 10

$= \left[-x^2\cos x + 2x\sin x + \cos x\right]_0^{\pi} = \pi^2 - 1 - 1 = \pi^2 - 2$

(b) $\displaystyle\int_2^3 \frac{x^2+2x+2}{x^2+x-2}\,dx.$ 15

2. Da li je integral $\displaystyle\int_{-1}^{1} \frac{x^2+2x+2}{x^2+x-2}\,dx$ nepravi i zašto? 5

3. Izračunati površinu lika omeđenog krivuljama: $y = 2x^2+1$ i $y = 3-x$. 20

4. Riješiti zadanu diferencijalnu jednadžbu uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. Uvrstiti rješenje u jednadžbu i provjeriti zadovoljenje jednakosti. 15+5

$$y'' + 2y' + y = 0$$

5. Istražiti ekstreme funkcije $f(x,y) = y^3 - 3xy + x^2$. 20

6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x) = 2x\cos x$ oko točke $x_0 = \dfrac{\pi}{2}$. 10

Ukupno:

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\vert\frac{a+x}{a-x}\right\vert + C$
$\int \frac{dx}{x} = \ln\vert x\vert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\vert\frac{x-a}{x+a}\right\vert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\vert x+\sqrt{x^2\pm a^2}\right\vert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\vert\cosh x\vert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\vert\sinh x\vert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\vert\cos x\vert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\vert\sin x\vert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

f	$\frac{df}{dx}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
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$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

~~(1b)~~ $\int \frac{x^2-1}{x^2+1} = (\cancel{x^2}-1):(x^2+1) = 1 - \frac{2}{x^2+1}$

$\frac{\cancel{x^2} \pm 1}{-2}$

$\int_0^1 \frac{x^2-1}{x^2+1}dx = \int_0^1 1\,dx - 2\int_0^1 \frac{dx}{x^2+1} = \left[x - 2\arctan x\right]_0^1$

$= 1 - 2\arctan 1 + 2\underbrace{\arctan 0}_{=0}$

$= 1 - 2\arctan 1$

(1b) $(\cancel{x^2}+2x+2):(x^2+x-2) = 1 + \frac{x+4}{(x-1)(x+2)}$

$\frac{\cancel{x^2} \pm x \mp 2}{x+4}$

$\int_2^3 \frac{x^2+2x+2}{x^2+x-2}dx = \int 1 + \frac{x+4}{(x-1)(x+2)}dx = \int 1 + \frac{5}{3}\frac{1}{x-1} - \frac{2}{3}\frac{1}{x+2}\,dx$

$= \left[x + \frac{5}{3}\ln|x-1| - \frac{2}{3}\ln|x+2|\right]_2^3$

$\frac{x+4}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \Rightarrow x+4 = A(x+2) + B(x-1) \Rightarrow \begin{cases} A = \frac{5}{3} \\ B = -\frac{2}{3} \end{cases}$

$= 3 + \frac{5}{3}\ln 2 - \frac{2}{3}\ln 5 - 2 - \frac{5}{3}\overset{0}{\cancel{\ln 1}} + \frac{2}{3}\ln 4$

$= 1 + \frac{5}{3}\ln 2 - \frac{2}{3}\ln 5 + \frac{2}{3}\ln 4$

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(2) ~~...~~ JEST NEPRAVI JER TOČKA $1 \in [-1,1]$ NIJE U DOMENI FUNKCIJE $1 + \frac{x+4}{(x-1)(x+2)}$ DAKLE DIO RAVNINE (NA GRAFU F-JE) IZMEĐU $[-1,1]$ JE NEOGRANIČEN, STOGA JE INTEGRAL NEPRAVI.

