MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: Lovre Kereš **BROJ INDEKSA:** 57933

VRIJEME POČETKA: 01:56 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

5

1. Riješiti jednadžbu: $z^3 + \overline{1+i} = 0$. 20

2. Riješi sustav Gaussovom metodom: 20

$$\begin{array}{rcrcrcrcr} 2x_1 & - & x_2 & + & x_3 & - & x_4 & = & -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 & = & 1 \\ 3x_1 & & & - & x_3 & + & x_4 & = & -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 & = & -1 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{4x^2+x} - 2x\right)$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{e^x}{x}$. 20(graf)

5. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2+4) + \sin(x-2)$. 5+15

Ukupno:

5.

$f(x) = \ln(x^2+4) + \sin(x-2)$

$\ln > 0$

$\sin [-1, 1]$

$x^2+4=0$

$x^2 = -4 / \sqrt{\ }$

$x = \sqrt{-4}$ nema rješenje

$x-2=0$

$x=2$

$D_f = \langle -\infty, +\infty \rangle \cup [-1, 1] \setminus \{2\}$

$f'(x) = \ln(x^2+4) + \sin(x-2) = \frac{1}{(x^2+4)} \cdot 2x + \cos x \cdot 1$

$= \frac{2x}{(x^2+4)} + \cos x$

3. $g(x) = (\sqrt{4x^2+x} - 2x)$

DOMENA

$\sqrt{\ } \geq 0$

$D(g) = \mathbb{R}$ ✓ 5

ASIMPTOTE

H. A.

$\lim_{x \to \infty} (\sqrt{4x^2+x} - 2x) = \infty - \infty = -\infty$

$\lim_{x \to -\infty} (\sqrt{4x^2+x} - 2x) = x \to -x = (\sqrt{4x^2-x} + 2x) = \infty + \infty$
$= \infty$

K. A.

$\lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{(\sqrt{4x^2+x} - 2x)}{x}$

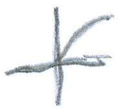
---

1. $z^3 + \overline{1+i} = 0 \qquad z^3 = -1-i$

Ime i prezime: Lovre Kereš   Broj indeksa: 57933

$h(x) = \frac{e^x}{x}$

DOMENA

$\frac{e^x}{x}$ e^x  $D(e^x) = \mathbb{R}$

$D(e) = \mathbb{R}$

NUL TOČKE

$h(x) = \frac{e^x}{x}$ NEMA NUL TOČAKA !!!

ASIMTOTE

$\boxed{h(x) = \frac{e^x}{x}}$ $\underline{x > 0}$

V.A. Nema

H.A

$\lim_{x \to \infty} \frac{e^x}{x} = \frac{\infty}{\infty} = \frac{e^\infty}{\infty} = \frac{0}{e^\infty} = 0$

$\lim_{x \to -\infty} \frac{e^x}{x} = \frac{x \to -x}{\infty \to -\infty} = \frac{e^{-x}}{-x}$ = Nema Asimptota

Ekstremi

$$h(x) = \frac{e^x}{x}$$

$$h'(x) = \frac{(e^x)' \cdot x - e^x \cdot (x)'}{2x} = \frac{\cancel{e^x} \cdot x - \cancel{e^x} \cdot 1}{2x} = \frac{-x}{2x}$$

$$h''(x) = \frac{-x}{2x} = \frac{(-x)' \cdot 2x - (-x) \cdot (2x)'}{2(2x)} = \frac{-1 \cdot 2x + x \cdot 2}{4x}$$

$$= \frac{-2x + 2x}{4x} = \frac{0}{4x} = 0$$

2.

$$2x_1 - x_2 + x_3 - x_4 = -1$$

$$2x_1 - x_2 \qquad - 3x_4 = 1$$

$$3x_1 \qquad - x_3 + x_4 = -1$$

$$2x_1 + 2x_2 - 2x_3 + 5x_4 = -1$$

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & 1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 1 \\ 0 & -1 & 2 & -3 & 1 \\ -1 & 0 & 2 & 1 & -1 \\ -2 & 2 & 2 & 5 & -1 \end{array}\right] \cdot 1, \cdot 2$$

$\Longrightarrow$

Ime i prezime: Lovre Koreš **Broj indeksa:** 57933

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 1 \\ 0 & -1 & 2 & -3 & 1 \\ 0 & -1 & 5 & 0 & 0 \\ 0 & 0 & 6 & 3 & 1 \end{array}\right] \sim$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ **IME I PREZIME:** MATEO BOBAČEK **BROJ INDEKSA:** **VRIJEME POČETKA:** **VRIJEME ZAVRŠETKA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova 55

1. Riješiti jednadžbu: $z^3 + \overline{1+i} = 0$. 20
2. Riješi sustav Gaussovom metodom: 20

$$
\begin{array}{rcrcrcrcr}
2x_1 & - & x_2 & + & x_3 & - & x_4 & = & -1 \\
2x_1 & - & x_2 & & & - & 3x_4 & = & 1 \\
3x_1 & & & - & x_3 & + & x_4 & = & -1 \\
2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 & = & -1
\end{array}
$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{4x^2+x} - 2x\right)$. 5+15
4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{e^x}{x}$. 20(graf)
5. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2+4) + \sin(x-2)$. 5+15

Ukupno:

5.

$f(x) = \ln(x^2+4) + \sin(x-2)$

$f'(x) = \frac{1}{x^2+4} \cdot (2x) + (-\cos(x-2)) \cdot 1$

$f'(x) = \frac{2x}{x^2+4} - \cos(x-2)$ 15

$x^2+4>0 /\sqrt{}$

$x+2>0$

$x>-2$

$\cancel{\boxed{D(f) = \langle -2, +\infty \rangle}}$

1.

