MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

POPUNJAVA NASTAVNIK

IME I PREZIME: RJEŠENJE 1 **BROJ INDEKSA:** **VRIJEME POČETKA:**

Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y)=x-y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20
2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,\mathrm{d}x\mathrm{d}y$. 20
3. Neka je C polukružna krivulja radijusa $r=1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\overset{\frown}{C}} \mathrm{d}x+\mathrm{d}y$.. 20
4. Izračunaj volumen prostora omeđenog plohama $y=x^2$, $y=x$, $z=-1$ i $z=2$. 20
5. Neka je K kugla radijusa $r=2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
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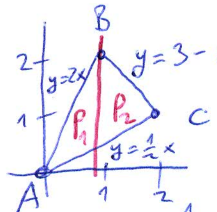
1

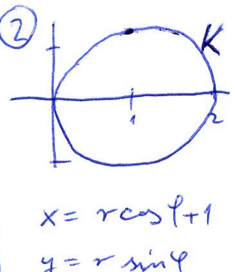


$$P_1 = \int_0^1 \int_{\frac12 x}^{2x} dy\,dx = \int_0^1 \frac32 x\,dx = \frac32\left[\frac{x^2}{2}\right]_0^1 = \frac34$$

$$P_2 = \int_1^2 \int_{\frac12 x}^{3-x} dy\,dx = \int_1^2 3-\frac32 x\,dx = \left[3x-\frac34 x^2\right]_1^2 = 6-3-3+\frac34 = \frac34$$

$$P = P_1 + P_2 = \frac32$$

2



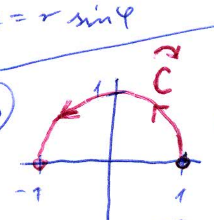
$$\iint_K 2-3y\,dxdy = \int_0^{2\pi}\int_0^1 (2-3r\sin\varphi)\,r\,dr\,d\varphi = 2\pi$$

$x = r\cos\varphi + 1$

$y = r\sin\varphi$

3

PARAMETRIZACIJA: $r(t) = \begin{bmatrix}\cos t\\ \sin t\end{bmatrix},\ t\in[0,\pi]$



VEKTORSKA FUNKCIJA $w(x,y) = \begin{bmatrix}1\\1\end{bmatrix}$

$r'(t) = \begin{bmatrix}-\sin t\\ \cos t\end{bmatrix}$

$$\int_{\overset{\frown}{C}} dx+dy = \int_0^\pi \begin{bmatrix}1\\1\end{bmatrix}\cdot\begin{bmatrix}-\sin t\\ \cos t\end{bmatrix} dt = -\int_0^\pi \sin t\,dt + \int_0^\pi \cos t\,dt$$

$$= \Big[\cos t + \sin t\Big]_0^\pi = -\frac12 + 0 - \frac12 - 0$$

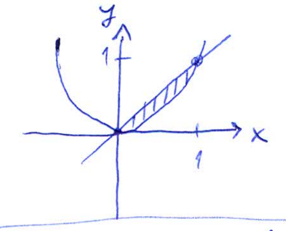
$$= 1$$

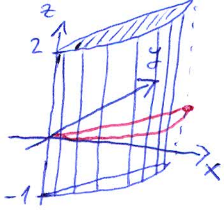
④ $y = x^2$

$y = x$

$z = -1$

$z = 2$





$$V = \int_{-1}^{2}\int_{0}^{1}\int_{x^2}^{x} 1 \, dy \, dx \, dz = 3 \cdot \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 3 \cdot \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{2}$$

---

⑤ K kugla, $r = 2$, centar u ishodište, $w = \begin{bmatrix} 0 \\ 0 \\ 3-2y \end{bmatrix}$

∂K rub kugle (sfera) usmjerena van.

TEOREM O DIVERGENCIJI:

$$\operatorname{div} w = \partial_x w_x + \partial_y w_y + \partial_z w_z = 0$$

$$\iint_{\partial K} w \cdot dS = \iiint_K \operatorname{div} w = \iiint_K 0 = 0$$

---

① OBZIROM DA SE NE TRAŽI POVRŠINA, VEĆ INTEGRAL FUNKCIJE NA TROKUTU RAČUN JE:

$$I = \int_0^1 \int_{\frac{1}{2}x}^{2x} x-y \, dy dx + \int_1^2 \int_{\frac{1}{2}x}^{3-x} x-y \, dy dx = \int_0^1 x \cdot \left(\tfrac{3}{2}x\right) - \frac{4x^2 - \frac{1}{4}x^2}{2} dx + \int_1^2 x \cdot \left(3 - \frac{3}{2}x\right) - \frac{(3-x)^2 - \frac{x^2}{4}}{2} dx$$

$$= \int_0^1 x^2\left(\frac{3}{2} - 2 + \frac{1}{8}\right) dx + \int_1^2 3x - \frac{3}{2}x^2 - \frac{9}{2} + 3x - \frac{1}{2}x^2 + \frac{1}{8}x^2 \, dx =$$

$t = \frac{1}{2}$ $t = 5$

$$= -\frac{3}{8}\left(\frac{x^3}{3}\right)\Big|_0^1 + \left(-\frac{15}{8}\frac{x^3}{3} + 6\frac{x^2}{2} - \frac{9}{2}x\right)\Big|_1^2 = -\frac{1}{8} \underbrace{- \frac{5}{8} \cdot 8}_{-5} \underbrace{+ 3 \cdot 4}_{12} \underbrace{- \frac{9}{2} \cdot 2}_{-9} + \frac{5}{8} - 3 + \frac{9}{2} = 0$$

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POPUNJAVA NASTAVNIK

IME I PREZIME: NIKOLA BOŠNJAK **BROJ INDEKSA:** 53799

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

	Broj ↓ bodova
1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$.	20
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Ukupno:	$\emptyset$

Tablica integrala

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4. $y = x^2$ $y = x$ $z = -1$ $z = 2$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: Mateja Mitrović **BROJ INDEKSA:** 0269037541

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y)=x-y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. ~~20~~
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Ukupno: ~~0~~

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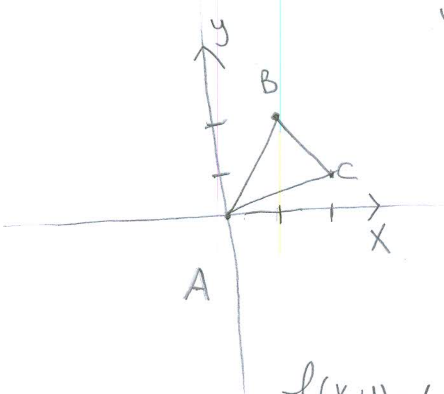
$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$



$A(0,0)\ B(1,2)$

$y-0=\frac{2-0}{1-0}(x-0)$

$y-0=2(x-0)$

$y=2x$

$-2x=y \,/\cdot(-2)$

$x=-2y$

$A(0,0)\ C(2,1)$

$y-0=\frac{1-0}{2-0}(x-0)$

$y-0=\frac{1}{2}x-0$

$y=\frac{1}{2}x$

$\frac{1}{2}x=y \,/\cdot\frac{1}{2}$

$x=\frac{1}{2y}$

$B(1,2)\ C(2,1)$

$y-2=\frac{1-2}{2-1}(x-1)$

$y-2=-1(x-1)$

$y-2=-x+1$

$y=-x+3$

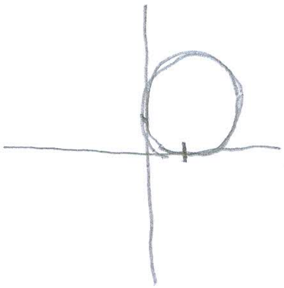
$x=-y+3$

$$f(x,y)=\left(-2y-\frac{1}{2}y+y+3\right)-\left(2x-\frac{1}{2}x+x+3\right)$$

$$f(x,y)=-\frac{3}{2}y-\frac{5x}{2}+3-3=-\frac{3}{2}y-\frac{5}{2}x$$

$$\iint_T \left(-\frac{3}{2}x-\frac{5}{2}y\right) dx\,dy =$$

2.



$$\int_0^{2\pi}\int_0^{1} (2-3y)\,dx\,dy =$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓

IME I PREZIME: IVA PEZEROVIĆ **BROJ INDEKSA:**

VRIJEME POČETKA: 07:50 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. ~~20~~
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Tablica integrala

Ukupno: 0

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4. $V = \iiint 1\,dx\,dz\,d\varphi$