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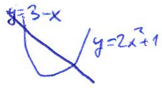
(3) $\begin{cases} y = 2x^2+1 \\ y = 3-x \end{cases}$ SJECIŠTA $\Rightarrow 2x^2+1 = 3-x \Rightarrow 2x^2+x-2=0$

$x_{1,2} = \frac{-1 \pm \sqrt{1+16}}{4} = \frac{-1\pm\sqrt{17}}{4}$

SKICA:



$P = \int_{\frac{-1-\sqrt{17}}{4}}^{\frac{-1+\sqrt{17}}{4}} (3-x)-(2x^2+1) = \int_{\frac{-1-\sqrt{17}}{4}}^{\frac{-1+\sqrt{17}}{4}} -2x^2 - x + 4\,dx$

$P = \left[-2\frac{x^3}{3} - \frac{x^2}{2} + 4x\right]_{\frac{-1-\sqrt{17}}{4}}^{\frac{-1+\sqrt{17}}{4}} = \ldots =$

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(4) $y'' + 2y' + y = 0$

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda_{1,2} = -1$

$y(x) = (A+Bx)e^{-x}$

$y(0) = 1 \Rightarrow Ae^0 = 1 \Rightarrow A = 1$

$y(x) = e^{-x} + Bxe^{-x}$

$y'(x) = -e^{-x} - Bxe^{-x} + Be^{-x}$

$0 = y'(0) = -1 + B \Rightarrow B = 1$

$\Rightarrow y(x) = (1+x)e^{-x}$, $y'(x) = -xe^{-x}$, $y''(x) = xe^{-x} - e^{-x}$

UVRŠTAVANJE: $xe^{-x} \cancel{-e^{-x}} - 2xe^{-x} \cancel{+e^{-x}} + xe^{-x} = 0$

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(5) $y^3 - 3xy + x^2 \Rightarrow \begin{cases} \partial_x f = -3y + 2x = 0 \\ \partial_y f = 3y^2 - 3x = 0 \end{cases} \Rightarrow 6y^2 - 9y = 0 \Rightarrow (2y-3)y = 0 \Rightarrow \begin{cases} y = 0,\ x = 0 \\ y = \frac{3}{2},\ x = \frac{9}{4} \end{cases}$

$\partial_{xx} f = 2$ $\quad \partial_{yy} f = 6y$

$\partial_{xy} f = \partial_{yx} f = -3$

$T_1(0,0)$, $A = 2$, $\Delta = \begin{vmatrix} 2 & -3 \\ -3 & 0 \end{vmatrix} = -9$ SEDLASTA TOČKA, $T_2\left(\frac{9}{4}, \frac{3}{2}\right)$, $A = 2$, $\Delta = \begin{vmatrix} 2 & -3 \\ -3 & 9 \end{vmatrix} = 9$ LOKALNI MINIMUM

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(6) $f(x) = 2x\cos x$

$f'(x) = 2\cos x - 2x\sin x$

$f''(x) = -2\sin x - 2\sin x - 2x\cos x$
$= -4\sin x - 2x\cos x$

$f'''(x) = -4\cos x - 2\cos x + 2x\sin x$
$= -6\cos x + 2x\sin x$

$f\left(\frac{\pi}{2}\right) = 0$

$f'\left(\frac{\pi}{2}\right) = -2\cdot\frac{\pi}{2} = -\pi$

$f''\left(\frac{\pi}{2}\right) = -4$

$f'''\left(\frac{\pi}{2}\right) = 2\cdot\frac{\pi}{2} = \pi$

TAYLOROV RAZVOJ:

$f(x) \approx \underset{0}{f\left(\frac{\pi}{2}\right)} + \underset{-\pi}{f'\left(\frac{\pi}{2}\right)}\left(x-\frac{\pi}{2}\right) + \underset{-4}{f''\left(\frac{\pi}{2}\right)}\frac{\left(x-\frac{\pi}{2}\right)^2}{2} + \underset{\pi}{f'''\left(\frac{\pi}{2}\right)}\frac{\left(x-\frac{\pi}{2}\right)^3}{6}$

$f(x) \approx -\pi\left(x-\frac{\pi}{2}\right) - 2\left(x-\frac{\pi}{2}\right)^2 + \frac{\pi}{6}\left(x-\frac{\pi}{2}\right)^3$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊

Ime i prezime: FRANO ŽIVKOVIĆ **Broj indeksa:** 54958-2007

Vrijeme početka: 08:40 **Vrijeme završetka:**

Popunjava nastavnik
Broj ↓ bodova

0

1. Riješiti integrale:

(a) $\displaystyle\int_0^{\pi} x^2 \sin(x)\, \mathrm{d}x$; 10

(b) $\displaystyle\int_2^{3} \frac{x^2+2x+2}{x^2+x-2}\, \mathrm{d}x.$ 15

2. Da li je integral $\displaystyle\int_{-1}^{1} \frac{x^2+2x+2}{x^2+x-2}\, \mathrm{d}x$ nepravi i zašto? 5

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$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
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⑤ $f(x,y) = y^3 - 3xy + x^2$