$z^3 + \overline{1+i} = 0$

$z^3 + 1 - i = 0$

$z^3 = -1 + i /\sqrt[3]{}$

$z = \sqrt[3]{-1+i}$

$z = \sqrt[3]{w}$

$w = -1+i$

$|w| = \sqrt{(-1)^2+1^2}$

$= \sqrt{2}$

$x=-1 \quad y=1$

$tg\varphi = \frac{1}{-1} = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$

$\varphi = \frac{3\pi}{4}$

$\frac{3\pi}{12} = \frac{\pi}{4}$

$k=0,1,2$

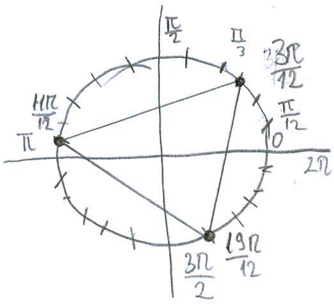
$z = \sqrt[3]{w}\left(\cos\frac{\frac{3\pi}{4}+2k\pi}{3} + i\sin\frac{\frac{3\pi}{4}+2k\pi}{3}\right)$

$k=0 \quad z_1 = \sqrt[3]{\sqrt{2}}\left(\cos\frac{\frac{3\pi}{4}+2\cdot 0\cdot\pi}{3} + i\sin\frac{\frac{3\pi}{4}+2\cdot 0\cdot\pi}{3}\right)$

$z_1 = \sqrt[6]{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ 20

$k=1 \quad z_2 = \sqrt[6]{2}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$

$k=2 \quad z_3 = \sqrt[6]{2}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$



(4) $h(x) = \frac{e^x}{x}$ $\qquad$ $NT(0,0)$

$x \neq 0$ $\qquad$ $S(0,1)$

$D(h) = \mathbb{R} \setminus \{0\}$

VA: $\lim_{x \to 0^-} h(x) = \frac{1}{0^-} = -\infty$

$\lim_{x \to 0^+} h(x) = \frac{1}{0^+} = +\infty$

$\boxed{VA \ldots x = 0}$

HA $\lim_{x \to \infty} \frac{e^x}{x} = \left\{\frac{\infty}{\infty}\right\}$

(3) $g(x) = \left(\sqrt{4x^2 + x} - 2x\right)$

$4x^2 + x \geq 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm 1}{8}$

$x_1 = -\frac{1}{4} \quad x_2 = 0$

	$-\infty$	$-\frac{1}{4}$	0	$+\infty$
$4x^2+x$	$+$	$-$		$+$

$D(g) = \langle -\infty, -\frac{1}{4}] \cup [0, +\infty\rangle$

VA: $\lim_{x \to 0^-} g(x) = {}_+0$

$\lim_{x \to 0^+} g(x) = {}_+0$

nema VA.

HA: $\lim_{x \to \infty} = \sqrt{4x^2 + x} - 2x \,/ : x^2 = [\infty]$

$\lim_{x \to \infty} = \sqrt{4 + \frac{1}{\cancel{x}}_0} - \frac{2}{\cancel{x}} = \sqrt{4} = 2$

$\boxed{HA: \ldots y = 2}$

nema K.A.

Ime i prezime: MATEO BOBAČEK **Broj indeksa:**

(2)

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \cdot(-1) \sim \left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 3 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \begin{array}{l} \cdot 3 \\ \cdot(-2) \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 0 & -3 & 5 & -5 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \cdot(-1) \sim \left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 0 & -3 & 5 & -5 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & -3 & 3 & -6 & 0 \end{array}\right] \cdot(-1)$$

$$\sim \left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 0 & -3 & 5 & -5 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 2 & 1 & -1 \end{array}\right] \cdot(-2) \sim \begin{array}{c} \\ \\ \\ IV_r \end{array} \left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 0 & -3 & 5 & -5 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & -3 & 3 \end{array}\right]$$

Iz IV_r

$-3x_4 = 3 \quad /:(-3)$

$x_4 = -1$

Iz III_r

$1x_3 + 2x_4 = -2$

$x_3 + 2\cdot(-1) = -2$

$x_3 - 2 = -2$

$x_3 = -2+2$

$x_3 = 0$

Iz II_r

$-3x_2 + 5x_3 - 5x_4 = -1$

$-3x_2 + 5\cdot 0 - 5\cdot(-1) = -1$

$-3x_2 + 5 = -1$

$-3x_2 = -6 \quad /:(-3)$

$x_2 = 2$

Iz I_r

$2x_1 - 1x_2 + 1x_3 - 1x_4 = -1$

$2x_1 - 2 + 0 + 1 = -1$

$2x_1 = -1 + 2 - 1$

$2x_1 = 0 \quad /:2$

$x_1 = 0$

$$X = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix}$$

✓ 20

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MARIO IVANAC **BROJ INDEKSA:** 17-1-0096-2011	
VRIJEME POČETKA: 07:55 **VRIJEME ZAVRŠETKA:** 09:15	5

1. Riješiti jednadžbu: $z^3 + \overline{1+i} = 0$. 20
2. Riješi sustav Gaussovom metodom: 20

$$\begin{array}{rcrcrcrcr} 2x_1 & - & x_2 & + & x_3 & - & x_4 & = & -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 & = & 1 \\ 3x_1 & & & - & x_3 & + & x_4 & = & -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 & = & -1 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{4x^2+x} - 2x\right)$. 5+15
4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{e^x}{x}$. 20(graf)
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Ukupno:

3) $g(x) = (\sqrt{4x^2+x} - 2x)$

Domena

$4x^2 + x - 2x = 0$

$x_{1,2} = \frac{1 \cdot \sqrt{1^2 + 4 \cdot 4 \cdot (-2)}}{4^2}$

$x_{1,2} = \frac{1 \cdot \sqrt{-31}}{16}$ => Nema nul točaka

DF $\langle -\infty, +\infty \rangle$ ✓ 5

4) $f(x) = \ln(x^2+4) + \sin(x-2)$

$f'(x) \ln(x^2+4) + \sin(x-2) = \frac{1}{(x^2+4)} \cdot 2x + \cos \cdot 1$

$= \frac{2x}{(x^2+4)} + \cos$

Domena

$f(x) = \ln(x^2+4) + \sin(x-2)$ $\ln > 0$

$x^2+4>0$

$x^2>4 / \sqrt{}$

$x > \pm 2$

$x-2=0$

$x=2$

	$-\infty$	$-2,5$	-2	$1,5$	2	$2,5$	$+\infty$
$x^2+4>0$		$+$		$-$		$+$	
$x-2=0$		$-$		$-$		$+$	
R		$-$		$+$		$+$	

