MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊

Ime i prezime: RJEŠENJE 3 **Broj indeksa:**

Vrijeme početka: **Vrijeme završetka:**

Popunjava nastavnik
Broj ↓ bodova

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. 20
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. 5+15
3. Ispitati domenu, (ne)parnost i drugu derivaciju funkcije $g(x) = \ln\left(x^2+1\right)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2-1}{x^2+1}$. 20(graf)
5. Gaussovom metodom riješiti matrični sustav: 20

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

Ukupno:

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① $z^3 + \sqrt{9+16} = -5i$ $|-5-5i| = \sqrt{25+25} = 2\sqrt{5}$

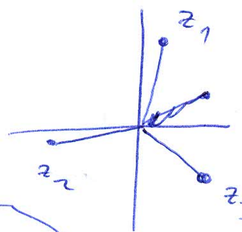
$z^3 = -5-5i$ $\varphi = Arg(-5-5i) = \pi + \arctan\frac{-5}{-5} = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$z_1 = \sqrt[3]{2\sqrt{5}}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) = \ldots$

$z_2 = \sqrt[3]{2\sqrt{5}}\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right) = \ldots$

$z_3 = \sqrt[3]{2\sqrt{5}}\left(\cos\frac{21\pi}{12} + i\sin\frac{21\pi}{12}\right) = \ldots$

SLIKA RJEŠENJA:



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② $\mathcal{D}(f) = \{x : x^2-1 \geq 0\} = \langle -\infty, -1] \cup [1, +\infty\rangle$

LIJEVO: $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to+\infty} -x - \sqrt{x^2-1} = -\infty - \infty = -\infty$

DESNO: $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} x - \sqrt{x^2-1} \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \lim\limits_{x\to+\infty} \frac{\cancel{x^2} - \cancel{x^2} + 1}{x+\sqrt{x^2-1}} = \frac{1}{\infty} = 0$ D.H.A. $y=0$

$\lim\limits_{x\to-\infty} \frac{f(x)}{x} = \lim\limits_{x\to+\infty} \frac{-x-\sqrt{x^2-1}}{-x} = \lim\limits_{x\to+\infty} 1 + \sqrt{\frac{x^2}{x^2} - \frac{1}{x^2}} = 2$ $k=2$

$\lim\limits_{x\to-\infty} f(x) - kx = \lim\limits_{x\to+\infty} \underbrace{-x-\sqrt{x^2-1}+2x}_{=x-\sqrt{x^2-1}} \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \lim\limits_{x\to+\infty} \frac{\cancel{x^2} - \cancel{x^2} + 1}{x+\sqrt{x^2-1}} = \frac{1}{\infty} = 0$ L.K.A. $y = 2x$

③ $g(x) = \ln(x^2+1)$

$$g'(x) = \frac{1}{x^2+1}\cdot 2x = \frac{2x}{x^2+1}$$

DOMENA $= \{x : \underbrace{x^2+1>0}_{\text{UVIJEK}}\} = \mathbb{R}$

$$g''(x) = \frac{2(x^2+1)-2x\cdot 2x}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2} = -2\,\frac{x^2-1}{(x^2+1)^2}$$

(NE) PARNOST :

$g(-x) = \ln((-x)^2+1) = \ln(x^2+1) = g(x)$ PARNA

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④ $h(x) = \dfrac{x^2-1}{x^2+1}$, $\mathcal{D}(h) = \langle -\infty, +\infty\rangle$

$$h'(x) = \frac{2x(x^2+1)-2x(x^2-1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

$\lim\limits_{x\to\pm\infty} h(x) = 1$ $y=1$ JE L.H.A i D.H.A.

