MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: RJEŠENJE 2 **BROJ INDEKSA:** **VRIJEME POČETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(-2,-3)$ i $C(-3,-2)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,0)$. Izračunati iz definicije $\int\limits_{\partial K} (3-2y)\,\mathrm{d}s$.. 20

3. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (x-y)\,\mathrm{d}y$. 20

4. Neka je X stožac. Baza stošca je krug postavljen je u $x-y$ ravnini s centrom u ishodištu i radijusom $r = 2$. Vrh stošca je u točki $T(0,0,3)$. Izračunati $\iiint\limits_X (2x+1)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20

5. Plohama $x = 0$, $y = 0$, $z = 0$ i $x + y + z = 1$ omeđena je piramida P. Plašt piramide usmjeren prema van označen je sa ∂P. Izračunati $\iint\limits_{\partial P} (x-y)\,\mathrm{d}y\mathrm{d}z$. 20

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

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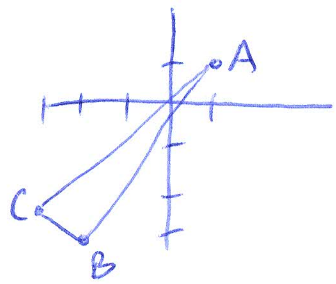


$BC: y = -5 - x$

$AB: (y-1)\underbrace{(-2-1)}_{=-3} = (x-1)\underbrace{(-3-1)}_{=-4}$

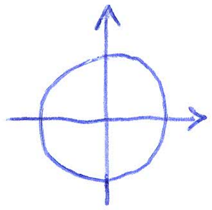
$y = \frac{4}{3}x - \frac{1}{3}$

$AC: (y-1)(-3-1) = (x-1)(-2-1)$

$y = \frac{3}{4}x + \frac{1}{4}$

$$I = \int_{-3}^{-2}\int_{-5-x}^{\frac{3}{4}x+\frac{1}{4}} (x-y)\,dy\,dx + \int_{-2}^{1}\int_{\frac{4}{3}x-\frac{1}{3}}^{\frac{3}{4}x+\frac{1}{4}} (x-y)\,dy\,dx = \ldots = 0$$

②



PARAMETRIZACIJA $r(t)=(\cos t, \sin t)$, $t\in[0,2\pi]$

$r'(t)=(-\sin t, \cos t)$

$\|r'(t)\|=\sqrt{\sin^2 t+\cos^2 t}=1$

$$\int_{\partial K} 3-2y\, ds=\int_0^{2\pi}(3-2\sin t)\cdot 1\, dt=6\pi$$

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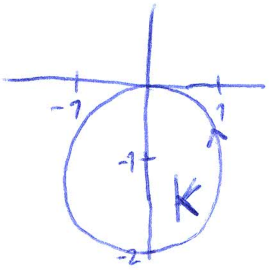
③



$$\int_{\partial K}(x-y)\,dy=\left\{\begin{matrix}\text{GREENOVA}\\ \text{FORMULA}\end{matrix}\right\}=\iint_K 1\,dx\,dy=\ldots=\pi$$

$$\left\{\begin{matrix}\text{PREKO PARAMETRIZACIJE}\\ \text{I DEFINICIJE}\\ r(t)=(\cos t, \sin t-1)\end{matrix}\right\}=\int_0^{2\pi}\underbrace{\begin{bmatrix}0\\ \cos t-\sin t+1\end{bmatrix}\cdot\begin{bmatrix}-\sin t\\ \cos t\end{bmatrix}}_{=\cos^2 t-\sin t\cos t+\cos t}dt=\ldots=\pi$$

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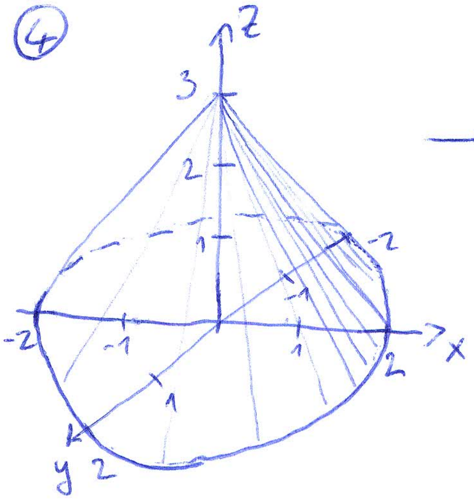
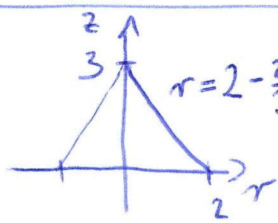
④