$D_f \, [-2,2] \cup [2,+\infty\rangle$

1) $z^3 + \overline{1+i} = 0$

$z^3 = -1 - i$ $i^2 = -1$

$z^3 = -1-1$

$z^3 = -2$

$z = \sqrt[3]{-2}$

Ime i prezime: MARIO IVANAC **Broj indeksa:** 17-1-0096-2011

2)

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] = \left[\begin{array}{cccc|c} 3 & 0 & -1 & 1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 2 & -1 & 1 & -1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \; R_1 - R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 2 & -1 & 0 & -3 & 1 \\ 2 & -1 & 1 & -1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] /\cdot(-1) \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ -2 & 1 & 0 & 3 & -1 \\ 2 & -1 & 1 & -1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] R_3 + R_2 \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ -2 & 1 & 0 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] R_2 + R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 3 & -2 & 8 & -2 \\ 0 & 0 & 1 & 2 & -2 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] R_4 - 2\cdot R_1 \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 3 & -2 & 8 & -2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 2 & 5 & -1 \end{array}\right] R_4 + R_1 \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 3 & -2 & 8 & -2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 5 & -1 \end{array}\right] /:5$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 3 & -2 & 8 & -2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] R_2 - 8\cdot R_4 \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & 3 & -2 & 0 & -\frac{18}{5} \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] R_2 - R_1 \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ -1 & 2 & 0 & 0 & -\frac{18}{5} \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] /\cdot(-1)$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 1 & -2 & 0 & 0 & \frac{18}{5} \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] R_2 - R_1 \left[\begin{array}{cccc|c} 1 & 1 & -2 & 0 & 0 \\ 0 & -3 & -2 & 0 & \frac{18}{5} \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] /\cdot 3 \left[\begin{array}{cccc|c} 3 & 3 & -6 & 0 & 0 \\ 0 & -3 & -2 & 0 & \frac{18}{5} \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] R_1 + R_2$$

$$\left[\begin{array}{cccc|c} 3 & 0 & -8 & 0 & \frac{18}{5} \\ 0 & -3 & -2 & 0 & \frac{18}{5} \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] 4R_2 - R_1 \left[\begin{array}{cccc|c} 3 & 0 & -8 & 0 & \frac{18}{5} \\ 0 & -12 & 0 & 0 & \frac{72}{5} \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] /:(-12) \left[\begin{array}{cccc|c} 3 & 0 & -8 & 0 & \frac{18}{5} \\ 0 & 1 & 0 & 0 & -\frac{6}{5} \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] /:2$$

$$\left[\begin{array}{cccc|c} 3 & 0 & -8 & 0 & \frac{18}{5} \\ 0 & 1 & 0 & 0 & -\frac{6}{5} \\ 0 & 0 & \frac{1}{2} & 1 & -1 \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] R_3 - R_4 \left[\begin{array}{cccc|c} 3 & 0 & -8 & 0 & \frac{18}{5} \\ 0 & 1 & 0 & 0 & -\frac{6}{5} \\ 0 & 0 & \frac{1}{2} & 0 & -\frac{6}{5} \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] /\cdot 2 \left[\begin{array}{cccc|c} 3 & 0 & -8 & 0 & \frac{18}{5} \\ 0 & 1 & 0 & 0 & -\frac{6}{5} \\ 0 & 0 & 1 & 0 & -\frac{12}{5} \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] R_1 + 8 \cdot R_3$$

$$\left[\begin{array}{cccc|c} 3 & 0 & 0 & 0 & -\frac{88}{5} \\ 0 & 1 & 0 & 0 & -\frac{6}{5} \\ 0 & 0 & 1 & 0 & -\frac{12}{5} \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right] /:3 \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{29.3}{5} \\ 0 & 1 & 0 & 0 & -\frac{6}{5} \\ 0 & 0 & 1 & 0 & -\frac{12}{5} \\ 0 & 0 & 0 & 1 & -\frac{1}{5} \end{array}\right]$$

$$\begin{bmatrix} 2 & -1 & 1 & -1 \\ 2 & -1 & 0 & -3 \\ 3 & 0 & -1 & 1 \\ 2 & 2 & -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} -\frac{29.3}{5} \\ -\frac{6}{5} \\ -\frac{12}{5} \\ -\frac{1}{5} \end{bmatrix} = \begin{bmatrix} 11.72 & 5.86 & -5.86 & 5.86 \\ -2.4 & 1.2 & 0 & 3.6 \\ -7.2 & 0 & 2.4 & -2.4 \\ -0.4 & -0.4 & 0.4 & -1 \end{bmatrix}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ **IME I PREZIME:** MATE LADIĆ **BROJ INDEKSA:** 17-1-0006-2010 **VRIJEME POČETKA:** 08.00 **VRIJEME ZAVRŠETKA:** 9,30	POPUNJAVA NASTAVNIK Broj ↓ bodova 30

1. Riješiti jednadžbu: $z^3 + \overline{1+i} = 0$. 20
2. Riješi sustav Gaussovom metodom: 20

$$\begin{aligned} 2x_1 - x_2 + x_3 - x_4 &= -1 \\ 2x_1 - x_2 - 3x_4 &= 1 \\ 3x_1 - x_3 + x_4 &= -1 \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 &= -1 \end{aligned}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{4x^2+x} - 2x\right)$. 5+15
4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{e^x}{x}$. 20(graf)
5. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2+4) + \sin(x-2)$. 5+15

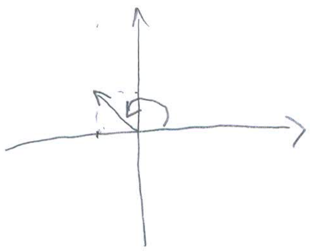
Ukupno:

(1) $z^3 + \overline{1+i} = 0$

$z^3 + 1 - i = 0$

$z^3 = -1 + i$

$z = \sqrt[3]{-1+i}$



$\tan \varphi = \frac{y}{x} = \frac{1}{-1} = 45°$

$\varphi = 180 - 45 = 135°$

$|z| = \sqrt{x^2+y^2} = \sqrt{(-1)^2+1^2} = \sqrt{1+1} = \sqrt{2}$

$k = 0, 1, 2$

$k = 0$

$k=0$

$z_1 = -1$

$z_2 = i$

$z_1 = \sqrt[3]{\sqrt{2}}\left(\cos\frac{\varphi + k\cdot 360}{3} + i\sin\frac{\varphi + k\cdot 360}{3}\right)$