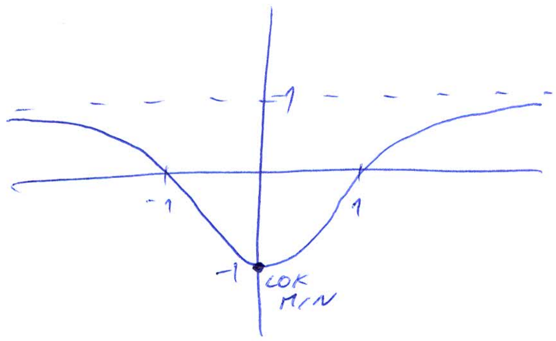
$h'(x)=0$ za $x=0$

nultočke $x^2-1=0 \Leftrightarrow x=1$ ili $x=-1$

	$-\infty$ … 0	0 … $+\infty$
$h'(x)$	$-$	$+$
$h(x)$	↘	↗

GLOBALNI MINIMUM

$h(0)=-1$



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⑤
$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & 14\\ 2 & 1 & 0 & -3 & 2\\ 1 & -1 & 2 & 1 & 3\\ 2 & 1 & 1 & -4 & 0\end{array}\right] \xrightarrow{R1-R4} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3\\ 0 & 0 & -1 & 1 & 2\\ 4 & -1 & 2 & 1 & 3\\ 2 & 1 & 1 & -4 & 0\end{array}\right]\begin{array}{l} \\ \\ R3-4R1\\ R4-2R1\end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3\\ 0 & 0 & -1 & 1 & 2\\ 0 & 3 & -6 & -3 & -9\\ 0 & 3 & -3 & -6 & -6\end{array}\right]\begin{array}{l} \\ \\ \cdot\frac{1}{3}\\ \cdot\frac{1}{3}\end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3\\ 0 & 1 & -1 & -2 & -2\\ 0 & 1 & -2 & -1 & -3\\ 0 & 0 & -1 & 1 & 2\end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1\\ 0 & 1 & -1 & -2 & -2\\ 0 & 0 & -1 & 1 & -1\\ 0 & 0 & -1 & 1 & 2\end{array}\right]\begin{array}{l} \\ \\ \\ R4-R3\end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 1\\ 0 & 1 & -1 & -2 & -2\\ 0 & 0 & -1 & 1 & -1\\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{0} & \boxed{3}\end{array}\right]$$

SUSTAV NEMA RJEŠENJA

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊	POPUNJAVA NASTAVNIK
IME I PREZIME: Mitrović Martin **BROJ INDEKSA:**	Broj ↓ bodova
VRIJEME POČETKA: 08.00 **VRIJEME ZAVRŠETKA:** 17-2-0033-2010	~~20~~

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. — 20
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. — 5+15
3. Ispitati domenu, (ne)parnost i drugu derivaciju funkcije $g(x) = \ln(x^2+1)$. — 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \frac{x^2-1}{x^2+1}$. — 20(graf)
5. Gaussovom metodom riješiti matrični sustav: — 20

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

Ukupno: 30

2. $f(x) = x - \sqrt{x^2-1}$ $\qquad D f(x) \in \mathbb{R} \setminus \{-1, 0, 1\}$

$x \neq 0 \qquad x^2 - 1 \neq 0 \qquad f(0) = 0 \qquad S(0,0)$

$x^2 \neq 1$

$x \neq \pm\sqrt{1}$

V. A

$\lim_{x\to -1^+} f(x) = x - \sqrt{x^2-1} = -1^-$

$\lim_{x\to 1^-} f(x) = x - \sqrt{x^2-1} = -1^-$

$\lim_{x\to 0^+} f(x) = x - \sqrt{x^2-1} = 0^+$

$\lim_{x\to 0^-} f(x) = x - \sqrt{x^2-1} = 0^-$

$\lim_{x\to 1^+} f(x) = x - \sqrt{x^2-1} = 1^+$

$\lim_{x\to 1^-} f(x) = x - \sqrt{x^2-1} = 1^+$

H. A.

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x - \sqrt{x^2-1}}{x}$

$= \lim_{x\to\infty} \frac{x - \sqrt{x^2-1}}{x} \cdot \frac{x + \sqrt{x^2-1}}{x + \sqrt{x^2-1}}$

$= \lim \frac{x^2 - x - 1 \quad /:x^2}{x^2 - x - 1 \quad /:x^2}$

$= \frac{1 - \frac{1}{x} - \frac{1}{x^2}}{1 - \frac{1}{x} - \frac{1}{x^2}} = \frac{1}{1} = 1$