$V = \iiint r\,dr\,d\varphi\,dz$

$V = \int_0^{2\pi}\int_0^{2}\int_{-1}^{2} r\,dz\,dr\,d\varphi$

$V = 2\pi \int_0^2 r(-1-2)\,dr$

$V = 2\pi \int_0^2 -3r\,dr$

$V = 2\pi (3(0-2))\Big|_0^2\,dr$

$V = 2\pi \cdot 6$

$V = 12\pi$

$x^2+y^2 = r^2$

$x^2+y^2 = z^2$

$z=-1 \quad z=2$

$r^2=-1 \quad r^2=2$

~~$r=1$~~ $\quad r=\sqrt{2}$

$y = x^2$

$y = r^2\sin\varphi$

$y = x^2 \Rightarrow x = \sqrt{y}$

$x = \sqrt{r\cos\varphi}$

$z\in[-1,2]$

$r\in[0,2]$,

$\varphi\in[0,2\pi]$

$-x+y=0$

$r=0$

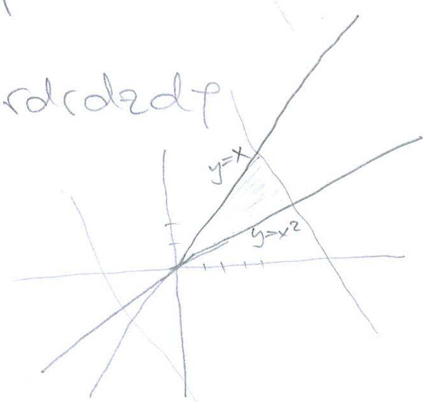
cilindrične koordinate:

$x = r\cos\varphi$

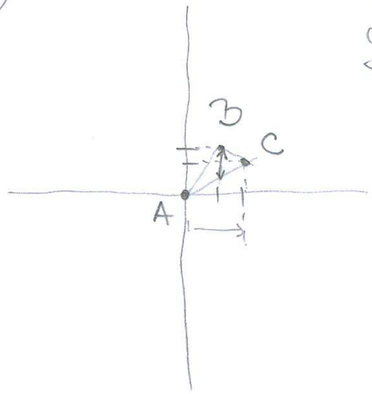
$y = r\sin\varphi$

$z = z$

$dx\,dy\,dz = r\,dr\,dz\,d\varphi$



1.



$x = r\cos\varphi$
$y = r\sin\varphi$

$f(x,y) = x - y$

$\iint f(x,y)\,dx\,dy$

$\int_0^{\frac{\pi}{2}} \int_{\frac{1}{2}x} f(x,y)\,dx\,dy$

$\int_0^2 \int_{x^2}^{\frac{1}{2}x} (x-y)\,dx\,dy$

A(0,0)
B(1,2)
C(2,1)

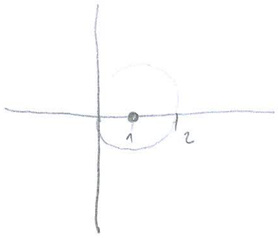
AC $\quad y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \dfrac{1-0}{2-0}(x-0)$

$y = \frac{1}{2}x$

$2\int_{x^2}^{\frac{1}{2}x} (r\cos\varphi - r\sin\varphi)\,dx\,dz = 2\int_{x^2}^{\frac{1}{2}x} r\cos\varphi\,dx - 2\int_{x^2}^{\frac{1}{2}x} -r\sin\varphi\,dy = 2\left(r\cos\varphi\left(x^2 - \frac{1}{2}x\right)\right)$

2. $r = 1 \qquad T(1,0)$



PRIJELAZ NA POLARNE NEDOSTAJE

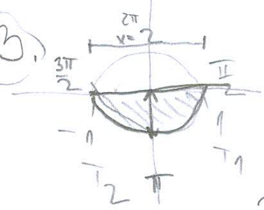
$\iint_K (2 - 3y)\,dx\,dy \qquad y = r\sin\varphi$

$\iint_{K} (2 - 3r\sin\varphi)\,r\,dx\,dy$

$\int_0^{2\pi} \int_0^1 (2 - 3r\sin\varphi)\,r\,dx\,dy$

$2\pi\left(2\cdot(1-0)\right)\Big|_0^1 dx - \underbrace{2\pi\left(3\sin(1-0)\right)dy}_{=0} = 4\pi$

3. $r = 1,\ T(0,0),\ T_1(1,0),\ T_2(-1,0) \qquad t \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$



$\oint dx + dy = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} x\,dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} y\,dy$

$y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \dfrac{0-0}{-1-1}(x-1)$

$y = x - 1$

$= 2\left(\frac{\pi}{2} - \frac{3\pi}{2}\right)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} dx + (x-1)\left(\frac{\pi}{2} - \frac{3\pi}{2}\right)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} dy$

$= 2\pi + x\pi - \pi$

$= \pi + x\pi$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: VICE VIŠIĆ **BROJ INDEKSA:** 57102

VRIJEME POČETKA: 0800 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. ~~20~~
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,\mathrm{d}x\mathrm{d}y$. ~~20~~ 15
3. Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\widehat{C}} \mathrm{d}x + \mathrm{d}y$.. ~~20~~
4. Izračunaj volumen prostora omeđenog plohama $y = x^2$, $y = x$, $z = -1$ i $z = 2$. ~~20~~
5. Neka je K kugla radijusa $r = 2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. ~~20~~

Ukupno: 15

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1. $A(0,0)$ $B(1,2)$ $C(2,1)$

AB $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y - 0 = \frac{2-0}{1-0}(x-0)$

$y = 2x$

AC $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y - 0 = \frac{1-0}{2-0}(x-0)$

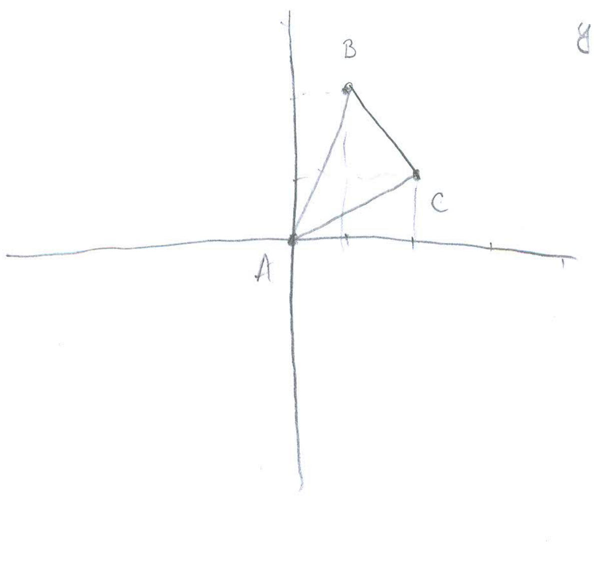
$y = \frac{1}{2}x$

BC $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y - 2 = \frac{1-2}{2-1}(x-1)$

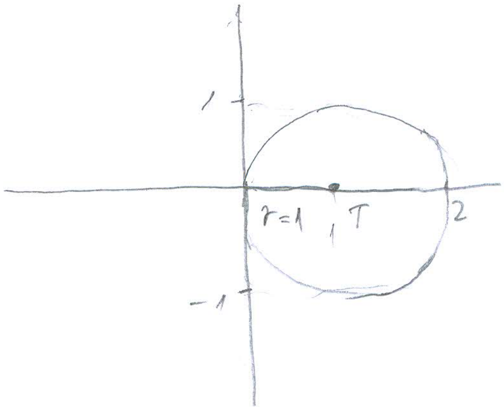
$y - 2 = -x+1$

$y = 3-x$



$$\int_{2x}^{3-x}\int_{\frac{1}{2}x}^{2} x-y \; dxdy$$

2. KRUG $r=1$ $T(1,0)$ Izračunati $\iint_K (2-3y)\,dx\,dy$

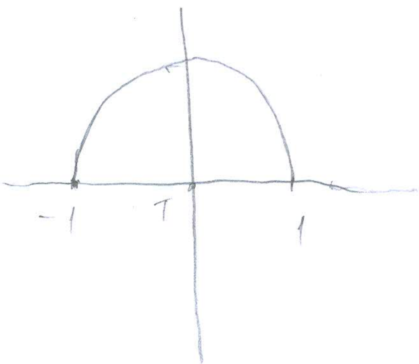


$x = r\cos\varphi + 1$ $\varphi \in [0, 2\pi]$

$y = r\sin\varphi$ $r \in [0, 1]$

$$\boxed{\int_0^{2\pi} d\varphi \int_0^1 \left[2-3(r\sin\varphi)\right] r\,dr}$$ ✓ 1,5

3. Polukružna krivulja $r=1$ $T(0,0)$ početak $T(1,0)$ kraj $T(-1,0)$ $\int_K dx+dy$



$\int_0^{\pi} d\varphi \int_0^1 r\,dr$

$r \in [0,1]$

$\varphi \in [0,\pi]$

Ø

4. Volumen prostora omeđen plohama $y=x^2$ $y=x$ $z=-1$ $z=2$

$V = \int_{?}^{?} dx \int_{x}^{x^2} dy \int_{-1}^{2} dz$ ✗ Ø

5. Kugla $r=2$ Izračunati $\iint_{\partial K} (3-2y)\,dx\,dy$

$\varphi \in [0, 2\pi]$

$r \in [0, 2]$

$\int_0^{2\pi}\int_0^{2} \left[3-2 r\sin\varphi\right] d\varphi\, r\,dr$ ✗ Ø

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊	POPUNJAVA NASTAVNIK
IME I PREZIME: ANTE GRUBIŠA **BROJ INDEKSA:** 57831	Broj ↓ bodova
VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**	