$= \sqrt[6]{2}\left(\cos 45° + i\sin 45°\right)$

$k=1$

$k=2$

$z_2 = \sqrt[3]{\sqrt{2}}\left(\cos\frac{\varphi + 1\cdot 360}{3} + i\sin\frac{\varphi + 1\cdot 360}{3}\right) = \sqrt[6]{2}\left(\cos 135° + i\sin 135°\right)$

10

(2)

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 1 & -2 & -3 & -4 & -2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & -3 & -4 & -2 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & -1 & 1 & -1 & -1 \end{array}\right] \sim$$

3.r − 4.r

1.r ⟺ 4.r

1.r·(−2) + 2.r
1.r·(−3) + 3.r
1.r·(−2) + 4.r

$$\left[\begin{array}{cccc|c} 1 & -2 & -3 & -4 & -2 \\ 0 & 3 & 6 & 5 & 5 \\ 0 & 6 & 8 & -11 & 5 \\ 0 & 3 & 7 & 7 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & -3 & -4 & -2 \\ 0 & 1 & 2 & 5/3 & 5/3 \\ 0 & 6 & 8 & -11 & 5 \\ 0 & 3 & 7 & 7 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & -3 & -4 & -2 \\ 0 & 1 & 2 & 5/3 & 5/3 \\ 0 & 0 & -4 & -21 & -5 \\ 0 & 0 & 1 & 2 & -1 \end{array}\right]$$

2.r : 3

2.r·(−6) + 3.r
2.r·(−3) + 4.r

3.r ⟺ 4.r

$$\begin{bmatrix} 1 & -2 & -3 & -4 & | & -2 \\ 0 & 1 & 2 & 5/3 & | & 5/3 \\ 0 & 0 & 1 & 2 & | & -1 \\ 0 & 0 & -4 & -11 & | & -5 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -3 & -4 & | & -2 \\ 0 & 1 & 2 & 5/3 & | & 5/3 \\ 0 & 0 & 1 & 2 & | & -1 \\ 0 & 0 & 0 & -13 & | & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -3 & -4 & | & -2 \\ 0 & 1 & 2 & 5/3 & | & 5/3 \\ 0 & 0 & 1 & 2 & | & -1 \\ 0 & 0 & 0 & 1 & | & 9/13 \end{bmatrix}$$

3. r · 4 + 4. r

4. r · (−13)

4. r · (−2) + 3. r

4. r · −5/3 + 2. r

4. r · 4 + 1. r

$$\begin{bmatrix} 1 & -2 & -3 & 0 & | & 10/13 \\ 0 & 1 & 2 & 0 & | & 20/39 \\ 0 & 0 & 1 & 0 & | & -31/13 \\ 0 & 0 & 0 & 1 & | & 9/13 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & 0 & | & -83/13 \\ 0 & 1 & 0 & 0 & | & 206/39 \\ 0 & 0 & 1 & 0 & | & -31/13 \\ 0 & 0 & 0 & 1 & | & 9/13 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & | & 163/39 \\ 0 & 1 & 0 & 0 & | & 206/39 \\ 0 & 0 & 1 & 0 & | & -31/13 \\ 0 & 0 & 0 & 1 & | & 9/13 \end{bmatrix}$$

3. r · (−2) + 2. r

3. r · 3 + 1. r

2. r · 2 + 1. r

(3) $g(x) = \sqrt{4x^2+x} - 2x$

$4x^2+x \geq 0$

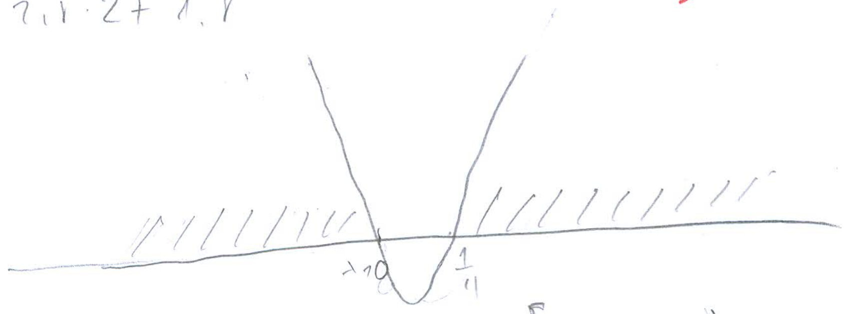
$4x^2+x = 0$

$x(4x+1)$

$x_1 = 0$

$4x+1 > 0 \; / :4$

$x = \frac{1}{4}$



$Dg) \; \langle -\infty, 0] \cup [\frac{1}{4}, +\infty \rangle$

V. A =

$\lim_{x \to 0} \sqrt{4x^2+x} - 2x = 0$

$x = 0$

$\lim_{x \to 1} \sqrt{4x^2+x} - 2x = \frac{\sqrt{4x^2+x} - 2x}{1} \cdot \frac{\sqrt{4x^2+x} + 2x}{\sqrt{x^2+x} + 2x} = \frac{(\sqrt{x^2+x})^2 - (2x)^2}{\sqrt{x^2+x} + (2x)^2}$

$= \frac{x^2+x-4x^2}{\sqrt{x^2+x} + \cancel{4}x^2} = \frac{-x^2+x \; / :x^2}{\sqrt{x^2+x} + 4x^2 \; / :x^2} = \frac{-1+\frac{1}{x}^{\to 0}}{\sqrt{1+\frac{1}{x}} + 4} = \frac{-1}{1+4} = -\frac{1}{5}$

NEMA

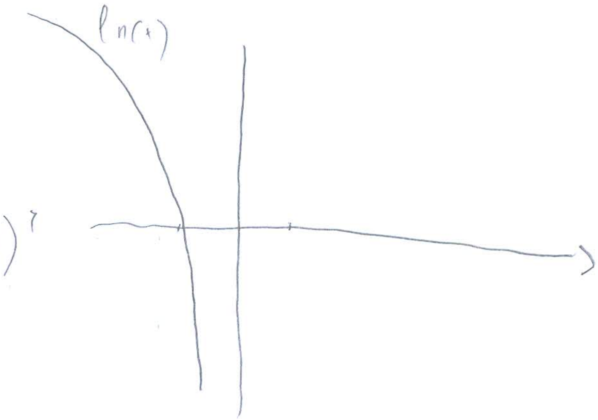
H. A. $\lim_{x \to +\infty} \sqrt{x^2+x} - 2x = \infty$ NEMA DESNE HORIZONTALNE ASIMPT.