$\underline{\underline{y = 1}}$

→ Nema kose asimptote

3. $\ln(x^2+1)$

a/ $f'(x) = \frac{1}{x^2+1} \cdot 2x$

b/ $f(-x) = x$

funkcija nije ni parna ni neparna

c/ I

$x^2+1 > 0$

$x^2 > -1$

$\cancel{x > \sqrt{-1}}$

II $\ln e^0 \quad e^0 = 1$

$x^2+1 > 1$

$x^2+1-1 > 0$

$x^2 > 0$

$x > 0$

$Df(x) \in \mathbb{R} \langle 0, +\infty \rangle$

d/ $f'(x) = \frac{2x}{x^2+1}$ $f''(x) = \frac{(2x)' \cdot (x^2+1) - (x^2+1)' \cdot (2x)}{(x^2+1)^2}$

$= \frac{2 \cdot (x^2+1) - (2x) \cdot (2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2}$

Ime i prezime: Mitrović Martin   Broj indeksa: 17-2-0033-2010

4\. $h(x) = \dfrac{x^2-1}{x^2+1}$ -N.S

N.T

$x^2-1=0$

$x^2=1$

$x=\pm\sqrt{1}$

$f(0)=-1$

$S(0,-1)$

D.D.

$x^2+1\neq 0$

$x^2\neq -1$

$\cancel{x\neq\sqrt{-1}}$

$\underline{Dh(x)\in R}$

✓

1/ V.A nema $Dh(x)\in R$

2/ H.A. $\lim\limits_{x\to\infty} h(x) = \lim\limits_{x\to\infty} \dfrac{x^2-1 \;/:x^2}{x^2+1\;/:x^2} = \dfrac{1-\frac{1}{x^2}\to 0}{1+\frac{1}{x^2}\to 0} = \dfrac{1}{1} = 1$ ✓

$\underline{y=1}$

3/ K.A. nema ima horizontalnu

4/ $h(x) = \dfrac{x^2-1}{x^2+1}$

$h'(x) = \dfrac{(x^2-1)'\cdot(x^2+1)-(x^2+1)'\cdot(x^2-1)}{(x^2+1)^2} = \dfrac{2x(x^2+1)-(2x)\cdot(x^2-1)}{(x^2+1)^2}$

$= \dfrac{\cancel{2x^3}+2x-\cancel{2x^3}+2x}{(x^2+1)^2} = \dfrac{4x}{(x^2+1)^2}$

	$-\infty$		1		$+\infty$
$f(x)$		$-$		$+$	
		$\searrow$		$\nearrow$	

$4x\neq 0$

Nema ekstrema KAKO NEMA!?

$h'(x) = \dfrac{4x}{(x^2+1)^2}$

$h''(x) = \dfrac{(4x)'\cdot(x^2+1)^2-((x^2+1)^2)'\cdot(4x)}{((x^2+1)^2)^2} = \dfrac{4\cdot(x^2+1)^2-(2(x^2+1)\cdot 2x\cdot(4x)}{(x^2+1)^4}$

$= \dfrac{4\cdot(x^2+1)^{\cancel{2}}-2x^2-2\cdot 8\cancel{x}}{(x^2+1)^{\cancel{4}}} = \dfrac{4\cdot(x^2+1)-2x^2-16x}{(x^2+1)^2} = \dfrac{4x^2+4-2x^2-16x}{(x^2+1)^2}$

$$f''(x) = \frac{2x^2 - 16x + 4}{(x^2+1)^2}$$

	$-\infty$	1	0,257	7,75	$+\infty$
$f''(x)$	+	+	−	+	
	∪	∪	∩	∪	

$$\underset{a}{2x^2} - \underset{b}{16x} + \underset{c}{4} = 0$$

$$x_{12} = \frac{-(b) \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{+16 \pm \sqrt{16^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2} = \frac{16 \pm \sqrt{256 - 32}}{4}$$