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y)=x-y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. ~~20~~

2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,\mathrm{d}x\mathrm{d}y$. ~~20~~ /15

3. Neka je C polukružna krivulja radijusa $r=1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\widehat{C}} \mathrm{d}x+\mathrm{d}y$.. ~~20~~

4. Izračunaj volumen prostora omeđenog plohama $y=x^2$, $y=x$, $z=-1$ i $z=2$. ~~20~~

5. Neka je K kugla radijusa $r=2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K}(3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

Ukupno: 15

Tablica integrala

$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a}+C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}+C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right)+C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$

1. $A(0,0)$ $B(1,2)$ $C(2,1)$

$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

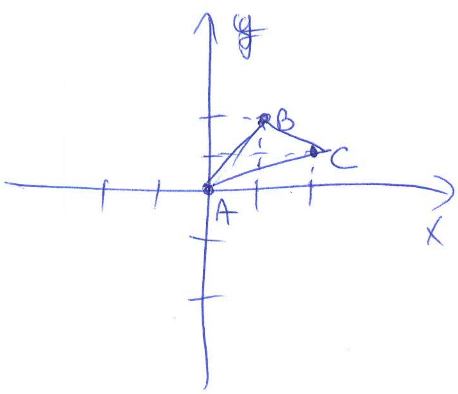
(AC, AB) x y

AB BC x y

AC BC

AB: $y-0=\frac{2-0}{1-0}(x-0)$

$y=2x$ ~~x=2y~~ ~~x=2y~~



BC $y-2=\frac{1-2}{2-1}(x-1)$

$y-2=-x+1$

$y=-x+3$ ~~x=y-1~~ ~~x=y-1~~

AC $y-0=\frac{1-0}{2-0}(x-0)$

$y=\frac{1}{2}x$

DRUGA STRANA ⟶

~~$\int\int x\,dx - y\,dy$~~

~~$\int\int x\,dx - y\,dy$~~

$$\int_{\frac{1}{2}x}^{-x+3}\int_{\frac{1}{2}x}^{2x} x\,dx - y\,dy = \int_{\frac{1}{2}x}^{2x} x\,dx - \int_{\frac{1}{2}x}^{-x+3} y\,dy = \cancel{\int_{\frac{1}{2}x}^{2x} x\,dx} -$$

3.

$\varphi \in [0, \pi]$

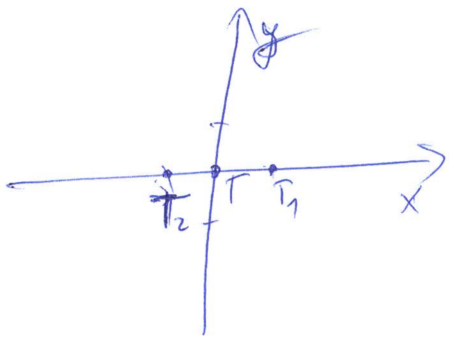
$\cancel{\int_{0}^{\pi}}$

$r \in [0, 1]$

$T(0,0)$

$T_1(1,0)$

$T_2(-1,0)$



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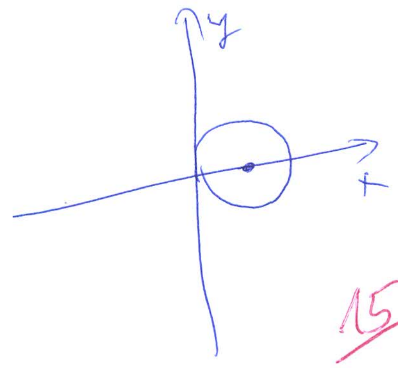
Broj indeksa: 57831

...e: ANTE GRUBIŠA

$\iint_K (2-3y)\,dx\,dy = ?$

$T(1,0)$

$r\cos\varphi + 1$ $\quad r\sin\varphi$



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$\ldots \int_0^1 (2-3r\sin\varphi)\, r\,dr = \int_0^{2\pi} d\varphi \left(2\int_0^1 r\,dr - 3\int_0^1 3r\sin\varphi\, r\,dr\right) = \int_0^{2\pi} d\varphi \left(2 + 3\cos\right)$

$= 2\pi \cdot \left(2\int_0^{2\pi} r\,dr + 3\int_0^{2\pi} \cos r\,dr\right) = 2\pi\left(4\pi + 3\right) = 8\pi^2 + 6\pi$

$y = x^2, \; y = x, \; z = -1, \; z = 2$ $\quad$ Pu44

$\varphi \in [0, 2\pi]$

$z \in [-1, 2]$

$y \in [x, x^2]$

$\int_0^{2\pi} d\varphi \int_{-1}^{2} dz \int_x^{x^2} dy = \int_0^2 d\varphi \int_{-1}^{2} dz \; x^2 - x =$

$\int_0^2 d\varphi \int_{-1}^{2} x^2 - x \, dz = \int_0^2 d\varphi \left(\frac{x^3}{3} - \frac{x^2}{2}\right)$

$\left(\frac{2^3}{3} - \frac{2^2}{2}\right) - \left(\frac{-1^3}{3} - \frac{-1^2}{2}\right)$

$= \frac{1}{2}\int_0^2 d\varphi = \frac{1}{2}\cdot 2 - 0$

$= 1$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: MAGDA MANDIĆ **BROJ INDEKSA:** 0268015993

VRIJEME POČETKA: 08:00 **VRIJEME ZAVRŠETKA:**

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)dxdy$. ~~20~~ 15
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,dxdy$. 20
3. Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\widehat{C}} dx + dy$.. ~~20~~
4. Izračunaj volumen prostora omeđenog plohama $y = x^2$, $y = x$, $z = -1$ i $z = 2$. ~~20~~
5. Neka je K kugla radijusa $r = 2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K} (3-2y)\,dxdy$. ~~20~~ 10

Ukupno: 45

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

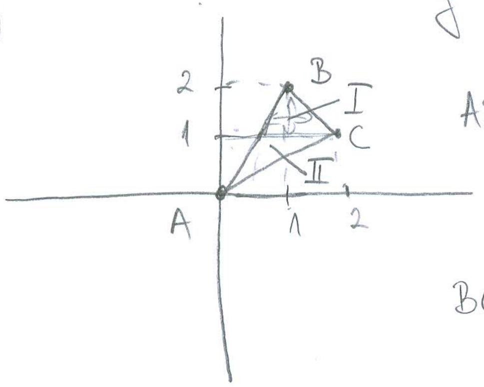
1)

$y - y_1 = \frac{(y_2-y_1)}{(x_2-x_1)}(x-x_1)$

$\iint\limits_T f(x,y)\,dx\,dy = ?$

$f(x,y) = x - y$

$\iint\limits_T (x-y)\,dx\,dy = ?$



AB: $y - 0 = \frac{(2-0)}{(1-0)}(x-0)$

$\boxed{y = 2x} \Rightarrow \boxed{x = \frac{y}{2}}$

BC: $y - 2 = \frac{(1-2)}{(2-1)}(x-1)$

$y = \frac{-1}{1}(x-1) + 2 = -x + 1 + 2$

$\boxed{y = -x + 3} \Rightarrow \boxed{x = 3 - y}$

$A(0,0)$, $B(1,2)$, $C(2,1)$

AC: $y - 0 = \frac{(1-0)}{(2-0)}(x-0)$ $\boxed{x = 2y}$

15

$\iint\limits_T (x-y)\,dxdy = \underbrace{\int_1^2\int_{\frac{y}{2}}^{3-y} (x-y)\,dx\,dy}_{I} + \underbrace{\int_0^1\int_{\frac{y}{2}}^{2y} (x-y)\,dxdy}_{II} \Rightarrow$

$$\mathrm{I}:\ \int_1^2\int_{\frac{y}{2}}^{3-y}(x-y)\,dx\,dy = \int_1^2\int_{\frac{y}{2}}^{3-y} x\,dx\,dy - \int_1^2 y\,dy\int_{\frac{y}{2}}^{3-y}dx =$$

$$= \int_1^2\left(\frac{x^2}{2}\right)\Big|_{\frac{y}{2}}^{3-y}dy - \int_1^2 y\,dy\,(x)\Big|_{\frac{y}{2}}^{3-y} = \int_1^2\left(\frac{(3-y)^2}{2} - \frac{\left(\frac{y}{2}\right)^2}{2}\right)dy - \int_1^2 y\,dy\left(3-y-\left(\frac{y}{2}\right)\right)$$

$$= \int_1^2\left(\frac{9-6y+y^2}{2} - \frac{\frac{y^2}{4}}{2}\right)dy - \int_1^2 y\left(\frac{6-2y-y}{2}\right)dy =$$

$$= \int_1^2\left(\frac{4(9-6y+y^2)-y^2}{8}\right)dy - \int_1^2\left(\frac{6y-3y^2}{2}\right)dy =$$