$\lim_{x \to -\infty} \sqrt{x^2+x} - 2x \begin{bmatrix} x \to +\infty \\ -\infty \to -\infty \end{bmatrix} \lim_{x \to \infty} \sqrt{(-x)^2 - x} + 2x \; \sqrt{x^2-x} + 2x \cdot \frac{\sqrt{x^2-x} - 2x}{\sqrt{x^2-x} - 2x} = \frac{(\sqrt{x^2-x})^2 - 4x^2}{\sqrt{x^2-x} - 2x}$

$\lim_{x \to \infty} \frac{x^2-x-4x^2}{\sqrt{x^2-x} - 2x} = \frac{-3x^2-x \; / :x^2}{\sqrt{x^2-x} - 2x \; / :x^2} = \frac{-3 - \frac{1}{x}^{\to 0}}{\sqrt{1-\frac{1}{x}} - 2\frac{1}{x}} = \frac{-3}{1} = -3$

Ime i prezime: MATE LADIĆ

Broj indeksa: 17-1-0006-2010



$$f(x) = \ln(x^2+4) + \sin(x-2)$$

$$f'(x) = \frac{1}{x^2+4} \cdot (x^2+4)' + \cos(x-2) \cdot (x-2)'$$

$$f'(x) = \frac{1}{x^2+4} \cdot 2x + \cos(x-2) \cdot 1$$

$$f'(x) = \frac{2x}{x^2+4} + \cos(x-2)$$

20

$x-2=0$

$x=2$

$x^2+4>0$

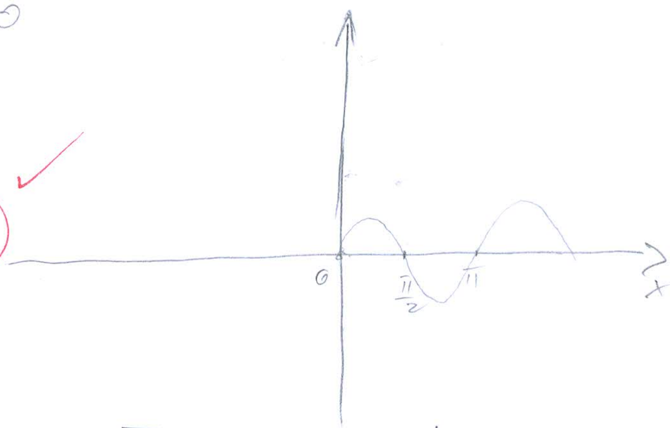
$x^2+4=0$

$x^2=-4$

$x=\sqrt[2]{-4}$

$x=2i$

$D(f)=\mathbb{R}$



$$0+\frac{2k\pi}{2} < x-2 < \frac{2k\pi}{2}$$

DESNA A.A

$y = kx + l$

$$k = \lim_{x \to +\infty} \frac{f(x)}{x} = \frac{\sqrt{x^2+x} - 2x}{x} \Big| :x = \frac{\sqrt{1+1} - 2}{1} = \sqrt{2} - 2$$

$$l = f(x) - k \cdot x$$

$$\lim_{x \to +\infty} \sqrt{x^2+x} - 2x \cdot \sqrt{2} - 2$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊	POPUNJAVA NASTAVNIK
IME I PREZIME: Branimir Pijevac BROJ INDEKSA: 17-2-0086-2011	Broj ↓ bodova
VRIJEME POČETKA: VRIJEME ZAVRŠETKA:	40

1. Riješiti jednadžbu: $z^3 + \overline{1+i} = 0$. — 20
2. Riješi sustav Gaussovom metodom: — 20

$$\begin{array}{rcrcrcrl} 2x_1 & - & x_2 & + & x_3 & - & x_4 = & -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 = & 1 \\ 3x_1 & & & - & x_3 & + & x_4 = & -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 = & -1 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{4x^2+x} - 2x\right)$. — 5+15
4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{e^x}{x}$. — 20(graf)
5. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2+4) + \sin(x-2)$. — 5+15

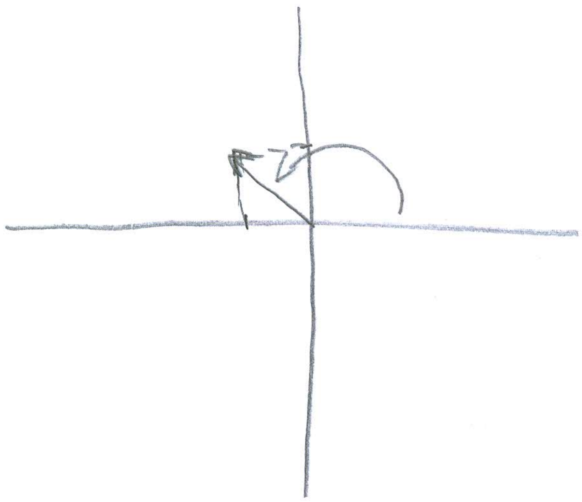
Ukupno:

1) $z^3 + \overline{1+i} = 0$

$z^3 + 1 - i = 0$

$z^3 = -1 + i$

$z = \sqrt[3]{-1+i}$



$|z| = \sqrt{(-1)^2 + (1)^2}$

$|z| = \sqrt{1+1}$

$= \sqrt{2}$

$\operatorname{tg}\varphi = \frac{y}{x} = \frac{1}{-1} = -1 = -45^\circ$

$\varphi = 180 - 45 = 135^\circ$

$z = \sqrt[3]{\sqrt{2}}\left(\cos\frac{\varphi + k\cdot 360^\circ}{3} + i\sin\frac{\varphi + k\cdot 360}{3}\right)$