$z_x = -3y + 2x = 0$

$z_y = 3y^2 - 3x = 0$

$z_{xx} = 2 = A$

$z_{xy} = -3 = B$

$z_{yy} = 6 = C$

$A \cdot C - B^2$

$2 \cdot 6 - (-3)^2$

$12 - 6 = 6 > 0$

$$\begin{aligned} -3y + 2x &= 0 \\ 3y^2 - 3x &= 0 \\ \hline y - 1x &= 0 \end{aligned}$$

$y - x = 0$

$T(0,0)$

$y = 0,\ x = 0$

nema stacionarnih točki

1) $\int_0^{\pi} x^2 \sin(x)\,dx = \left| \begin{array}{l} x^2 = t \\ 2x\,dx = dt \ /:2 \\ x\,dx = \frac{1}{2}\,dt \end{array} \right| = \int_0^{\pi} t \sin \frac{1}{2}\,dt = \frac{1}{2} \int_0^{\pi} \sin t\,dt =$

$= \frac{1}{2}(-\cos)t = \frac{1}{2}(-\cos x^2) =$

$= \frac{1}{2}(-\cos \pi) - \frac{1}{2}(-\cos 0) =$

$= \frac{1}{2}(-\cos \pi) - \frac{1}{2}(-1) = \frac{1}{2}(-\cos \pi) + \frac{2}{2} = \frac{1}{2}(-\cos \pi)$ //

Ime i prezime: FRANO ŽIVKOVIĆ **Broj indeksa:**

1) b) $\int_2^3 \frac{x^2+2x+2}{x^2+x-2}\,dx = \int \frac{A}{\quad}\,dx + \int \frac{Bx+C}{\quad}\,dx =$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{1 \pm \sqrt{1-8}}{2}$

$= \cancel{\frac{1 \pm \sqrt{-7}}{2}}$

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IME I PREZIME: LUKA STIPIC **BROJ INDEKSA:** 17-2-0083-2011

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

0

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$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

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$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

Ime i prezime: Luka Stipić **Broj indeksa:**

2. DA! IZ RAZLOGA ŠTO JE NAJVEĆA POTENCIJA U BROJNIKU JEDNAKA NAJVEĆOJ POTENCIJI U ~~...~~ NAZIVNIKU

1. $\int_0^{\pi} x^2 \sin x \, dx$

$\int x^2 \sin x \, dx$

$u = x^2 / d$ $\quad v = \sin x \, dx$

$du = 2x\,dx$ $\quad dv = \int \sin x \, dx$

$dv = -\cos x$

$u \cdot v - \int v \cdot du$

$x^2 \cdot \sin x - \int \sin x \cdot 2x\,dx$

$x^2 \sin x - \int 2x \sin x \, dx$

$\int 2x \sin x$

$u = 2x$ $\quad v = \sin x$

$du = 2\,dx$ $\quad d = \int \sin x \, dx$

$dv = -\cos x \, dx$

$2x \cdot \sin x - \int \sin x \, 2dx$

$2x \sin x - 2 \int \sin x \, dx$

$2x \sin x + 2\cos x$

$= \left(x^2 \sin x - (2x \sin x + 2\cos x) \right) \Big|_0^{\pi}$

$= \pi^2 \sin \pi - 2\pi \sin \pi - 2\cos \pi - (0^2 - \sin 0 - 2 \cdot 0 \sin 0$

