

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME:

RJEŠENJE 1

BROJ INDEKSA:

VRIJEME

POČETKA:

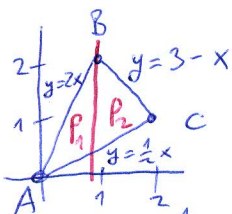
- Zadan trokut T sa vrhovima: $A(0,0)$, $B(1,2)$ i $C(2,1)$ i funkcija $f(x,y) = x - y$. Odrediti $\iint_T f(x,y) dx dy$. 20
- Neka je K krug radijusa $r = 1$ sa centrom u točki $T(1,0)$. Izračunati $\iint_K (2 - 3y) dx dy$. 20
- Neka je C polukružna krivulja radijusa $r = 1$ sa centrom u točki $T(0,0)$, koji spaja početak $T_1(1,0)$ i kraj $T_2(-1,0)$. Iz definicije zračunati $\int_C dx + dy$. 20
- Izračunaj volumen prostora omeđenog plohama $y = x^2$, $y = x$, $z = -1$ i $z = 2$. 20
- Neka je K kugla radijusa $r = 2$ centrirana u ishodištu, a ∂K njen rub usmjeren prema van. Izračunati $\iint_{\partial K} (3 - 2y) dx dy$. 20

Tablica integrala

Ukupno:

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

①

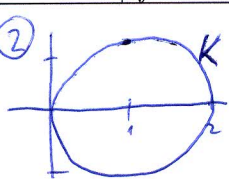


$$P_1 = \int_0^1 \int_{\frac{1}{2}x}^{2x} dy dx = \int_0^1 \frac{3}{2}x dx = \frac{3}{2} \left[\frac{x^2}{2} \right]_0^1 = \frac{3}{4}$$

$$P_2 = \int_1^2 \int_{\frac{1}{2}x}^{3-x} dy dx = \int_1^2 \left(3 - \frac{3}{2}x \right) dx = \left[3x - \frac{3}{4}x^2 \right]_1^2 = 6 - 3 - 3 + \frac{3}{4} = \frac{3}{4}$$

$$P = P_1 + P_2 = \frac{3}{2}$$

②

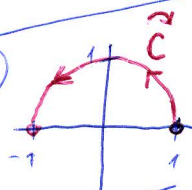


$$x = r \cos \varphi + 1$$

$$y = r \sin \varphi$$

$$\iint_K (2 - 3y) dx dy = \int_0^{2\pi} \int_0^1 (2 - 3r \sin \varphi) r dr d\varphi = 2\pi$$

③



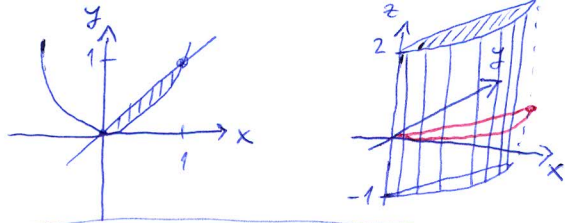
PARAMETRIZACIJA: $r(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}, t \in [0, \pi]$

VEKTORSKA FUNKCIJA $w(x,y) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $r'(t) = \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix}$

$$\int_C dx + dy = \int_0^\pi \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix} dt = -\int_0^\pi \sin t dt + \int_0^\pi \cos t dt$$

$$= \left[\cos t + \sin t \right]_0^\pi = -\frac{1}{2} + 0 - \frac{1}{2} - 0 = -1$$

④ $y = x^2$
 $y = x$
 $z = -1$
 $z = 2$



$$V = \int_{-1}^2 \int_0^1 \int_{x^2}^x 1 \, dy \, dx \, dz$$

$$\rightarrow = 3 \cdot \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = 3 \cdot \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{1}{2}$$

⑤ K kugla, $r = 2$, centar u ishodište, $w = \begin{bmatrix} 0 \\ 0 \\ 3-2y \end{bmatrix}$

∂K radi kugle (sfera) usmjerenom van.

TEOREM O DIVERGENCIJI:

$$\operatorname{div} w = d_x w_x + d_y w_y + d_z w_z = 0$$

$$\iint_{\partial K} w \cdot ds = \iiint_K \operatorname{div} w = \iiint_K 0 = 0$$

① OBTIROM DA SE NE TRAŽI POVRŠINA, VEĆ INTEGRAL FUNKCIJE NA

TROKUTU RAČUN JE:

$$I = \int_0^1 \int_{\frac{1}{2}x}^{2x} (x-y) \, dy \, dx + \int_{\frac{1}{2}}^1 \int_{\frac{1}{2}x}^{3-x} (x-y) \, dy \, dx = \int_0^1 x \cdot \left(\frac{3}{2}x \right) - \frac{4x^2 - \frac{1}{4}x^2}{2} \, dx + \int_1^2 x \cdot \left(3 - \frac{3}{2}x \right) - \frac{(3-x)^2 - \frac{x^2}{4}}{2} \, dx$$

$$= \int_0^1 x^2 \left(\frac{3}{2} - 2 + \frac{1}{8} \right) \, dx + \int_1^2 \left(3x - \frac{3}{2}x^2 - \frac{9}{2} + 3x - \frac{1}{2}x^2 + \frac{1}{8}x^2 \right) \, dx =$$

$$= -\frac{1}{8} \left(\frac{x^3}{3} \right)_0^1 + \left(-\frac{5}{8} \frac{x^2}{2} + \frac{3}{2} \frac{x^2}{2} - \frac{9}{2} x \right)_1^2 = \left(-\frac{1}{8} - \frac{5}{8} + 3 - \frac{9}{2} \right) + \left(\frac{5}{8} - 3 + \frac{9}{2} \right) = 0$$