$k=0 \quad = \sqrt[6]{2}\left(\cos\frac{135+0\cdot 360}{3} + i\sin\frac{135+0\cdot 360}{3}\right)$

$= \sqrt[6]{2}\,(\cos\frac{135+0}{3} + i\sin\frac{135+0}{3}$

$k=1 \quad = \sqrt[6]{2}\left(\cos\frac{135+1\cdot 360^\circ}{3} + i\sin\frac{135+1\cdot 360}{3}\right)$

$k=2 \quad = \sqrt[6]{2}\,(\cos\frac{135+2\cdot 360^\circ}{3} + i\sin\frac{135+2\cdot 360}{3}$

✓ 20

$$
\begin{aligned}
2x_1 - x_2 + x_3 - x_4 &= -1 \\
2x_1 - x_2 \phantom{{}+x_3} - 3x_4 &= 1 \\
3x_1 \phantom{{}-x_2} - x_3 + x_4 &= -1 \\
2x_1 + 2x_2 - 2x_3 + 5x_4 &= -1
\end{aligned}
$$

$$
\left[\begin{array}{cccc|c}
2 & -1 & 1 & -1 & -1 \\
2 & -1 & 0 & -3 & 1 \\
3 & 0 & -1 & 1 & -1 \\
2 & 2 & -2 & 5 & -1
\end{array}\right]
$$

1r : 2

$$
\sim \left[\begin{array}{cccc|c}
1 & -1/2 & 1/2 & -1/2 & -1/2 \\
2 & -1 & 0 & -3 & 1 \\
3 & 0 & -1 & 1 & -1 \\
2 & 2 & -2 & 5 & -1
\end{array}\right]
\sim \left[\begin{array}{cccc|c}
1 & -1/2 & 1/2 & -1/2 & -1/2 \\
0 & 0 & -1 & -2 & 2 \\
0 & 3/2 & -5/2 & 5/2 & 1/2 \\
0 & 3 & -3 & 6 & 0
\end{array}\right]
$$

1r·(−2)+2r
1r·(−3)+3r
1r·(−2)+4r

4r <-> 2r

$$
\sim \left[\begin{array}{cccc|c}
1 & -1/2 & 1/2 & -1/2 & -1/2 \\
0 & 3 & -3 & 6 & 0 \\
0 & 3/2 & -5/2 & 5/2 & 1/2 \\
0 & 0 & -1 & -2 & 2
\end{array}\right]
\sim \left[\begin{array}{cccc|c}
1 & -1/2 & 1/2 & -1/2 & -1/2 \\
0 & 1 & -1 & 2 & 0 \\
0 & 3/2 & -5/2 & 5/2 & 1/2 \\
0 & 0 & -1 & -2 & 2
\end{array}\right]
$$

2r : 3

2r·(−3/2)+3r
2r·(1/2)+1r

$$
\sim \left[\begin{array}{cccc|c}
1 & 0 & 0 & 1/2 & -1/2 \\
0 & 1 & -1 & 2 & 0 \\
0 & 0 & -1 & -\frac{1}{2} & 1/2 \\
0 & 0 & -1 & -2 & 2
\end{array}\right]
\sim \left[\begin{array}{cccc|c}
1 & 0 & 0 & 1/2 & -1/2 \\
0 & 1 & -1 & 2 & 0 \\
0 & 0 & 1 & 1/2 & -1/2 \\
0 & 0 & -1 & -2 & 2
\end{array}\right] \sim
$$

3r : (−1)

3r·(−1)+4r

$$
\left[\begin{array}{cccc|c}
1 & 0 & 0 & 1/2 & -1/2 \\
0 & 1 & 0 & 4 & -1/2 \\
0 & 0 & 1 & 2 & -1/2 \\
0 & 0 & 0 & 0 & 5/2
\end{array}\right]
\sim \left[\begin{array}{cccc|c}
1 & 0 & 0 & 1/2 & -1/2 \\
0 & 1 & -1 & 2 & 0 \\
0 & 0 & 1 & 2 & -1/2 \\
0 & 0 & 0 & 0 & 2
\end{array}\right]
$$

$$
\sim \left[\begin{array}{cccc|c}
1 & 0 & 0 & 1/2 & -1/2 \\
0 & 1 & -1 & 2 & 0 \\
0 & 0 & -1 & -1/2 & 1/2 \\
0 & 0 & -1 & -2 & 2
\end{array}\right]
\sim \left[\begin{array}{cccc|c}
1 & 0 & 0 & 1/2 & -1/2 \\
0 & 1 & -1 & 2 & 0 \\
0 & 0 & 1 & 1/2 & -1/2 \\
0 & 0 & -1 & -2 & 2
\end{array}\right]
\sim \left[\begin{array}{cccc|c}
1 & 0 & 0 & 1/2 & -1/2 \\
0 & 1 & 0 & 5/2 & -1/2 \\
0 & 0 & 1 & 1/2 & -1/2 \\
0 & 0 & 0 & -3/2 & 3/2
\end{array}\right]
$$

3 : (−1)

3r·1+2r
3r·1+2r

4r·(−1/2)+3r
4r·(−5/2)+2r
4r·$\frac{(-1)}{2}$+1r

IME I PREZIME: Branimir Pijaca BROJ INDEKSA: 17-2-0086-2011

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 1 & 0 & 5/2 & -1/2 \\ 0 & 0 & 1 & 1/2 & -1/2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