$$= \frac{1}{8}\cdot\int_1^2\left(36-24y+\underbrace{4y^2-y^2}_{3y^2}\right)dy - \frac{1}{2}\int_1^2\left(6y-3y^2\right)dy =$$

$$= \frac{1}{8}\cdot\left(36y-\cancel{24}^{12}\frac{y^2}{\cancel{2}}+\cancel{3}\frac{y^3}{\cancel{3}}\right)\Big|_1^2 - \frac{1}{2}\left(\cancel{6}^{3}\frac{y^2}{\cancel{2}}-\cancel{3}\frac{y^3}{\cancel{3}}\right)\Big|_1^2 =$$

$$= \frac{1}{8}\cdot\left[(36\cdot2-12\cdot4+8)-(36-12+1)\right] - \frac{1}{2}\cdot\left[(3\cdot4-8)-(3-1)\right] =$$

$$= \frac{1}{8}\cdot(7) - \frac{1}{2}\cdot(2) = \frac{8}{7}-1 = \frac{8-7}{7} = \frac{1}{7}$$

$$\mathrm{II}:\ \int_0^1\int_{\frac{y}{2}}^{2y}(x-y)\,dx\,dy = \int_0^1\int_{\frac{y}{2}}^{2y} x\,dx\,dy - \int_0^1 y\,dy\int_{\frac{y}{2}}^{2y}dx =$$

$$= \int_0^1\left(\frac{x^2}{2}\right)\Big|_{\frac{y}{2}}^{2y}dy - \int_0^1 y\,dy\,(x)\Big|_{\frac{y}{2}}^{2y} = \int_0^1\left(\frac{2y^2}{2} - \frac{\left(\frac{y}{2}\right)^2}{2}\right)dy - \int_0^1 y\left(2y-\frac{y}{2}\right)dy =$$

$$= \int_0^1\left(y^2-\frac{y^2}{8}\right)dy - \int_0^1\left(2y^2-\frac{y^2}{2}\right)dy = \int_0^1\left(\frac{8y^2-y^2}{8}\right)dy - \int_0^1\left(\frac{4y^2-y^2}{2}\right)dy =$$

$$= \int_0^1\frac{1}{8}\cdot7y^2\,dy - \int_0^1\frac{1}{2}\cdot3y^2\,dy = \frac{1}{8}\cdot\left(7\frac{y^3}{3}\right)\Big|_0^1 - \frac{1}{2}\cdot\left(\cancel{3}\frac{y^3}{\cancel{3}}\right)\Big|_0^1 =$$

$$= \frac{1}{8}\cdot\frac{7}{3} - \frac{1}{2}\cdot1 = \frac{7}{24}-\frac{1}{2} = \frac{7-12}{24} = \frac{-5}{24}$$

$$\mathrm{I}+\mathrm{II} = \frac{1}{7}+\left(\frac{-5}{24}\right) = \frac{24-35}{168} = \frac{-11}{168}$$

Ime i prezime: Magda Mandić **Broj indeksa:** 0269015993

2) $r = 1$

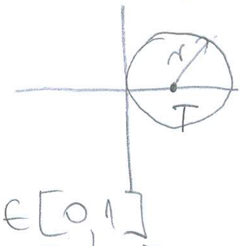
$T(p,q) = T(1,0)$

$y = r\sin\varphi + 0$

$y = r\sin\varphi$

$(x-p)^2+(y-q)^2 = r^2$

$(x-1)^2+(y)^2 = 1$



$r \in [0,1]$

$\varphi \in [0,2\pi]$

$$\iint_K (2-3y)\,dx\,dy = \boxed{\int_0^{2\pi}\int_0^1 (2-3\cdot r\sin\varphi)\,r\,dr\,d\varphi} =$$

$$\int_0^{2\pi}\int_0^1 2r\,dr\,d\varphi - \int_0^{2\pi}\int_0^1 3r\sin\varphi\, r\,dr\,d\varphi = \quad 3r^2dr$$

$$\int_0^{2\pi}\left(\cancel{2}\frac{r^2}{\cancel{2}}\right)\Big|_0^1 d\varphi - \int_0^{2\pi}\sin\varphi\,d\varphi\cdot\left(\cancel{3}\cdot\frac{r^3}{\cancel{3}}\right)\Big|_0^1 =$$

$$\int_0^{2\pi} d\varphi - \int_0^{2\pi}\sin\varphi\,d\varphi = (\varphi)\Big|_0^{2\pi} + (\cos\varphi)\Big|_0^{2\pi} = 2\pi + (\underbrace{\cos 2\pi - \cos 0}_{0})$$

$= 2\pi + 0 = 2\pi$

20

---

4) $y = x^2 \rightarrow x = \sqrt{y}$

$y = x \qquad x = y$

$z = -1$

$z = 2$

$z \in [-1, 2]$

$\varphi \in [0, 2\pi]$

$r \in [$

$\underbrace{x^2+y^2}_{r^2}+z^2 = R^2$

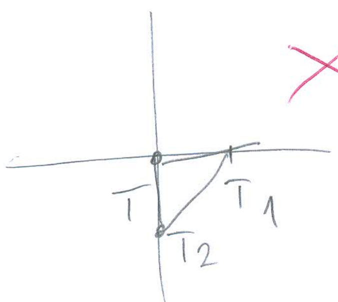
$$V = \int_0^{2\pi}\int_{-1}^{2}\int_{r_1}^{r_2} r\,dr\,d\varphi\,dz$$

---

3) $T(0,0)$

$T_1(1,0)$

$T_2(-1,0)$



$r = 1$

5.) $r = 2$ $\quad T(0,0,0)$ $\quad \iint_{\partial K} (3-2y)\,\underbrace{dx\,dy}_{2.\ \text{vrsta}} = ?$ $\quad w = \begin{pmatrix} 0 \\ 0 \\ 3-2y \end{pmatrix}$

↳ plošni integral

$r(u,v) = (u, \underset{y=v}{v}, \sqrt{1-u^2-v^2})$

$\vec{n} = \left(\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v}\right)$

$$\vec{n} = \begin{bmatrix} 1 \\ 0 \\ \frac{-u}{\sqrt{1-u^2-v^2}} \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ \frac{-v}{\sqrt{1-u^2-v^2}} \end{bmatrix} = \begin{bmatrix} 0 - \frac{-u}{\sqrt{1-u^2-v^2}} \\ 0 - \frac{-v}{\sqrt{1-u^2-v^2}} \\ 1 - 0 \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} \frac{u}{\sqrt{1-u^2-v^2}} \\ \frac{v}{\sqrt{1-u^2-v^2}} \\ 1 \end{bmatrix}$$

$$\int_0^{2\pi}\int_0^{2} \vec{n} \cdot w \, du\,dv = \int_0^{2\pi}\int_0^{2} \begin{pmatrix} \frac{u}{\sqrt{1-u^2-v^2}} \\ \frac{v}{\sqrt{1-u^2-v^2}} \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 3-2\cdot 3\sin v \end{pmatrix} du\,dv$$

$y = r\sin v$

10

$$= \int_0^{2\pi}\int_0^{2} (3-6\sin v)\,du\,dv = \int_0^{2\pi}\int_0^{2} 3\,du\,dv - \int_0^{2\pi}\int_0^{2} 6\sin v\,du\,dv$$

$$= 3\int_0^{2\pi} dv\,(u)\Big|_0^2 - 6\int_0^{2\pi} \sin v\,dv\,(u)\Big|_0^2$$

$$= 3\int_0^{2\pi} 2\,dv - 12(-\cos v)\Big|_0^{2\pi} = 6\cdot(v)\Big|_0^{2\pi} + 12\underbrace{(\cos 2\pi - \cos 0)}_{0}$$

$$= 12\pi$$

OVO JE SAMO GORNJA POLOVICA

$v = r\sin\varphi$

$$= \int_0^{2\pi}\int_0^{2} (3 - 2r\sin\varphi)\,r\,dr\,d\varphi = \ldots$$

JOŠ + DONJA POLOVICA ... VIDI RJEŠENJE 1

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: TONI SESTAN **BROJ INDEKSA:** 55283-2007	
VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**	

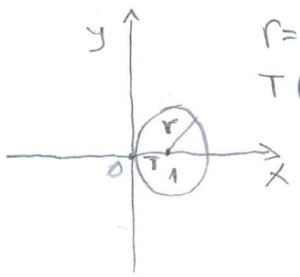
1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y)=x-y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20
2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,\mathrm{d}x\mathrm{d}y$. 20
3. Neka je C polukružna krivulja radijusa $r=1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\widehat{C}} \mathrm{d}x+\mathrm{d}y$.. 20
4. Izračunaj volumen prostora omeđenog plohama $y=x^2$, $y=x$, $z=-1$ i $z=2$. 20
5. Neka je K kugla radijusa $r=2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

Tablica integrala

Ukupno:

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

2.