$r=2-\frac{2}{3}z$

$z\in[0,3]$ $\qquad x=r\cos\varphi$

$\varphi\in[0,2\pi]$ $\qquad y=r\sin\varphi$

$r\in[0,2-\frac{2}{3}z]$ $\qquad z=z$

$$\iiint_X 2x+1\,dx\,dy\,dz=\int_0^{2\pi}\int_0^3\int_0^{2-\frac{2}{3}z}(2\cos\varphi\cdot r+1)\cdot r\,dr\,dz\,d\varphi=$$

$$=\underbrace{2\int_0^{2\pi}\cos\varphi}_{=0}\int_0^3\int_0^{2-\frac{2}{3}z}r^2\,dr\,dz\,d\varphi+2\pi\int_0^3\int_0^{2-\frac{2}{3}z}r\,dr\,dz=\cancel{2}\pi\int_0^3\frac{(2-\frac{2}{3}z)^2}{\cancel{2}}dz=$$

$=0$

$$=\pi\left(4z-\frac{\cancel{8}\,4}{3}\cdot\frac{z^2}{\cancel{2}}+\frac{4}{9}\frac{z^3}{3}\right)\Big|_0^3=4\pi$$

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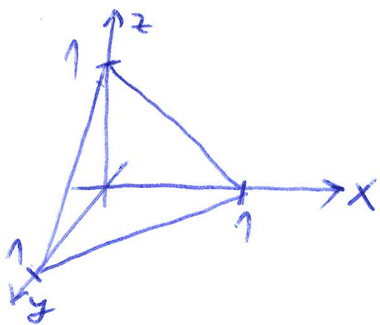
⑤ P piramida, ∂P plašt usmjeren van, $w=\begin{bmatrix}x-y\\0\\0\end{bmatrix}$, $\operatorname{div} w=1$

TEOREM O DIVERGENCIJI

$$\iint_{\partial P}(x-y)\,dy\,dz=\iiint_P 1\,dx\,dy\,dz=\int_0^1\int_0^{1-x}\int_0^{1-x-y}1\,dz\,dy\,dx=\int_0^1\int_0^{1-x}1-x-y\,dy\,dx=$$

$$=\int_0^1 1-x-x(1-x)-\frac{(1-x)^2}{2}dx=\int_0^1\frac{1}{2}-x+\frac{1}{2}x^2\,dx$$

$$=\frac{1}{2}-\frac{1}{2}+\frac{1}{6}=\frac{1}{6}$$



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ **IME I PREZIME:** MARKO VULELIJA **BROJ INDEKSA:** 57660 **VRIJEME POČETKA:** 8:40 **VRIJEME ZAVRŠETKA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(-2,-3)$ i $C(-3,-2)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. ~~20~~

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Ukupno: 0

Tablica integrala

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$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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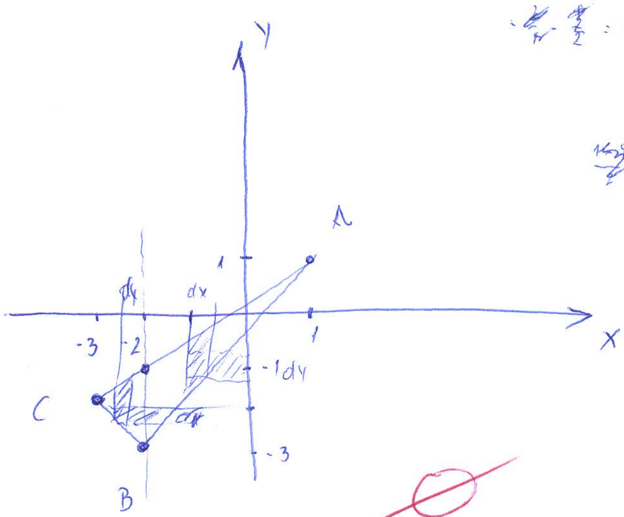
Ime i prezime: MARKO VULELIJA **Broj indeksa:** 57660