$+ 2\cos 0) = 5{,}47$

$\approx 1.14.$

$x^2+2x+2 : (x^2+x-2) = 1$
$-x^2 \mp x \pm 2$

$x+4$

$=2$

J_2

x^2+x-2

$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$

$x_1 = \frac{-1+3}{2} = \frac{2}{2} = 1$

$x_2 = \frac{-1-3}{2} = \frac{-4}{2} = -2$

$-2 = \int \frac{x+4}{(x-1)(x+2)} = \int \frac{A}{(x-1)} + \frac{B}{(x+2)}$

$\frac{A}{(x-1)} + \frac{B}{(x+2)} \Big| (x-1)\cdot(x+2)$

$A(x+2) + B(x-1)$

$Ax + 2A + Bx - B$

$A + B = 1 \Rightarrow A = 1 - B \Rightarrow A = 1+1 = 2$

$2A - B = 4 \Rightarrow 2\cdot(1-B) - B = 4$

$2 - B - B = 4$

$-2B = 4 - 2$

$-2B = 2 \,|\, : -2$

$B = -1$

$J_2 = 2\int \frac{dx}{x-1} - 1\int \frac{B}{x+2}$

$J_2 = 2 \ln|x-1| - 1 \ln|x+2|$

$J_2 = 2\left(\ln|x-1| - 1\ln|x+2|\right)\Big|_2^3$

$\left(x + 2\ln|x-1| - 1\ln|x+2|\right) - \left(2 + 2\ln|2-1| - 1\ln|2+2|\right)$

$\left(3 - 2\ln|3-1| - 1\ln|3+2|\right)$

$= -0{,}6094$

IME I PREZIME: LUKA ŠTIPIĆ **BROJ INDEKSA:**

5. $f(x,y) = y^3 - 3xy + x^2$

$z_x = 0 - 3\cdot(1\cdot y + 0\cdot x) + 2x = -3y + 2x \Rightarrow z_{xx} = 2 > 0$

$z_y = 3y^2 - 3\cdot(0\cdot y + 1\cdot x) + 0 = 3y^2 - 3x \Rightarrow \cancel{z_{xy}}\ z_{yy} = 6y$

$z_{xy} = -3$

$z_{yx} = -3$

NUŽAN UVJET

$z_x = -3y + 2x = 0 \Rightarrow$

$z_y = 3y^2 - 3x = 0$

$2x = 3y$

$x = \frac{3}{2}y \Rightarrow \frac{3}{2}\cdot\frac{1}{2} = \frac{3}{4}$

$3y^2 - 3\cdot\left(\frac{3}{2}y\right) = 0$

$3y^2 - \frac{9}{2}y = 0$

$y\cdot\left(3 - \frac{9}{2}y\right) = 0$

$y_1 = 0$

$3 - \frac{9}{2}y = 0$

$-\frac{9}{2}y = -3 \quad / \cdot\left(-\frac{2}{9}\right)$

$y = \frac{1}{2}$

$T\left(\frac{3}{4}, \frac{1}{2}\right)$

DOVOLJAN UVJET

$$\begin{vmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{vmatrix} > 0$$

$$\begin{vmatrix} 2 & -3 \\ -3 & 6y \end{vmatrix}$$

$= 9 - 2\cdot 6\cdot\left(\frac{1}{2}\right)$

$= 9 - 2\cdot 3$

$= 9 - 6 = 3 > 0$

$MIN\left(\frac{3}{4}, \frac{1}{2}\right)$

4. $y''+2y'+y=0$

$\Vert$

$\lambda^2+2\lambda+1=0$

$\lambda_{1,2}=\frac{-2\pm\cancel{\sqrt{4-4}}}{2}=-1$

$y=C_1e^{-x}+C_1e^{-x}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊

Ime i prezime: IVAN DADOVIĆ **Broj indeksa:** 57230

Vrijeme početka: 8:39 **Vrijeme završetka:**

Popunjava nastavnik
Broj ↓ bodova
10

1. Riješiti integrale:

(a) $\displaystyle\int_0^{\pi} x^2 \sin(x)\,\mathrm{d}x$; 10

(b) $\displaystyle\int_2^{3} \frac{x^2+2x+2}{x^2+x-2}\,\mathrm{d}x.$ 15

2. Da li je integral $\displaystyle\int_{-1}^{1} \frac{x^2+2x+2}{x^2+x-2}\,\mathrm{d}x$ nepravi i zašto? 5

3. Izračunati površinu lika omeđenog krivuljama: $y = 2x^2+1$ i $y = 3-x$. 20

4. Riješiti zadanu diferencijalnu jednadžbu uz početne uvjete $y(0) = 1$ i $y'(0) = 0$. Uvrstiti rješenje u jednadžbu i provjeriti zadovoljenje jednakosti. 15+5