$r=1$

$T(1,0)$
$\quad x_1\ y_1$

$\varphi \in (0, 2\pi)$

$r \in (0, 1)$

$y = r\sin\varphi + y_1$

$dxdy = r dr d\varphi$

$\iint 2 - 3r\sin\varphi \cdot r dr d\varphi$

$\int_0^{2\pi} 2-\sin\varphi\, d\varphi \int_0^1 3r^2 dr$

$\int_0^{2\pi} 2-\sin\varphi\, d\varphi \left(\frac{3r^3}{3}\right)\Big|_0^1$

$9\int_0^{2\pi} 2-\sin\varphi\, d\varphi$

$\cancel{0}$

4. $y=x^2$, $y=x$, $z=-1$, $z=2$

$x \in [y, \sqrt{y}]$

$z \in [-\ \ ]$

$y \in [\ \ ]$

IME I PREZIME: Toni Šestan **BROJ INDEKSA:** 55283-2007

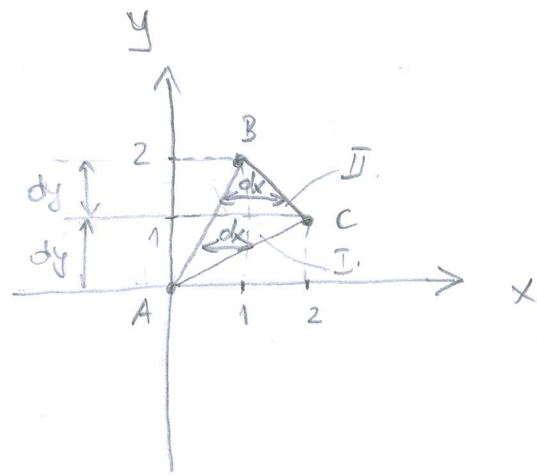
1. $A(0,0)$ $B(1,2)$ $C(2,1)$

$$\iint_T (x-y)\,dx\,dy$$

$$\iint_T x\,dx\,dy - \iint_T y\,dx\,dy$$

$$\text{I:} \boxed{\int_0^1 dy \int_y^y x\,dx - \int_0^1 y\,dy \int_y^y dx}$$

$$\text{II:} \int_1^2 dy \int_{3-y}^{y} x\,dx - \int_1^2 y\,dy \int_{3-y}^{y} dx$$



AB:

$$y-y_1 = \frac{x_2-x_1}{y_2-y_1}(x-x_1)$$

$$y-0 = \frac{1-0}{2-0}(x-0)$$

$$y = \frac{1}{2}x \,/\cdot \frac{2}{1} \quad x=y$$

AC:

$$y-0 = \frac{2-0}{1-0}(x-0)$$

$$y = 2x \Rightarrow x=y$$

BC:

$$y-2 = \frac{2-1}{1-2}(x-1)$$

$$y-2 = -1(x-1)$$

$$y-2 = -x+1$$

$$x = 3-y$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ **IME I PREZIME:** MARKO JARIN **BROJ INDEKSA:** JJ 708 - 2008 0269016121 **VRIJEME POČETKA:** **VRIJEME ZAVRŠETKA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. ~~20~~ 10

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,\mathrm{d}x\mathrm{d}y$. 20

3. Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\widehat{C}} \mathrm{d}x + \mathrm{d}y$.. 20

4. Izračunaj volumen prostora omeđenog plohama $y = x^2$, $y = x$, $z = -1$ i $z = 2$. 20

5. Neka je K kugla radijusa $r = 2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

Ukupno: 30

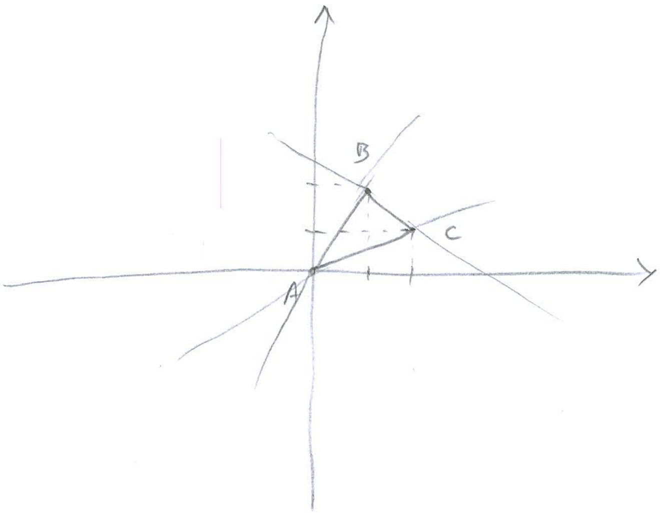
Tablica integrala		
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1.

A (0,0)
B (1,2)
C (2,1)

$f(x,y) = x - y$

$\iint_T f(x,y)\,dx\,dy$



p_1 .... kroz A i B

$y - 0 = \frac{2-0}{1-0}(x-0)$

$y = 2x$

p_3 kroz B i C

$y - 2 = \frac{1-2}{2-1}(x-1)$

$y - 2 = \frac{-1}{1}(x-1)$

$y = -1x + 1 + 2$

$y = -x + 3$

p_2 ... kroz A i C

$y - 0 = \frac{1-0}{2-0}(x-0)$

$y = \frac{1}{2}x$

$$\iint_{P_1} (x-y)\,dx\,dy = \int_0^1 \int_{2x}^{\frac{1}{2}x} (x-y)\,dy\,dx = \int_0^1 \left(xy - \frac{y^2}{2}\right)\Big|_{2x}^{\frac{1}{2}x} dx =$$

$$= \int_0^1 \Big(x\big(\underbrace{\tfrac{1}{2}x - 2x}_{-\frac{3}{2}x}\big) - \frac{1}{2}\big(\tfrac{1}{4}x^2 - 4x^2\big)\Big)\,dx =$$

$$= \int_0^1 \left(-\frac{3}{2}x^2 + \frac{15}{8}x^2\right) dx = \int_0^1 \frac{-6+15}{8}x^2\,dy =$$

$$= \frac{9}{8}\int_0^1 x^2\,dx = \frac{9}{8}\cdot\frac{x^3}{3}\Big|_0^1 = \frac{9}{8}\cdot\frac{1}{3}\cdot(1-0) = \frac{3}{8}$$

Ime i prezime: MARKO JARIN **Broj indeksa:** 55708-2008 0269016121

(1.) NASTAVAK

$$\iint_{P_2}(x-y)\,dx\,dy = \int_1^2\int_{\frac{1}{2}x}^{-x+3}(x-y)\,dy\,dx = \int_1^2 \left(xy-\frac{y^2}{2}\right)\Big|_{\frac{1}{2}x}^{-x+3} dx =$$

$$= \int_1^2 \left(x\left(-x+3-\frac{1}{2}x\right)-\frac{1}{2}\left(x^2-6x+9-\frac{1}{4}x^2\right)\right)dx =$$

$$= \int_1^2 \left(\left(-\frac{3x^2}{2}+3x\right)-\frac{1}{2}\left(\frac{3x^2}{4}-6x+9\right)\right)dx =$$

$$= \int_1^2 -\frac{3x^2}{2}+3x-\frac{3x^2}{8}+3x-\frac{9}{2}\,dx =$$

$$= \int_1^2 \left(\frac{-15x^2}{8}+6x-\frac{9}{2}\right)dx = -\frac{15}{8}\cdot\frac{x^3}{3}\Big|_1^2+6\frac{x^2}{2}\Big|_1^2-\frac{9}{2}x\Big|_1^2 =$$

$$= \frac{-5}{8}(8-1)+3(4-1)-\frac{9}{2}(2-1) = \frac{-35}{8}+9-\frac{9}{2} =$$

$$= \frac{-35+72-36}{8} = \frac{1}{8}$$

10

$$P = P_1+P_2 = \frac{3}{8}+\frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

(2.) $T(1,0)$ $\rightarrow$ $(x-1)^2+(y-0)^2=1^2$

$r=1$ $\qquad (x-1)^2+y^2=1$

$f(x,y) = 2-3y$

$x = r\cos\varphi+1$

$y = r\sin\varphi+0 = r\sin\varphi$

$\varphi \in (0,2\pi)$

$$\iint_K (2-3y)\,dx\,dy =$$

$$\int_0^{2\pi}\int_0^1 (2-3\cdot r\sin\varphi)\,r\,dr\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^1 (2r-3r^2\sin\varphi)\,dr\,d\varphi = \int_0^{2\pi}\left(2\cdot\frac{r^2}{2}-3\frac{r^3}{3}\sin\varphi\right)\Big|_0^1 d\varphi =$$

$$= \int_0^{2\pi}\left((1-0)-(1-0)\sin\varphi\right)d\varphi = \rightarrow$$

2\. NASTAVAK

$$= \int_0^{2\pi} (1 - \sin \varphi)\, d\varphi = \int_0^{2\pi} 1\, d\varphi - \int_0^{2\pi} \sin \varphi\, d\varphi =$$

$$= \varphi \Big|_0^{2\pi} - (-\cos \varphi) \Big|_0^{2\pi} =$$

$$= (2\pi - 0) + (\cos 2\pi - \cos 0) =$$

$$= 2\pi + (1 - 1) = 2\pi \checkmark$$

20

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: GREGOR HAMARIĆ **BROJ INDEKSA:** 57650

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. ~~20~~
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,\mathrm{d}x\mathrm{d}y$. ~~20~~ 15
3. Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\widehat{C}} \mathrm{d}x + \mathrm{d}y$.. 20
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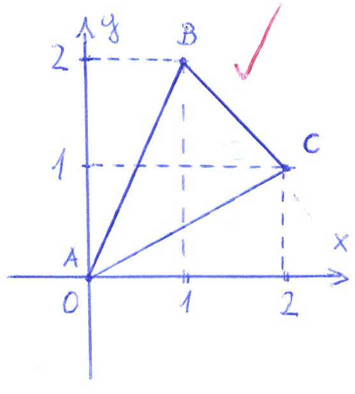
Ukupno: 15

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1.