① $A(1,1)$ $\quad f(x,y) = x - y$

$B(-2,-3)$

$C(-3,-2)$ $\quad \iint_T f(x,y)\,dx\,dy$



$$\iint_T (x-y)\,dx\,dy$$

$$\int_{-3}^{-1} x\,dx \int_{-3}^{-2} -y\,dy = \int_{-3}^{-1} x\,dx \left| -\frac{y^2}{2} \right|_{-3}^{-2} = \int_{-3}^{-1} x\,dx \left(-\frac{4}{2} - \frac{9}{2}\right) = \int_{-3}^{-1} x\,dx \left(-2 - \frac{9}{2}\right) = -\int_{-3}^{-1} \frac{13}{2} x\,dx$$

$$= -\frac{13}{2}\int_{-3}^{-1} x\,dx = -\frac{13}{2}\,\frac{x^2}{2}\Big|_{-3}^{-1} = -\frac{13}{2}\left(\frac{1}{2} - \frac{9}{2}\right) = -\frac{13}{2}\cdot 4 = -\frac{52}{2} = 26 \quad =$$

$$\int_{-2}^{1} x\,dx \int_{-3}^{1} -y\,dy = \int_{-2}^{1} x\,dx \left| -\frac{y^2}{2} \right|_{-3}^{1} = \int_{-2}^{1} x\,dx \left(-\frac{1}{2} + \frac{9}{2}\right) = \int_{-2}^{1} x\,dx\,4 = 4\int_{-2}^{1} x\,dx = 4\,\frac{x^2}{2}\Big|_{-2}^{1}$$

$$= 4\left(\frac{1}{2} - \frac{2}{2}\right) = 4\left(\frac{1}{2} - 1\right) = 4\cdot\frac{1}{2} = 2$$

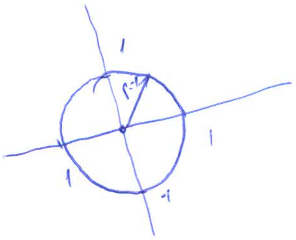
$26 + 2 = 28$

BROJ INDEKSA: 5 ...

...ELIJA

$x = r\cos\varphi + 1$

$y = r\sin\varphi$



X RAČUNATE? DVOSTRUKI.

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... $r^2 \sin\varphi\, dr$ X

$\ldots \sin\varphi \int_0^1 r^2 dr) = \int_0^{2\pi} d\varphi \left[3\frac{r^2}{2}\Big|_0^1 - 2\sin\varphi \frac{r^3}{3}\Big|_0^1 \right]$

$= \int_0^{2\pi} d\varphi \left[\frac{3}{2} - 2\sin\varphi \frac{1}{3}\right]$

$= \int_0^{2\pi} d\varphi \left(\frac{3}{2} - 2\sin\varphi \frac{1}{3}\right) = \int_0^{2\pi} d\varphi - \int_0^{2\pi} \frac{3}{2} - 2\sin\frac{1}{3}\varphi$

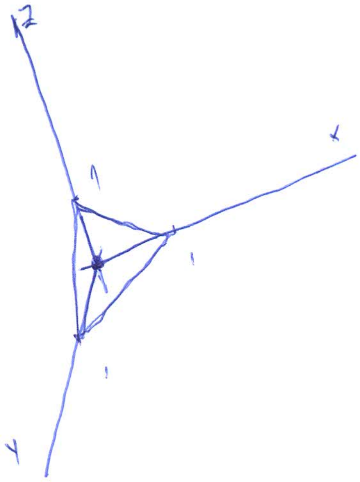
$= \int_0^{2\pi} d\varphi - \frac{3}{2}\int_0^{2\pi} -2\sin\frac{1}{3}\varphi$

$\int_0^{2\pi} d\varphi - \frac{3}{2}\cdot(-2)\int \sin\frac{1}{3}\varphi$

$2\pi + 3(-\cos\frac{1}{3})$

$2\pi + \cos 2\pi = 2\pi$

$\iint_{OP} (x-y)\, dy\, dz$



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