$$x_{1,2} = \frac{16 \pm \sqrt{224}}{4} = \frac{16 \pm 14.97}{4}$$

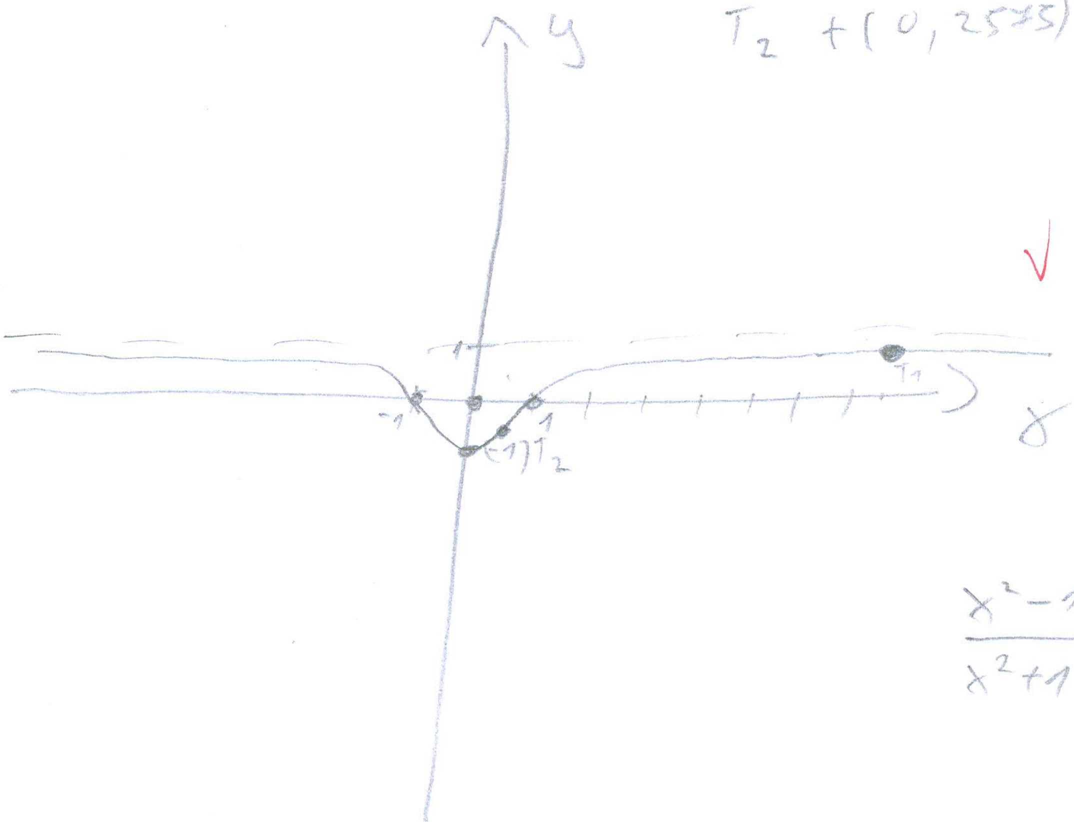
$x_1 = 7,75$

$x_2 = 0,2575$

$T_1 \quad f(7.75) = 0.967$

$T_2 \quad f(0,2575) = -0.875$

2



$$\frac{x^2 - 1}{x^2 + 1} =$$

−0.93369

1,0663

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: ROMANO FUZUL **BROJ INDEKSA:** 171-0070-2011

VRIJEME POČETKA: 07:55 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

40

1. Pronaći sve kompleksne brojeve z takve da je $z^3 + |3+4i| = \frac{5}{i}$. — 20
2. Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$. — 5+15
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5. Gaussovom metodom riješiti matrični sustav: — 20

$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

Ukupno:

2.) $f(x) = x - \sqrt{x^2-1}$

$x^2 - 1 \geq 0$

$x^2 \geq 1$

$x \geq \sqrt{1}$

$x \neq \pm 1$

$D(f) = \mathbb{R} \setminus \{-1, +1\}$

$\cancel{5}$

VERTIKALNA ASIMPTOTA

$x_1 = -1 \quad x_2 = 1$

HORIZONTALNA ASIMPTOTA

$$\lim_{x\to\infty} x - \sqrt{x^2-1} \cdot \frac{-x+\sqrt{x^2-1}}{-x+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{x^2-1 \,:x^2}{-x+\sqrt{x^2-1}\,:x^2} =$$

$$= \lim_{x\to\infty} \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{-\frac{x}{x^2} + \sqrt{\frac{x^2}{x^2} - \frac{1}{x^2}}} = \frac{1}{1} = 1$$