$A(0,0)$
$B(1,2)$
$C(2,1)$



$\overline{AB}\ldots\quad y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y = \frac{2}{1}x$

$\underline{y = 2x}$

$\overline{BC}\ldots\quad y-2 = \frac{1-2}{2-1}(x-1)$

$y-2 = -1(x-1)$

$y-2 = -x+1$

$\underline{y = -x+3}$

$\overline{AC}\ldots\quad \underline{y = \frac{1}{2}x}$

$$\iint_T f(x,y)\,dx\,dy =$$

$(-x+3)\cdot(-x+3) = x - 3x - 3x$

$$\int_0^1\int_{\frac{1}{2}x}^{2x} dx\,dy + \int_1^2\int_{\frac{1}{2}x}^{-x+3} dx\,dy = \int_0^1\int_{\frac{1}{2}x}^{2x} dy\,dx + \int_1^2\int_{\frac{1}{2}x}^{-x+3} dy\,dx =$$

$$= \int_0^1 \frac{y^2}{2}\Big|_{\frac{1}{2}x}^{2x} dx + \int_1^2 \frac{y^2}{2}\Big|_{\frac{1}{2}x}^{-x+3} dx =$$

$$= \int_0^1 \left(\frac{4x^2}{2} - \frac{1}{4}x^2\right) dx + \int_1^2 \left(\frac{x^2-6x+9}{2} - \frac{1}{4}x^2\right) = \int_0^1 \frac{8x^2 - x^2}{4} dx + \int_1^2 \frac{2x^2-12x+18-x^2}{4}$$

$$= \int_0^1 \frac{7}{4}x^2\,dx + \int_1 \frac{x^2-12x+18}{4} dx$$

$$= \int_0^1\int_{\frac{1}{2}x}^{2x} dy\,dx + \int_1^2\int_{\frac{1}{2}x}^{-x+3} dy\,dx = \int_0 y\Big|_{\frac{1}{2}x}^{2x} dx + \int_1 y\Big|_{\frac{1}{2}x}^{-x+3} dx =$$

$$= \int_0^1 \left(2x - \frac{1}{2}x\right) dx + \int_1^2 \left(-x+3-\frac{1}{2}x\right) dx = \int_0 \frac{3}{2}x\,dx + \int_1 \left(-\frac{3}{2}x+3\right) dx =$$

$$= 3\int_0^1 \frac{1}{2}x\,dx - 3\int_1^2 \left(\frac{1}{2}x - 1\right) dx = 3\cdot\left(\frac{1}{2}\cdot\frac{x^2}{2}\right)\Big|_0^1 - 3\left[\left(\frac{1}{2}\cdot\frac{x^2}{2}\right) - x\right]\Big|_1^2 =$$

$$= \frac{3}{4}x^2\Big|_0^1 - \frac{3}{4}x^2 - 3x\Big|_1^2 = \frac{3}{4} - \left(3 - 6 - \left(\frac{3}{4} - 3\right)\right) =$$

$$= \frac{3}{4} - \left(3 - 6 - \frac{3}{4} + 3\right) = \frac{3}{4} - \cancel{3} + \cancel{6} + \frac{3}{4} - \cancel{3} = \frac{6}{4} = \frac{3}{2}$$

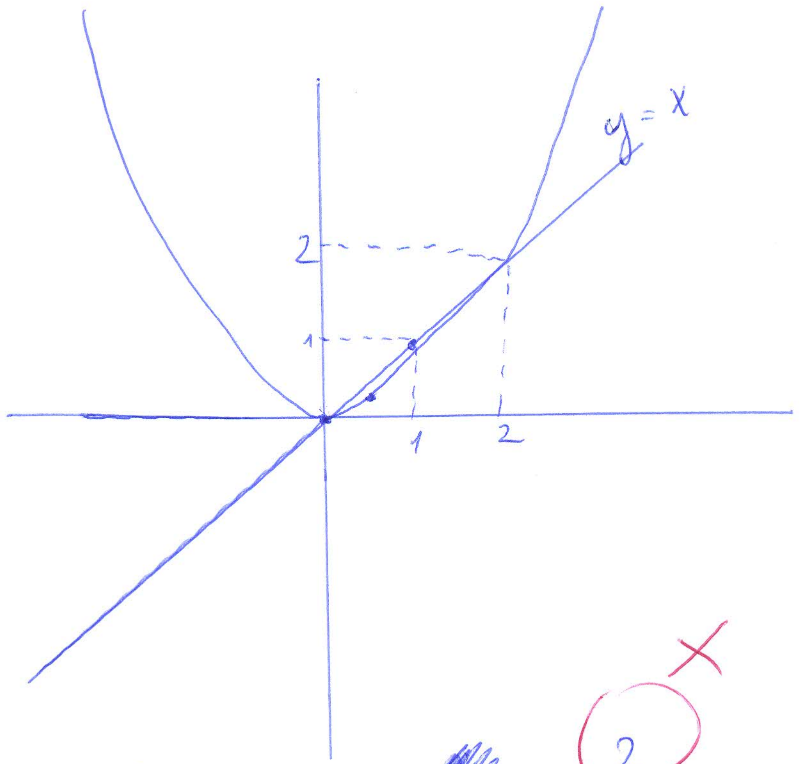
∅

NE TRAŽI SE POVRŠINA, VEĆ RAČUN INTEGRALA ZADANE FUNKCIJE NA TROKUTU!

4. $y = x^2$, $y = x$, $z = -1$, $z = 2$

$y = x^2$

x	0	1	2	3
y	0	1	4	9



$$P = \int_0^{2} (x - x^2)\,dx = \frac{x^2}{2} - \frac{x^3}{3} \Big|_0^2 =$$

X

$$= \frac{4}{2} - \frac{8}{3} = \frac{12 - 16}{6} = -\frac{4}{6} \overset{!!!!}{=} \frac{4}{6} = \frac{2}{3}$$

NEGATIVNO?

$P = \frac{2}{3}$

$V = P \cdot 3 = \frac{2}{3} \cdot 3 = 2$

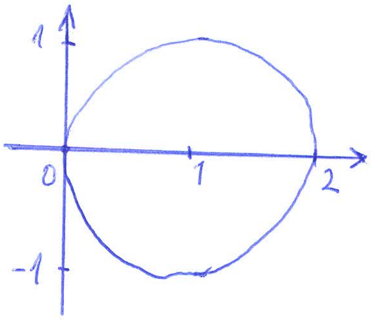
VOLUMEN je 2

Integriranjem smo izračunali površinu presjeka tijela na ravnini xy, a zatim je bilo dovoljno tu površinu pomnožiti s 3 jer nam tijelo na z osi ograničavaju 2 ravnine ($z=-1$, $z=2$) međusobno udaljenosti 3 jedinične duljine.

$$\boxed{-\frac{4}{6} \neq \frac{4}{6}}$$

Ime i prezime: GREGOR HAMARIĆ **Broj indeksa:** 57650

2. $\overset{}{r=1,\ T(1,0),\ \iint\limits_K (2-3y)\,dx\,dy}$ PQ



$r \in [0,1]$

$\varphi \in [0, 2\pi]$

15

$$\iint\limits_K (2-3y)\,dx\,dy = \int_0^{2\pi}\int_0^1 (2-3y)\,dx\,dy =$$

?

$$= \int_0^{2\pi}\int_0^1 2-3(r\sin\varphi + Q)\,r\,dr\,d\varphi = \int_0^{2\pi}\int_0^1 2r - 3r^2\sin\varphi\,dr\,d\varphi =$$

$$= \int_0^{2\pi} \cancel{2}\cdot\frac{r^2}{\cancel{2}} - \cancel{3}\cdot\frac{r^3}{\cancel{3}}\sin\varphi \Big|_0^1 d\varphi = \int_0^{2\pi}(1-\sin\varphi)\,d\varphi =$$

$$= \varphi + \cos\varphi \Big|_0^{2\pi} = 2\pi + \cos 2\pi = 2\pi + 1 = 2\cdot 3.14 + 1 \approx \underline{\underline{7.28}}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: BORIS DURBIĆ

