

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME:

RJEŠENJE 2

BROJ INDEKSA:

VRIJEME POČETKA:

VRIJEME ZAVRŠETKA:

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. 20

2. Riješi sustav Gaussovom metodom: 20

$$\begin{array}{rccccrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & 0 \\ & & x_2 & - & x_3 & + & x_4 & = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & 7 \\ & & - & 7x_2 & + & 3x_3 & + & x_4 & = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = (\sqrt{x^2 - 5x + 1} - x)$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{9-x^2}$. 20(graf)

5. Odrediti domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x)$. 2+5+4+9

Ukupno:

① $|z| = 3i \cdot 2(z+i)$ $z = x+iy$

$$\sqrt{x^2+y^2} = 6i \cdot (x+iy+i)$$

$$\sqrt{x^2+y^2} = 6ix - 6y - 6$$

Realna: $\sqrt{x^2+y^2} = -6y - 6$

Imagin: $0 = 6x \Rightarrow x = 0$

$\sqrt{y^2} = -6y - 6$

$y^2 = y$ za $y > 0$
 $y^2 = -y$ za $y < 0$

$\pm y = -6y - 6 \Rightarrow$ ili $7y = -6 \Rightarrow y = -\frac{6}{7}$ $\& y > 0$ ✗
ili $5y = 6 \Rightarrow y = \frac{6}{5}$ $\& y < 0$ ✗

SAMO 1 RJEŠENJE: $z = \frac{-6}{5}i$

②

$$\begin{bmatrix} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{bmatrix} \xrightarrow{R3-R1} \begin{bmatrix} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{bmatrix} \xrightarrow{R3-5R2, R4+7R2} \begin{bmatrix} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & 8 & -8 & -8 \end{bmatrix} \xrightarrow{\cdot \frac{1}{2}} \begin{bmatrix} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 8 & -8 & -8 \end{bmatrix}$$

~~$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 8 & -14 \end{bmatrix}$~~

~~$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -14 \end{bmatrix}$~~

~~$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -14 \end{bmatrix}$~~

$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -4 \end{bmatrix}$ **SUSTAV NEMA RJEŠENJA**

③ $g(x) = \sqrt{x^2 - 5x + 1} - x$

LIJEVA ASIMPTOTA: $\lim_{x \rightarrow -\infty} g(x) = +\infty + \infty = +\infty$ NEMA

DOMENA: $x^2 - 5x + 1 \geq 0$

$$x_{1,2} = \frac{5 \pm \sqrt{25-4}}{2}$$

$$x_1 = \frac{5 - \sqrt{21}}{2} \quad x_2 = \frac{5 + \sqrt{21}}{2}$$

DOMENA = $\left\{ x \in \left(-\infty, \frac{5 - \sqrt{21}}{2} \right] \cup \left[\frac{5 + \sqrt{21}}{2}, +\infty \right) \right\}$

DESNA ASIMPTOTA: $\lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 - 5x + 1} - x) \cdot (\sqrt{x^2 - 5x + 1} + x)}{\sqrt{x^2 - 5x + 1} + x} =$

$$= \lim_{x \rightarrow \infty} \frac{x^2 - 5x + 1 - x^2}{\sqrt{x^2 - 5x + 1} + x} = \dots = -\frac{5}{2} \quad \boxed{x = -\frac{5}{2}}$$

LIJEVA KOSA: $\lim_{x \rightarrow \infty} g(x) = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 5x + 1} + x}{-x} =$

$$= \lim_{x \rightarrow \infty} -\sqrt{\frac{x^2 - 5x + 1}{x^2}} - 1 = -2 = k$$

$$\lim_{x \rightarrow -\infty} g(x) - kx = \lim_{x \rightarrow -\infty} \sqrt{x^2 - 5x + 1} + x - 2x = \lim_{x \rightarrow -\infty} (\sqrt{x^2 - 5x + 1} - x) \cdot \frac{\sqrt{x^2 - 5x + 1} + x}{\sqrt{x^2 - 5x + 1} + x}$$

$$= \lim_{x \rightarrow -\infty} \frac{x^2 - 5x + 1 - x^2}{\sqrt{x^2 - 5x + 1} + x} = \frac{5}{2}$$

L.K.A. $y = -2x + \frac{5}{2}$

$$(4) f(x) = \frac{5-x}{9-x^2} = \frac{5-x}{(3-x)(3+x)}$$

$$\text{DOMĚNA} = \mathbb{R} \setminus \{-3, 3\}$$

$$f'(x) = \frac{-9+x^2+2x(5-x)}{(9-x^2)^2} = \frac{-9+10x-x^2}{(9-x^2)^2} = 0$$

$$\text{za } -9+10x-x^2=0$$

$$x^2-10x+9=0$$

$$x_{1,2} = \frac{10 \pm \sqrt{100-36}}{2} = \frac{10 \pm 8}{2} \rightarrow x_1=1, x_2=9$$

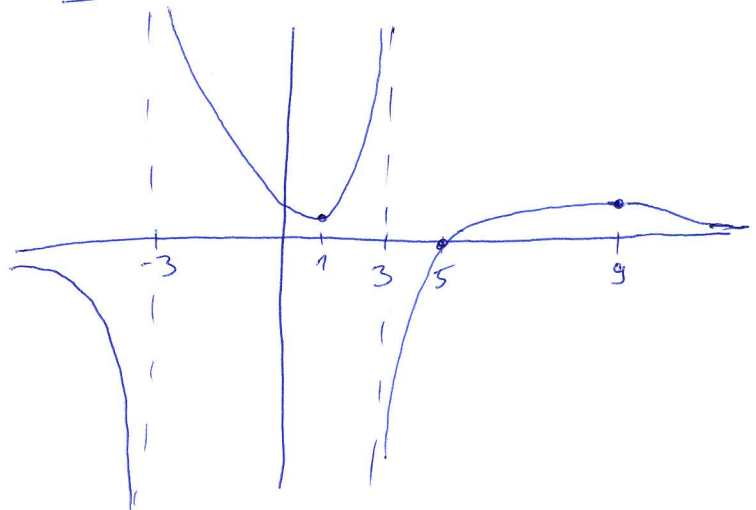
	$-\infty$	4	0	2	4	10	$+\infty$
		-3		3			
$f'(x)$		-	-	+	+	-	
$f(x)$		↘	↘	↗	↗	↘	

LOK
MIN
 $f(1) = \frac{1}{2}$

LOK
MAX
 $f(9) =$

NULOČKA $x=5$

SKICA GRAFA:



$$\lim_{x \rightarrow +\infty} f(x) = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

$$(5) h(x) = \cos(4x)$$

$$D(h) = \mathbb{R}$$

$$h(x) = h(x+P) \Leftrightarrow \cos(4x) = \cos(4x+4P) \quad \Rightarrow \quad 4P = 2\pi \Rightarrow P = \frac{\pi}{2} \text{ PERIOD}$$

ZNAMO $\cos(x) = \cos(x+2\pi) \Rightarrow \cos(4x) = \cos(4x+2\pi)$

$$h(-x) = \cos(-4x) = \cos(4x) \quad \Rightarrow \quad g(x) \text{ JE PARNÁ}$$

$\cos(x) = \cos x$

$$h'(x) = 4 \cos'(4x) = -4 \sin(4x)$$

$$h''(x) = -16 \cos(4x)$$