KOSE ASIMPTOTE NEMA JER IMAMO HORIZONTALNU

3.) $g(x) = \ln(x^2+1)$

Domena

$x^2+1>0$

$x^2>-1$

~~$x>\sqrt{-1}$~~

$D(f)=\mathbb{R}$

$g'(x) = \frac{1}{x^2+1} \cdot (x^2+1)'$

$g'(x) = \frac{1}{x^2+1} \cdot (2x)$

$g'(x) = \frac{2x}{x^2+1}$

$g''(x) = \frac{(2x)' \cdot (x^2+1) - (2x) \cdot (x^2+1)'}{(x^2+1)^2}$

$g''(x) = \frac{(2) \cdot (x^2+1) - (2x) \cdot (2x)}{(x^2+1)^2}$

$g''(x) = \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$g''(x) = \frac{-2x^2+2}{(x^2+1)^2}$

15

IME I PREZIME: ROMANO FUZUL **BROJ INDEKSA:** 171-0070-2011

4.) $h(x) = \frac{x^2-1}{x^2+1}$

1.) Asimptote

VA

$x^2+1=0$

$x^2=-1$

$\cancel{x=\sqrt{-1}}$

Nema vertikalne asimptote

2.)

HA

$\lim\limits_{x\to\infty} \frac{x^2-1}{x^2+1} = \frac{1}{1} = 1$

KA

kose nemamo

2.) ekstremi

$h'(x) = \frac{(x^2-1)'\cdot(x^2+1)-(x^2-1)\cdot(x^2+1)'}{(x^2+1)^2}$

$= \frac{(2x)\cdot(x^2+1)-(x^2-1)\cdot(2x)}{(x^2+1)^2}$

$= \frac{\cancel{2x^3}+2x-\cancel{2x^3}+2x}{(x^2+1)^2}$

$\boxed{h'(x) = \frac{4x}{(x^2+1)^2}}$

$4x=0 \;/:4$

$x=\frac{0}{4}=0$

$x=0$

$h(x)=\frac{0^2-1}{0^2+1}$

$=\frac{-1}{+1}=-1$

minimum

$E_1(0,-1)$

	$-\infty$	0	$+\infty$
y'	$-$	$+$	
$f(x)$	$\searrow$	$\nearrow$	

min

OSX

$x^2-1=0$

$x^2=1$

$x=\sqrt{1}$

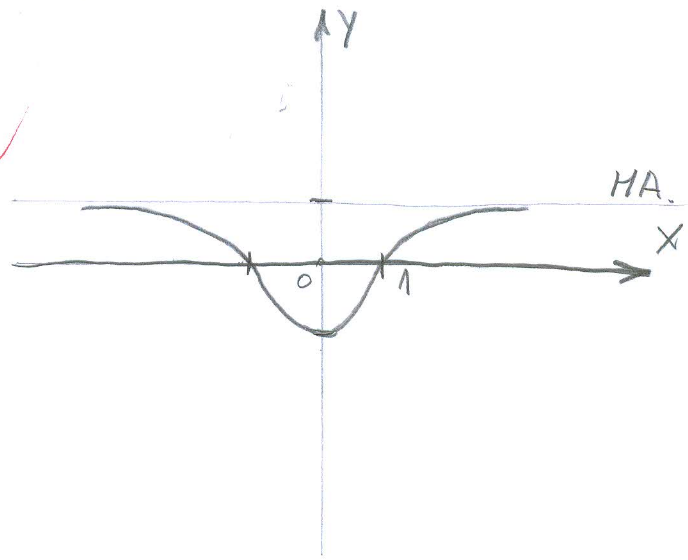
$x=\pm 1$

OSY

$x=0$

$h(x)=\frac{x^2-1}{x^2+1}$

$h(x)=\frac{0^2-1}{0^2+1}=-1$



5.)