20

$4r \cdot (-1/2) + 3r$

$4r \cdot (-5/2) + 2r$

$4r \cdot (-1/2) + 1r$

$2x_1 - x_2 + x_3 - x_4 = -1$

$2 \cdot 0 - 2 + 0 - 1 = -1$

$2 - 2 - 1 = -1$

$-1 = -1$ ✓

$2x_1 - x_2 - 3x_4 = 1$

$2 \cdot 0 - 2 + 3 = 1$

$1 = 1$ ✓

$3x_1 - x_3 + x_4 = -1$

$0 - 0 - 1 = -1$

$-1 = -1$ ✓

$2x_1 + 2x_2 - 2x_3 + 5x_4 = -1$

$0 + 4 - 0 - 5 = -1$

$-1 = -1$ ✓

3) Domena i asimptote

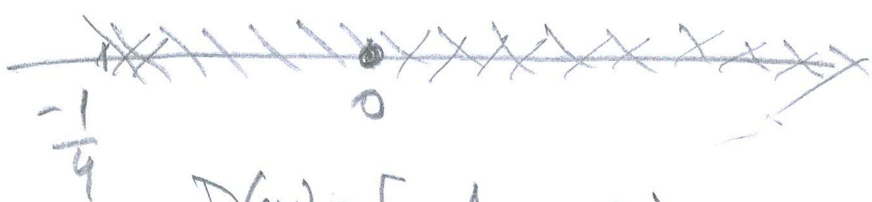
$g(x) = \sqrt{4x^2 + x} - 2x$

$D\,g(x) =$

$\sqrt{4x^2+x} \geq 0 \quad /^2$ $\qquad x \geq 0 \qquad 4x+1 \geq 0$

$4x^2 + x \geq 0 \qquad\qquad\qquad 4x \geq -1$

$x \cdot (4x+1) \geq 0 \qquad\qquad x \geq -\frac{1}{4}$



$D(gx) = \left[-\frac{1}{4}, +\infty \right\rangle$

V.A

$\lim_{x \to 0} \sqrt{4x^2+x} - 2x = \sqrt{0^2+0} - 0 = 0$

$\lim_{x \to -\frac{1}{4}} \sqrt{4x^2+x} - 2x = \frac{1}{2}$ $\qquad x = \frac{1}{2}$ Vertikalna asimptota

H.A

$\lim_{x \to \infty} \sqrt{4x^2+x} - 2x = \infty$

$\lim_{x \to -\infty} \sqrt{4x^2+x} - 2x = \overset{-\infty \mapsto +\infty}{x \mapsto -x} = \lim_{x \to \infty} \sqrt{4 \cdot (-x)^2 - x} + 2x$

$\lim_{x \to \infty} \sqrt{4x^2 - x} + 2x = \infty$ Nema horizontalnih asimptota

IME I PREZIME: Branimir Pijuca BROJ INDEKSA: 17-2-0086-2011

3) Kose asimptote

$y = kx + l$

$k = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{\sqrt{4x^2+x} - 2x}{\cancel{x}} \cdot \frac{\sqrt{4x^2+x} + 2x}{1}$

$= \lim_{x \to \infty} \frac{(\sqrt{4x^2+x})^2 - (2x)^2}{\cancel{x}} = \lim_{x \to \infty} \frac{\cancel{4x^2}+x-\cancel{4x^2}}{x} \; /:x = \boxed{\frac{1}{1} = 1}$

$l = \lim_{x \to \infty} f(x) - k \cdot x = \lim_{x \to \infty} \sqrt{4x^2+x} - 2x - x$

$\lim_{x \to \infty} \sqrt{4x^2+x} - 3x \cdot \frac{\sqrt{4x^2+x} + 3x}{1}$

$\lim_{x \to \infty} \frac{(\sqrt{4x^2+x})^2 - (3x)^2}{1} = \lim_{x \to \infty} \frac{4x^2+x-9x^2}{1} = \lim_{x \to \infty} \frac{-5x^2+x}{1} \; /:x^2$

$\lim_{x \to \infty} \frac{\frac{-5x^2}{x^2} + \frac{x}{x^2}}{\frac{1}{x^2}} = \frac{-5+0}{0} = \infty$ Nema kosih asimptota

⑤ $D(f(x)) = \langle 2, +\infty \rangle$

$f(x) = \ln(x^2+4) + \sin(x-2)$

$x^2+4>0$

$x^2=-4$

$x^2 = \pm\sqrt{-4}$

$x_1 = 2 > 0$

$f(x) = \ln(x^2+4)+\sin(x-2)$

$f'(x) = \frac{1}{x^2+4} + \cos(x-2)\cdot 1$

$f'(x) = \frac{1}{x^2+4}+\cos(x-2)$

---

4)

$h(x)=\frac{e^x}{x}$

Domena $(h) = \mathbb{R}\setminus\{0\}$

$x\neq 0$

Nul točke

$e^x \neq 0$

nema nul točaka

Asimptote

V.A $\lim_{x\to\infty} \frac{e^x}{x} = \frac{e^0}{0} = e^1$ e = vertikalna asimptota

H.A $\lim_{x\to\infty}\frac{e^x}{x} = \frac{e^\infty}{\infty}=\infty$ nema horizontalnih asimptota

I derivacija $h(x)=\frac{e^x}{x}$

$h'(x) = \frac{e^x\cdot x - e^x\cdot 1}{x^2} = \frac{e^x\cdot x - e^x}{x^2} = \frac{x}{x^2}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: RINO KURTIN **BROJ INDEKSA:** 17-2-0112-2011

VRIJEME POČETKA: 08:00 **VRIJEME ZAVRŠETKA:** 10:00

POPUNJAVA NASTAVNIK

Broj ↓ bodova

30

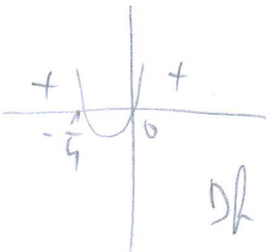
1. Riješiti jednadžbu: $z^3 + \overline{1+i} = 0$.	20
2. Riješi sustav Gaussovom metodom:	20

$$\begin{array}{rcrcrcrl} 2x_1 & - & x_2 & + & x_3 & - & x_4 = & -1 \\ 2x_1 & - & x_2 & & & - & 3x_4 = & 1 \\ 3x_1 & & & - & x_3 & + & x_4 = & -1 \\ 2x_1 & + & 2x_2 & - & 2x_3 & + & 5x_4 = & -1 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{4x^2+x} - 2x\right)$.	5+15
4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{e^x}{x}$.	20(graf)
5. Odrediti domenu i prvu derivaciju funkcije: $f(x) = \ln(x^2+4) + \sin(x-2)$.	5+15
	Ukupno:

3. $f(x) = \sqrt{4x^2+x} - 2x$

$4x^2 + x \geq 0$



$4x^2 + x = 0$

$x(4x+1) = 0$

$\downarrow$

$x = 0$

$4x + 1 = 0$

$4x = -1 \; / :4$

$x = -\frac{1}{4}$

$D_f = \langle -\infty, -\frac{1}{4}] \cup [0, +\infty\rangle$

V.A.