BROJ INDEKSA: 57640

VRIJEME POČETKA:

VRIJEME ZAVRŠETKA:

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. ~~20~~

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,\mathrm{d}x\mathrm{d}y$. ~~20~~ 15

3. Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\widehat{C}} \mathrm{d}x + \mathrm{d}y$.. 20

4. Izračunaj volumen prostora omeđenog plohama $y = x^2$, $y = x$, $z = -1$ i $z = 2$. ~~20~~

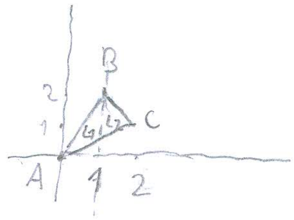
5. Neka je K kugla radijusa $r = 2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

Ukupno: 15

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1. $A(0,0)$ $B(1,2)$ $C(2,1)$ $f(x,y) = x-y$



$AB \dots \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{2-0}{1-0}(x-0)$

$y = 2x$

$AC \dots \quad y - 1 = \frac{1-0}{2-0}(x-0)$

$y - 1 = \frac{1}{2}x$

$y = \frac{1}{2}x + 1$

$BC \dots \quad y - 2 = \frac{1-2}{2-1}(x-1)$

$y - 2 = \frac{-1}{1}(x-1)$

$y - 2 = -x + 1$

$y = 3 - x$

$$I = \int_0^1 \int_{\frac{1}{2}x+1}^{2x} (x-y)\,dx\,dx + \int_1^2 \int_{\frac{1}{2}x+1}^{3-x} (x-y)\,dy\,dx = \int_0^1 xy - \frac{y^2}{2} \Big]_{\frac{1}{2}x+1}^{2x} + \int_1^2 xy - \frac{y^2}{2} \Big]_{\frac{1}{2}x+1}^{3-x}$$

$$= \int_0^1 \left(2x \cdot x - \frac{2x^2}{2} - \left(x \cdot \left(\frac{1}{2}x+1\right) - \frac{\left(\frac{1}{2}x+1\right)^2}{2}\right) + \left(x \cdot (3-x) - \frac{(3-x)^2}{2}\right) - \left(x\left(\frac{1}{2}x+1\right) - \frac{\left(\frac{1}{2}x+1\right)^2}{2}\right)\right) dx$$

$$= \int_0^1 \left(2x^2 - \frac{2x^2}{2} - \left(\frac{1}{2}x + x - \frac{\frac{1}{4}x^2 + x + 1}{2}\right) + \left(3x - x^2 - \frac{9 - 6x - x^2}{2}\right) - \left(\frac{1}{2}x^2 + x + \frac{\frac{1}{4}x^2 + x + 1}{2}\right)\right) dx$$

$$= \int_0^1 \left[2x^2 - x^2 - \left(\frac{1}{2}x + x - \frac{1}{8}x^2 + \frac{x}{2} - \frac{1}{2}\right) + \left(3x - x^2 - \frac{9}{2} - 3x - \frac{x^2}{2}\right) - \left(\frac{1}{2}x + x - \frac{1}{8}x^2 - \frac{x}{2} - \frac{1}{2}\right)\right] dx$$

$$= \int_0^1 \left(\cancel{2x^2} - \cancel{x^2} - \cancel{x} + \cancel{\frac{1}{8}x^2} + \frac{1}{2} + \cancel{3x} - \cancel{x^2} - \frac{9}{2} - \cancel{3x} - \cancel{\frac{x^2}{2}} - x + \cancel{\frac{1}{8}x^2} + \frac{1}{2}\right) dx$$

$$= \left(\cancel{2x^2} - \cancel{x^2} + \frac{1}{8}x^2 - \cancel{x^2} - \frac{x^2}{2} + \frac{1}{8}x^2\right) = -\frac{1}{4}x \quad (-x + 3x - 3x) = -x$$

$$\int_0^1 \left(-\frac{1}{4}x^2 - x + 1\right) dx = -\frac{1}{4} \cdot \frac{1^3}{3} - \frac{1^2}{2} + 1 = -\frac{1}{12} - \frac{1}{2} + 1 = \frac{-1-6+12}{12} = \frac{5}{12}$$

0

2. $r=1$ $\quad T(1,0)$ $\quad \iint_K (2-3y)\,dx\,dy$ $\quad r[0,1]$, $\varphi[0,2\pi]$

$$1+x = r\cos\varphi \Rightarrow x = r\cos\varphi - 1$$
$$y = r\sin\varphi$$

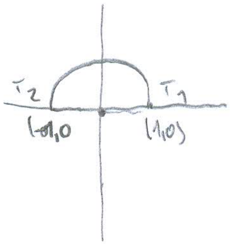
1 -

1.5

$$\boxed{\int_0^{2\pi}\int_0^1 (2-3r\sin\varphi)\,r\,dr\,d\varphi} = \int_0^{2\pi}\int_0^1 (2r-3r^2\sin\varphi)\,dr\,d\varphi = \int_0^{2\pi} d\varphi \left[\cancel{2}\frac{r^2}{\cancel{2}} - \cancel{3}\frac{r^3}{\cancel{3}}\sin\varphi\right]_0^1$$

$$= \int_0^{2\pi} d\varphi\,(1-\sin\varphi) = 2\pi + \cos 2\pi = 2\pi + 1$$

3. $r=1$

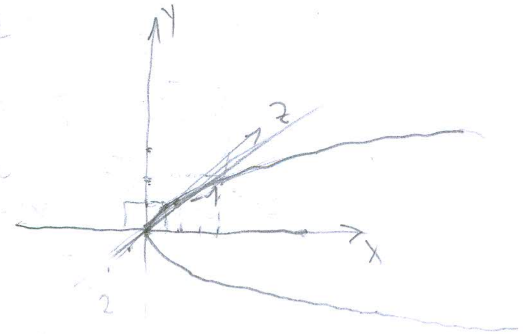


$$\int_C dx + dy$$

BORIS ĐURBIĆ
Ime i prezime:

57640
Broj indeksa:

4.



$x^2=x$

$x^2-x=0$

$x(x-1)=0$

$x_1=0,\ x_2=1$

$$P=\int dz\,dx\,dy=\int_{-1}^{2}dz\int_{0}^{1}dx\int_{x}^{x^2}dy=\int_{-1}^{2}dz\int_{0}^{1}(x^2-x)\,dx=\int_{-1}^{2}dz\left(\frac{x^3}{3}-\frac{x^2}{2}\right)\Big|_{0}^{1}$$

$$=\int_{-1}^{2}dz\left(\frac{1}{3}-\frac{1}{2}\right)=\int_{-1}^{2}-\frac{1}{6}\,dz=-\frac{1}{6}\cdot 2-\frac{1}{6}\cdot(-1)=-\frac{2}{6}+\frac{1}{6}=\frac{1-2}{6}=-\frac{1}{6}$$

$$=\frac{1}{6}$$

?

5. $r=2$

$$\begin{bmatrix}0\\0\\3-2y\end{bmatrix}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MATE BALJAK **BROJ INDEKSA:** 57115	
VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**	

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,\mathrm{d}x\mathrm{d}y$. 20
3. Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\widehat{C}} \mathrm{d}x + \mathrm{d}y$.. 20
4. Izračunaj volumen prostora omeđenog plohama $y = x^2$, $y = x$, $z = -1$ i $z = 2$. 20
5. Neka je K kugla radijusa $r = 2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
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$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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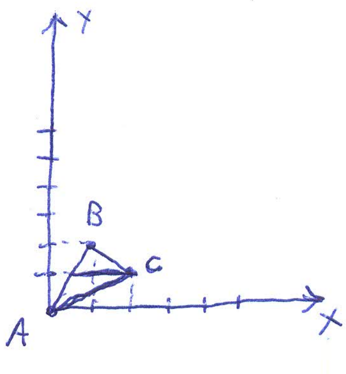

$BC = y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$AB = y - 0 = \frac{-2}{-1}$

$y = 2$

$y_0 - 2 = \frac{2-1}{1-2}$

$y_0 = -1 + 2$

$y_0 = 1$

5.) $\iint\limits_{\partial K} (3-2y)\, dx\, dy$ $r=2$ $r \in [0, 2]$

$\varphi \in [0, 2\pi]$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: DINO KURIĆ **BROJ INDEKSA:** 56192-2008

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20
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3. Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\widehat{C}} \mathrm{d}x + \mathrm{d}y$.. 20
4. Izračunaj volumen prostora omeđenog plohama $y = x^2$, $y = x$, $z = -1$ i $z = 2$. 20
5. Neka je K kugla radijusa $r = 2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: Igor Brajica **BROJ INDEKSA:** 52803-2005

VRIJEME POČETKA: 08:00 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓
bodova