$$\left[\begin{array}{cccc|c} 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 0 & -3 & 2 \\ 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 2 & 1 & 0 & -3 & 2 \\ 4 & -1 & 1 & 2 & 14 \\ 2 & 1 & 1 & -4 & 0 \end{array}\right] \begin{array}{l} \cdot(-2)\ \cdot(-4)\ \cdot(-2) \\ + \\ + \\ + \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 3 & -7 & -2 & 2 \\ 0 & 3 & -3 & -6 & -6 \end{array}\right] \begin{array}{l} \\ \cdot(-1)\ \cdot(-1) \\ + \\ + \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 0 & -3 & 3 & 6 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{array}{l} \\ \\ \cdot\left(-\frac{1}{3}\right) \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & -1 & -2 \end{array}\right] \begin{array}{l} \\ \\ \cdot(-1) \\ + \end{array} \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 3 \\ 0 & 3 & -4 & -5 & -4 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} c \\ b \\ a \\ d \end{array}$$

$0d = 0$

$\boxed{d = 0}$

$1a - \cancel{1 \cdot 0} = -2$

$\boxed{a = -2}$

$3b - 4 \cdot (-2) - \cancel{5 \cdot 0} = -4$

$3b + 8 = -4$

$3b = -12$

$b = -\frac{12}{3}$

$\boxed{b = -4}$

$1c \underbrace{- 1 \cdot (-4)}_{4} \underbrace{+ 2 \cdot (-2)}_{-4} + \cancel{1 \cdot 0} = 3$

$1c + \cancel{4} - \cancel{4} = 3$

$1c = 3$

$\boxed{c = 3}$

$a = -2$

$b = -4$

$c = 3$

$d = 0$

✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: NIKOLINA KOMLJENOVIĆ **BROJ INDEKSA:** 17-2-OMY-2011

VRIJEME POČETKA: 07:45 **VRIJEME ZAVRŠETKA:** 8:30

POPUNJAVA NASTAVNIK

Broj ↓ bodova

25

1.	Pronaći sve kompleksne brojeve z takve da je $z^3 + \|3+4i\| = \frac{5}{i}$.	20
2.	Odrediti domenu i sve asimptote funkcije $f(x) = x - \sqrt{x^2-1}$.	5+15
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$$\begin{bmatrix} 4 & -1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

Ukupno:

1.

$z^3 + |3+4i| = \frac{5}{i} \cdot \frac{-i}{-i}$ $\quad z = x+yi$

$z^3 + |3+4i| =$

(2.) $f(x) = x - \sqrt{x^2 - 1}$

$x^2 - 1 \geq 0$

$x^2 = 1 \;|\sqrt{\ }$

$x_{1,2} = \pm 1$

	$-\infty$	-1	0	1	$+\infty$
x^2-1	$+$	$\circ$	$-$	$\circ$	$+$

5

$D\,f(x)\quad x \in \langle -\infty, -1] \cup [1, +\infty\rangle$

V.A. $\lim\limits_{x \to -1} -1 - \sqrt{(-1)^2 - 1} = \lim\limits_{x \to -1} -1 - \sqrt{0} = -1$ nema V.A.

$\lim\limits_{x \to 1} 1 - \sqrt{1^2 - 1} = 1$ nema V.A.

H.A. $\lim\limits_{x \to \infty} \dfrac{x - \sqrt{x^2 - 1}}{1} \cdot \dfrac{x - \sqrt{x^2 + 1}}{x - \sqrt{x^2 + 1}} = \lim\limits_{x \to \infty} \dfrac{x^2 - \sqrt{x^2 - 1} \;/ : x^2}{x - \sqrt{x^2 + 1} \;/ : x^2} = \lim\limits_{x \to \infty} \dfrac{\frac{x^2}{x^2} - \sqrt{\frac{x^2}{x^2} - \frac{1}{x^2}}}{\frac{x}{x^2} - \sqrt{\frac{x^8}{x^4} + \frac{1}{x^2}}} =$

$\lim\limits_{x \to \infty} \dfrac{1 - 1}{0} =$ neodređen oblik

$\lim\limits_{x \to -\infty} -x - \sqrt{(x)^2 - 1} = \lim\limits_{x \to -\infty} -x - \sqrt{x^2 - 1} \cdot \dfrac{-x - \sqrt{x^2 + 1}}{-x - \sqrt{x^2 + 1}} =$ nema H.A.