$\lim_{x \to 0} \sqrt{4x^2+x} - 2x = 0$

$\lim_{x \to -\frac{1}{4}} \sqrt{4x^2+x} - 2x = \frac{1}{2}$

15

NEMA V.A

H.A.

$\lim_{x \to \infty} \sqrt{4x^2+x} - 2x = \infty - \infty$

$\lim_{x \to \infty} \sqrt{4x^2+x} - 2x \cdot \frac{\sqrt{4x^2+x} + 2x}{\sqrt{4x^2+x+2x}}$

$\lim_{x \to \infty} \frac{4x^2 + x - 4x^2}{\sqrt{4x^2+x} + 2x}$

$\lim_{x \to \infty} \frac{x}{\sqrt{4x^2+x} + 2x} \; / :x$

$\lim_{x \to \infty} \frac{\frac{x}{x}}{\sqrt{\frac{4x^2}{x^2} + \frac{x}{x^2}} + \frac{2x}{x}} = \frac{1}{2+2} = \frac{1}{4}$

$y = \frac{1}{4}$ je H.A

→

2. $\left[\begin{array}{cccc|c} 2 & -1 & 1 & -1 & -1 \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right] /:2 \left[\begin{array}{cccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 2 & -1 & 0 & -3 & 1 \\ 3 & 0 & -1 & 1 & -1 \\ 2 & 2 & -2 & 5 & -1 \end{array}\right]$ $\begin{array}{l} R_2-2R_1 \\ R_3-3R_1 \\ R_4-2R_1 \end{array}$

$\left[\begin{array}{cccc|c} 1 & -1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -1 & -2 & 2 \\ 0 & \frac{3}{2} & -\frac{5}{2} & \frac{5}{2} & \frac{1}{2} \\ 0 & 3 & -3 & 6 & 0 \end{array}\right] /:3 \left[\begin{array}{cccc|c} 1 & -1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -1 & -2 & 2 \\ 0 & \frac{3}{2} & -\frac{5}{2} & \frac{5}{2} & \frac{1}{2} \\ 0 & 1 & -1 & 2 & 0 \end{array}\right] \updownarrows$

$\left[\begin{array}{cccc|c} 1 & -1 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & -1 & 2 & 0 \\ 0 & \frac{3}{2} & -\frac{5}{2} & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & -1 & -2 & 2 \end{array}\right]$ $\begin{array}{l} R_1+R_2 \\ \\ R_3-\frac{3}{2}R_2 \end{array}$ $\left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & -1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 & -2 & 2 \end{array}\right] /\cdot(-1)$

$\left[\begin{array}{cccc|c} 1 & 0 & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -1 & -2 & 2 \end{array}\right]$ $\begin{array}{l} R_1+\frac{1}{2}R_3 \\ R_2+R_3 \\ \\ R_4+R_3 \end{array}$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{7}{4} & -\frac{3}{4} \\ 0 & 1 & 0 & \frac{5}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & -\frac{3}{2} & \frac{3}{2} \end{array}\right] /:\left(-\frac{3}{2}\right)$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{7}{4} & -\frac{3}{4} \\ 0 & 1 & 0 & \frac{5}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$ $\begin{array}{l} R_1-\frac{7}{4}R_4 \\ R_2-\frac{5}{2}R_4 \\ R_3-\frac{1}{2}R_4 \end{array}$ $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right] = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$ ✓

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3. K.A $\quad k=4 \quad y=4x$

$k = \frac{f(x)}{x}$

$\lim\limits_{x\to\infty} \frac{\sqrt{4x^2+x}+2x}{x} \;\; /:x$

$\lim\limits_{x\to\infty} \frac{\sqrt{\frac{4x^2}{x^2}+\frac{x}{x^2}}+\frac{2x}{x}}{\frac{x}{x}} = \frac{2+2}{1} = 4$

$y = f(x) - kx$

$\lim\limits_{x\to\infty} \sqrt{4x^2+x} + 2x - 4x$

$\infty - \infty$

$\lim\limits_{x\to\infty} \sqrt{4x^2+x} - 2x \qquad \frac{x}{\sqrt{4x^2+x}-2x} \;\; /:x$

$l = 0 \qquad \frac{1}{2-2} = \frac{1}{0} = +\infty$

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BROJ INDEKSA: 17-2-0112-2011

$x \neq 2$

5. $f(x) = \ln(x^2+4) + \sin(x-2)$

$x^2+4>0$

$x^2>-4$

$\cancel{D_f = \mathbb{R} \setminus \{2\}}$

$f'(x) = \dfrac{1}{x^2+4} \cdot (x^2+4)' + \cos(x-2) \cdot (x-2)'$

$f'(x) = \dfrac{2x}{x^2+4} + \cos(x-2)$

25

4. $f(x) = \dfrac{e^x}{x}$

$x \neq 0$ $\quad D_f = \mathbb{R} \setminus \{0\}$

V.A

pravac $x=0$ je V.A

$\lim\limits_{x \to 0} \dfrac{e^x}{x} = +\infty$

H.A

$\lim\limits_{x \to \infty} \dfrac{e^x}{x} = \dfrac{\infty}{\infty}$ L'H

$f(x) = \dfrac{(e^x)'}{(x)'}$

$\lim\limits_{x \to \infty} e^x = \infty$

$f(x) = \dfrac{e^x}{1}$

NEMA H.A

K.A. NEMA KOSE

$k = \dfrac{f(x)}{x}$

$\lim\limits_{x \to \infty} \dfrac{\frac{e^x}{x}}{\frac{x}{1}}$

$\lim\limits_{x \to \infty} \dfrac{e^x}{x^2}$

$\lim\limits_{x \to \infty} \dfrac{e^x}{2x}$

$\lim\limits_{x \to \infty} \dfrac{e^x}{2} = +\infty$

$f'' = e^x$

$f'(x) = \dfrac{e^x}{1} = e^x$

$e^x = 0$ NEMA MIN. NI MAX.