MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: RJEŠENJE 3 **BROJ INDEKSA:**

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. 20
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. 5+15
3. Ispitati domenu, (ne)parnost i drugu derivaciju funkcije $g(x) = \ln(x^2+1)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2-1}{x^2+1}$. 20(graf)
5. Gaussovom metodom riješiti matrični sustav: 20

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

Ukupno:

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① $z^3 + \sqrt{9+16} = -5i$ $\quad |-5-5i| = \sqrt{25+25} = 2\sqrt{5}$

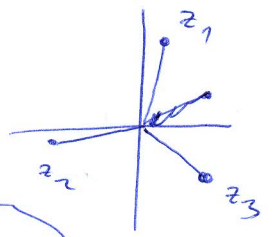
$z^3 = -5-5i$ $\quad \varphi = \text{Arg}(-5-5i) = \pi + \arctan\frac{-5}{-5} = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$z_1 = \sqrt[3]{2\sqrt{5}}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) = \ldots$

$z_2 = \sqrt[3]{2\sqrt{5}}\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right) = \ldots$

$z_3 = \sqrt[3]{2\sqrt{5}}\left(\cos\frac{21\pi}{12} + i\sin\frac{21\pi}{12}\right) = \ldots$

SLIKA RJEŠENJA:



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② $\mathcal{D}(f) = \{x : x^2-1 \geq 0\} = \langle -\infty, -1] \cup [1, +\infty\rangle$

LIJEVO: $\lim_{x\to-\infty} f(x) = \lim_{x\to+\infty} -x - \sqrt{x^2-1} = -\infty - \infty = -\infty$

DESNO: $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x - \sqrt{x^2-1} \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \lim_{x\to+\infty} \frac{x^2 - x^2 + 1}{x+\sqrt{x^2-1}} = \frac{1}{\infty} = 0$ D.H.A. $y = 0$

$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{-x-\sqrt{x^2-1}}{-x} = \lim_{x\to+\infty} 1 + \sqrt{\frac{x^2}{x^2} - \frac{1}{x^2}} = 2 \quad k = 2$

$\lim_{x\to-\infty} f(x) - kx = \lim_{x\to+\infty} \underbrace{-x-\sqrt{x^2-1}+2x}_{= x-\sqrt{x^2-1}} \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \lim_{x\to+\infty} \frac{x^2 - x^2 + 1}{x+\sqrt{x^2-1}} = \frac{1}{\infty} = 0$ L.K.A. $y = 2x$

③ $g(x) = \ln(x^2+1)$

$g'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

$g''(x) = \frac{2(x^2+1) - 2x \cdot 2x}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2} = -2\frac{x^2-1}{(x^2+1)^2}$

DOMENA $= \{x : \underbrace{x^2+1 > 0}_{\text{UVIJEK}}\} = \mathbb{R}$

(NE) PARNOST:

$g(-x) = \ln((-x)^2+1) = \ln(x^2+1) = g(x)$ PARNA

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④ $h(x) = \frac{x^2-1}{x^2+1}$, $\mathcal{D}(h) = \langle -\infty, +\infty \rangle$

$h'(x) = \frac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$

$\lim_{x \to \pm\infty} h(x) = 1$ $\quad y=1$ JE L.H.A i D.H.A.

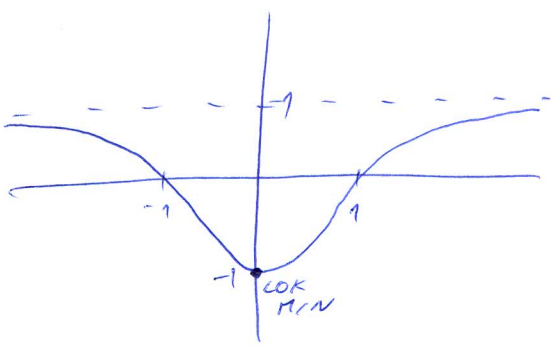
$h'(x) = 0$ za $x = 0$

nultočke $x^2 - 1 = 0 \Leftrightarrow x = 1$ ili $x = -1$

	$-\infty$	-1	0	1	$+\infty$
$h'(x)$		$-$	0	$+$	
$h(x)$		↘		↗	

GLOBALNI MINIMUM

$h(0) = -1$



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⑤

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \overset{R1 \leftrightarrow R4}{\sim} \left[\begin{array}{cccc|c} \textcircled{1} & -1 & 2 & 1 & 3 \\ 0 & 0 & -1 & 1 & 2 \\ 4 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \begin{array}{l} \\ \\ R3-4R1 \\ R4-2R1 \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 0 & -1 & 1 & 2 \\ 0 & 3 & -6 & -3 & -9 \\ 0 & 3 & -3 & -6 & -6 \end{array}\right] \begin{array}{l} \\ \\ \cdot\frac{1}{3} \\ \cdot\left(-\frac{1}{3}\right) \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & \textcircled{1} & -1 & -2 & -2 \\ 0 & 1 & -2 & -1 & -3 \\ 0 & 0 & -1 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & -1 & 1 & 2 \end{array}\right] \begin{array}{l} \\ \\ \\ R4-R3 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -2 & -2 \\ 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 3 \end{array}\right]$$

SUSTAV NEMA RJEŠENJA