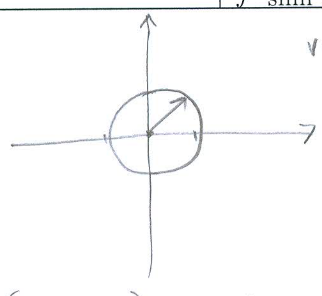
1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\,\mathrm{d}x\mathrm{d}y$. 20 0
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,\mathrm{d}x\mathrm{d}y$. 20 0
3. Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\widehat{C}} \mathrm{d}x + \mathrm{d}y$.. 20
4. Izračunaj volumen prostora omeđenog plohama $y = x^2$, $y = x$, $z = -1$ i $z = 2$. 20
5. Neka je K kugla radijusa $r = 2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

(2)



$r = 1$

$r \in [0, 2]$

$\varphi \in [0, 2\pi]$

0

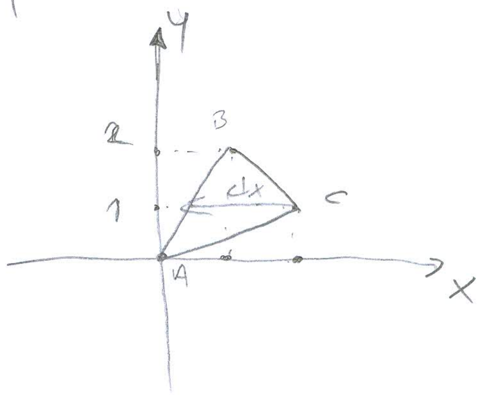
$\iint (2-3y)\,dx\,dy$

$\int (2)\,dx\,dy - (3y)\,dx\,dy$

7. $A(\overset{x_1}{0};\overset{y_1}{0})$ $\quad f(x,y) = x - y$

$B(\overset{x_2}{1},\overset{y_2}{2})$

$C(2,1)$



$$\iint f(x,y)\,dx\,dy$$

$$\int_0^1 y\,dy \int_1^2 x\,dx$$

$AB: y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$AB: y - 0 = \frac{2-0}{1-0}(x-0)$

$AB: y = \frac{2}{1}(x-0)$

$y = 2x$

$BC: y - 2 = \frac{1-2}{2-1}(x-1)$

$y - 2 = -x + 1$

$y = -x + 3$

$AC: y - 0 = \frac{1-0}{2-1}(x-0)$

$y - 0 = \frac{1}{2}(x-0)$

$y = \frac{1}{2}x$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: LUKA KURILIĆ **BROJ INDEKSA:** 58076

VRIJEME POČETKA: $08^h 00$ min **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint_T f(x,y)\,dxdy$. 20

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(1,0)$. Izračunati $\iint_K (2-3y)\,dxdy$. 20

3. Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int_{\widehat{C}} dx + dy$.. 20

4. Izračunaj volumen prostora omeđenog plohama $y = x^2$, $y = x$, $z = -1$ i $z = 2$. 20

5. Neka je K kugla radijusa $r = 2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint_{\partial K} (3-2y)\,dxdy$. ~~20~~

Ukupno: 20

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

2) $r = 1 \quad T(\underset{p}{1}, \underset{q}{0})$

$$x = r\cos\varphi + p \Rightarrow \boxed{x = r\cos\varphi + 1}$$
$$y = r\sin\varphi + q \Rightarrow \boxed{y = r\sin\varphi}$$

$$\iint_K (2-3y)\,dxdy$$

$$\int_0^{2\pi} d\varphi \int_0^1 (2-3y)\,r\,dr$$

$$\int_0^{2\pi} d\varphi \int_0^1 (2-3r\sin\varphi)\,r\,dr$$

$$\int_0^{2\pi} d\varphi \int_0^1 2r\,dr - \int_0^1 3r^2\sin\varphi\,dr$$

$$\int_0^{2\pi} d\varphi \left[2\int_0^1 r\,dr - 3\sin\varphi\int_0^1 r^2\,dr\right]$$

$$\int_0^{2\pi} d\varphi \left[\cancel{2}\frac{r^2}{\cancel{2}}\Big|_0^1 - \cancel{3}\sin\varphi\,\frac{r^3}{\cancel{3}}\Big|_0^1\right]$$

$$\int_0^{2\pi} d\varphi \left[(1^2-0^2) - \sin\varphi\,(1^3-0)\right]$$

$$\int_0^{2\pi} d\varphi \left[1 - \sin\varphi\right]$$

$$\int_0^{2\pi} d\varphi - \int_0^{2\pi} \sin\varphi\,d\varphi$$

$$\varphi\Big|_0^{2\pi} + \cos\varphi\Big|_0^{2\pi}$$

$$(2\pi - 0) + (\cos 2\pi - \cos 0)$$

$$2\pi + 1 - 1 = \boxed{2\pi}$$

20

⑤ $r=2$ $\quad S(0,0)$

$$\iint_{\partial K}(3-2y)\,dx\,dy$$

$$\boxed{\begin{array}{l} x = r\cos\varphi \\ y = r\sin\varphi \end{array}}$$

$$\int_0^{2\pi} d\varphi \int_0^2 (3-2y)\,r\,dr$$ X

$$\int_0^{2\pi} d\varphi \int_0^2 (3-2r\sin\varphi)\,r\,dr$$

$$\int_0^{2\pi} d\varphi \int_0^2 3r\,dr - \int_0^2 2r^2\sin\varphi\,dr$$

$$\int_0^{2\pi} d\varphi \left[3\int_0^2 r\,dr - 2\sin\varphi\int_0^2 r^2\,dr\right]$$

$$\int_0^{2\pi} d\varphi \left[3\frac{r^2}{2}\Big|_0^2 - 2\sin\varphi\,\frac{r^3}{3}\,dr\Big|_0^2\right]$$

$$\int_0^{2\pi} d\varphi \left[\frac{3}{2}r^2\Big|_0^2 - 2\sin\varphi\,\frac{1}{3}r^3\,dr\Big|_0^2\right]$$

$$\int_0^{2\pi} d\varphi \left[\frac{3}{2}(2^2-0^2) - 2\sin\varphi\,\frac{1}{3}(2^3-0^3)\right]$$

$$\int_0^{2\pi} d\varphi \left[\frac{\cancel{12}6}{\cancel{2}} - 2\sin\varphi\,\frac{8}{3}\right]$$

$$\int_0^{2\pi} d\varphi \left(6 - 2\sin\varphi\,\frac{8}{3}\right)$$

$$6\int_0^{2\pi} d\varphi - 2\cdot\frac{8}{3}\int \sin\varphi\,d\varphi$$

$$6\int_0^{2\pi} d\varphi - \frac{16}{3}\int \sin\varphi\,d\varphi$$

$$6\varphi\Big|_0^{2\pi} - \frac{16}{3}(-\cos\varphi)\Big|_0^{2\pi}$$

$$6\varphi\Big|_0^{2\pi} + \frac{16}{3}\cos\varphi\Big|_0^{2\pi}$$

$$6(2\pi-0) + \frac{16}{3}(\cos 2\pi - \cos 0)$$

$$12\pi + \frac{16}{3}(\cancel{1}-\cancel{1})$$

$$12\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: BRUNO LIPOTICA **BROJ INDEKSA:**

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(1,0)$. Izračunati $\iint\limits_K (2-3y)\,\mathrm{d}x\mathrm{d}y$. 20

3. Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int\limits_{\overset{\frown}{C}} \mathrm{d}x + \mathrm{d}y$.. 20

4. Izračunaj volumen prostora omeđenog plohama $y = x^2$, $y = x$, $z = -1$ i $z = 2$. 20 15

5. Neka je K kugla radijusa $r = 2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

Ukupno: 15

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

4.

$y = x^2 \quad y = x \quad z = -1,\ z = 2$

$$\int_{-1}^{2} \mathrm{d}z \int_{0}^{1} x\,\mathrm{d}x \int_{x^2}^{x} \mathrm{d}y$$

15

$x^2 = x$

$x(x-1) = 0$

$x = 0$

$x = 1$

$$\boxed{\begin{array}{l} y\ (x^2, x) \\ x\ (0,1) \\ z\ (-1,2) \end{array}}$$

→

1

$$\int_{-1}^{2} dz \int_{0}^{1} x \, dx \int_{x^2}^{x} dy$$

$$\int_{-1}^{2} dz \int_{0}^{1} x \,(x - x^2)\, dx$$

$$\int_{-1}^{2} dz \int_{0}^{1} x^2 - x^3 \, dx$$

$$\int_{-1}^{2} dz \left. \frac{x^3}{3} - \frac{x^4}{4} \right|_{0}^{1}$$

$$\int_{-1}^{2} dz \quad \frac{1^3}{3} - \frac{1^4}{4}$$

$$\int_{-1}^{2} dz \quad \frac{1}{3} - \frac{1}{4}$$

$$\int_{-1}^{2} \frac{1}{12} \, dz$$

$$\left. \frac{1}{12} z \right|_{-1}^{2}$$

$$\frac{1}{12} \cdot 2 - \left(\frac{1}{12} \cdot (-1) \right)$$

$$\frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$$