K.A

$k \ldots \dfrac{fx}{x}$ $\lim\limits_{x \to \infty} \dfrac{x - \sqrt{x^2 - 1} \;/ : x}{x \;/ : x}$ $\lim\limits_{x \to \infty} \dfrac{1 - \sqrt{1 - \frac{1}{x}}}{1} = \dfrac{0}{1}$

$l = f(x) - x \;\ldots\; x - \sqrt{x^2 - 1} - x$

Ime i prezime: NIKOLINA KOMLJENOVIĆ **Broj indeksa:** 17-2-0114-2011

(3.) $g(x) = \ln(x^2+1)$ $\quad x^2+1 \geq 0$

$x^2 = -1$ $\quad D\ f(x)\ x \in \mathbb{R}$

$f(-x) = \ln((-x)^2+1)$

$f(-x) = \ln(x^2+1)$ parna je

$f(x) = \ln(x^2+1)$

$f'(x) = \frac{1}{x^2+1} \cdot (x^2+1)'$

$f'(x) = \frac{1}{x^2+1} \cdot 2x$

$f'(x) = \frac{2x}{x^2+1}$

$f(x)'' = \frac{(2x)'(x^2+1) - 2x(x^2+1)'}{(x^2+1)^2}$

$f(x)'' = \frac{2(x^2+1) - 2x \cdot 2x}{(x^2+1)^2}$

$f(x)'' = \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$f''(x) = \frac{-2x^2+2}{(x^2+1)^2}$

4. $f(x) = \frac{x^2-1}{x^2+1}$

$x^2+1 \neq 0$
$x^2 = -1$ $\quad D f(x)\ x \in \mathbb{R}$

$f(-x) = \frac{(-x)^2-1}{(-x)^2+1} = \frac{x^2-1}{x^2+1}$ parzysta

mult.
$x^2-1=0$
$x^2=1 \mid \sqrt{\ }$
$x_{1,2} = \pm 1$ $\quad (1,0)$ $\quad (-1,0)$

H.A. $\lim_{x\to\infty} \frac{x^2-1}{x^2+1} \frac{/:x^2}{/:x^2} = \lim \frac{1}{1} = 1$ $\quad y=1$

$\lim_{x\to-\infty} \frac{(-x)^2-1}{(-x)^2+1} = \lim_{x\to-\infty} \frac{x^2-1}{x^2+1} = 1$ $\quad y=1$

$f'(x) = \frac{(x^2-1)'(x^2+1)-(x^2-1)(x^2+1)'}{(x^2+1)^2}$

$f'(x) = \frac{2x(x^2+1)-(x^2-1)2x}{(x^2+1)^2}$

$f'(x) = \frac{\cancel{2x^3}+2x-\cancel{2x^3}+2x}{(x^2+1)^2}$

$f'(x) = \frac{4x}{(x^2+1)^2}$

$4x=0$ nowa ekstrema

$f''(x) = \frac{(4x)'(x^2+1)^2 - 4x((x^2+1)^2)'}{(x^2+1)^4}$

$f''(x) = \frac{4(x^2+1)^2 - 4x \cdot 2(x^2+1)\cdot(x^2+1)'}{(x^2+1)^4}$

$f''(x) = \frac{4(x^2+1)^2 - 8x^3 - 8x \cdot 2x}{(x^2+1)^4}$

$f''(x) = \frac{4(x^2+1)^2 - 8x^3 - 16x}{(x^2+1)^4}$

$4(x^2+1)^2 - 8x^3 - 16x = 0 \quad /:4$

$(x^2+1)^2 - 2x^3 - 4x = 0$

$x^2+1 - 2x^3 - 4x = 0$

$x^2(1+1-2x-4x$