$$y'' + 2y' + y = 0$$

5. Istražiti ekstreme funkcije $f(x,y) = y^3 - 3xy + x^2$. 20

6. Odrediti početak (prva 4 člana) Taylorovog razvoju funkcije $f(x) = 2x\cos x$ oko točke $x_0 = \dfrac{\pi}{2}$. 10

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

$\int = u \cdot dv = uv - \int v\,du$

Ime i prezime: IVAN RADOVIĆ **Broj indeksa:** 57230

$$\int_0^{\pi} x^2 \sin(x)\,dx = \int_0^{\pi} x^2 \cdot \int_0^{\pi} \sin x\,dx = \frac{x^3}{3}(-\cos x)\Big|_0^{\pi} = \frac{3.14^3}{3}\cdot(-\cos 3.14) - \left(\frac{0^3}{3}\cdot(-\cos 0)\right)$$

$= 9.32$

3. $y = 2x^2 + 1$

$y = 3 - x$

$2x^2 + 1 = 3 - x$

$2x^2 + x - 2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1 + 16}}{4}$

$x = \frac{-1 \pm \sqrt{17}}{4}$

$x_1 = \frac{-1 + 4.1}{4}$ $x_2 = \frac{-1 - 4.1}{4}$

$x_1 = 0.77$ $x_2 = -1.275$

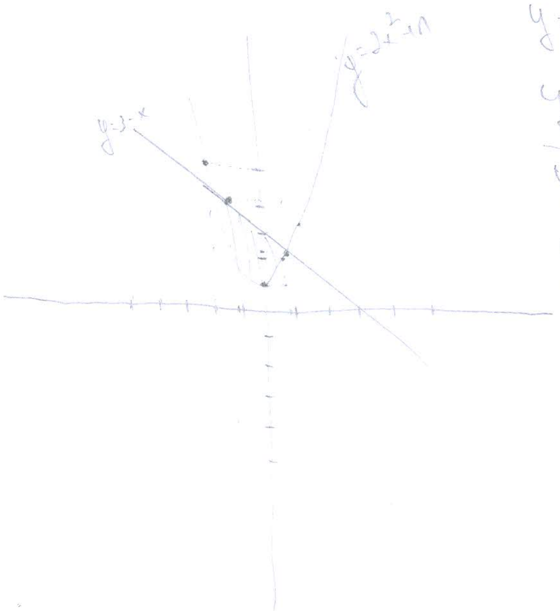
$y_1 = 2.23$ $y_2 = 4.27$

$y = 3 - 0.77$

$y = 2.23$

$y = 3 + 1.27$

$y = 4.27$



$y = 2x^2 + 1 = 0$

$2x^2 + 1 = 0$

$2x^2 = -1$

$x^2 = -\frac{1}{2}$

$x = \sqrt{-\frac{1}{2}}$

$x = 0.7$

$y = 2(-0.7)^2 + 1$

$y = 0.98 + 1$

$y = 1.98$

x	-2	0	1
y	5	1	3

$$P = \int_{-2}^{0.7} (3 - x) - (2x^2 + 1) = \int_{-2}^{0.7} 3 - x - 2x^2 - 1 = \int_{-2}^{0.7} -2x^2 - x + 2 = -2\frac{x^3}{3} - \frac{x^2}{2}\Big|_{-2}^{0.7}$$

$$= \left(-2\frac{0.7^3}{3} - \frac{0.7^2}{2}\right) - \left(-2\frac{-2^3}{3} - \frac{-2^2}{2}\right) = -0.016 - ($$

$f(x) = 2x \cos x$ $\quad f(\frac{\pi}{2}) =$

$f(x)' = 2(-\sin x)$ $\quad f(\frac{\pi}{2}) = -0.0548$

$f(x)'' = 2 \cos x$ $\quad f(\frac{\pi}{2}) = 1.999$

$f(x)''' = 2(-\sin x)$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: IVAN SKOBLAR **BROJ INDEKSA:** 56203

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓
bodova

0

1. Riješiti integrale:

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