MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓

IME I PREZIME: RJEŠENJE 3 **BROJ INDEKSA:**

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti integrale:

   (a) $\displaystyle\int_0^{\pi} \sin^3 x \,\mathrm{d}x.$; 10

   $x^2 : (x^2-2) = 1$
   $\underline{-x^2+2}$
   2

   $\dfrac{2}{x^2-2} = \dfrac{A}{x-\sqrt{2}} + \dfrac{B}{x+\sqrt{2}} \Rightarrow 2 = A(x+\sqrt{2}) + B(x-\sqrt{2}) \Rightarrow \begin{cases} A = \frac{\sqrt{2}}{2}, \; B = -\frac{\sqrt{2}}{2} \end{cases}$

   (b) $\displaystyle\int_0^1 \frac{x^2}{x^2-2}\,\mathrm{d}x. = \int_0^1 \left(1 + \frac{2}{x^2-2}\right) dx = \int_0^1 \left(1 + \frac{\sqrt{2}}{2}\frac{1}{x-\sqrt{2}} - \frac{\sqrt{2}}{2}\frac{1}{x+\sqrt{2}}\right) dx = \left[x + \frac{\sqrt{2}}{2}\ln|x-\sqrt{2}| - \frac{\sqrt{2}}{2}\ln|x+\sqrt{2}|\right]_0^1 = \ldots$ 15

2. Izračunati nepravi integral $\displaystyle\int_1^{\infty} \frac{1}{x^2}\,\mathrm{d}x. = \left[-\frac{1}{x}\right]_1^{\infty} = \lim_{x\to\infty}\left(-\frac{1}{x}\right) + \frac{1}{1} = 0 + 1 = 1$ 5

3. Izračunati površinu lika omeđenog omeđenog parabolom $y = 9 - x^2$ i pravcem $y = 3 - x$. 20

4. Riješiti diferencijalnu jednadžbu: $y'' + y' - 2y = x^2$. Uvrstiti izračunato rješenje u jednadžbu i provjeriti da li je zadovoljena. 15+5

5. Istražiti ekstreme funkcije $f(x,y) = x^3 + x^2 + y^3 + y^2$. 20

6. Razviti funkciju $f(x) = \dfrac{1}{x}$ u Taylorov red po potencijama od $x - 1$. Izračunati barem prva 4 člana. 10

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

(1a) $\int_0^{\pi} \sin^3 x\,dx = \left\{ \begin{array}{l} \text{1. način} \\ \sin^3 x = \sin^2 x \cdot \sin x \\ = (1-\cos^2 x)\cdot \sin x \end{array} \right\} = \int_0^{\pi} \sin x\,dx - \underbrace{\int_0^{\pi} \cos^2 x \sin x\,dx}_{\text{SUPSTITUCIJOM}} = \left[-\cos x + \frac{\cos^3 x}{3}\right]_0^{\pi} = 1 - \frac{1}{3} + 1 - \frac{1}{3} = \frac{4}{3}$

SUPSTITUCIJOM $\boxed{u = \cos x} \Rightarrow \int u^2 du = \frac{u^3}{3}$
$\boxed{du = -\sin x}$

$\left\{ \begin{array}{l} \text{2. način} \\ \text{PARCIJALNOM} \\ \text{INTEGRACIJOM} \end{array} \quad \begin{array}{ll} u = \sin^2 x & du = 2\sin x \cos x \\ dv = \sin x dx & v = -\cos x \end{array} \right\} = -\sin^2 x \cos x \Big|_0^{\pi} + 2\int_0^{\pi} \sin x \cos^2 x\,dx = \left[-\sin^2 x \cos x - 2\frac{\cos^3 x}{3}\right]_0^{\pi} = \frac{4}{3}$

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(3) $\left.\begin{array}{l} y = 9 - x^2 \\ y = 3 - x \end{array}\right\}$ SJECIŠTA: $9 - x^2 = 3 - x$, $x^2 - x - 6 = 0 \Rightarrow \left\{\begin{array}{l} x_1 = -3 \\ x_2 = 2 \end{array}\right.$

SKICA: 

$P = \int_{-3}^{2} \left((9-x^2) - (3-x)\right) = \left[6x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-3}^{2} = \ldots =$

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(4) $y'' + y' - 2y = x^2$

HOMOGENA:
$y'' + y' - 2y = 0$
$\lambda^2 + \lambda - 2 = 0$
$\lambda_{1,2} = -2, 1$
$y_H(x) = C_1 e^{-2x} + C_2 e^{+x}$

PARTIKULARNO
$y_p(x) = A + Bx + Cx^2$
$y_p'(x) = B + 2Cx$
$y_p''(x) = 2C$

UVRSTITI
$2C + B + 2Cx - 2A - 2Bx - 2Cx^2 = x^2$
uz x^2: $-2C = 1 \Rightarrow C = -\frac{1}{2}$
uz x: $-2B + 2C = 0 \Rightarrow -2B = +1 \Rightarrow B = -\frac{1}{2}$
~~bez~~ x: $2C + B - 2A = 0 \Rightarrow -1 - \frac{1}{2} - 2A = 0$
$\Rightarrow -2A = \frac{3}{2} \Rightarrow A = -\frac{3}{4}$

RJEŠENJE: $y = C_1 e^{-2x} + C_2 e^{+x} - \frac{1}{2}x^2 - \frac{1}{2}x - \frac{3}{4}$

$y' = -2C_1 e^{-2x} + C_2 e^{+x} - x - \frac{1}{2}$

$y'' = 4C_1 e^{-2x} + C_2 e^{+x} - 1$

UVRŠTAVANJE: $\cancel{+4C_1 e^{-2x}} + \cancel{C_2 e^{-x}} - 1 \cancel{- 2C_1 e^{-2x}} + \cancel{C_2 e^{+x}} - x - \frac{1}{2} \cancel{- 2C_1 e^{-2x}} \cancel{- 2C_2 e^{+x}} + x^2 + x + \frac{3}{2} = x^2$

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(5) $f = x^3 + x^2 + y^3 + y^2$

$\partial_x f = 3x^2 + 2x = (3x+2)x = 0 \Rightarrow x_1 = 0, x_2 = -\frac{2}{3}$
$\partial_y f = 3y^2 + 2y = (3y+2)y = 0 \Rightarrow y_1 = 0, y_2 = -\frac{2}{3}$

$\partial_{xx} f = 6x + 2 \quad \partial_{yy} f = 6y + 2$
$\partial_{xy} f = \partial_{yx} f = 0$
$A = 6x + 2$
$\Delta = \begin{vmatrix} 6x+2 & 0 \\ 0 & 6y+2 \end{vmatrix}$

$T_1(0,0)$	$T_2(0,-\frac{2}{3})$	$T_3(-\frac{2}{3},0)$	$T_4(-\frac{2}{3},-\frac{2}{3})$
$A = 2$	$A = 2$	$A = -2$	$A = -2$
$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$	$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = -4$	$\Delta = \begin{vmatrix} -2 & 0 \\ 0 & 2 \end{vmatrix} = -4$	$\Delta = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4$
~~NEODREĐENO~~ MINIMUM LOK.	SEDLASTA TOČKA	SEDLASTA TOČKA	~~NEODREĐENO~~ LOK. MAX

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(6) $f(x) = \frac{1}{x}$ $\quad f(1) = 1$
$f'(x) = -\frac{1}{x^2}$ $\quad f'(1) = -1$
$f''(x) = \frac{2}{x^3}$ $\quad f''(1) = 2$
$f'''(x) = \frac{-6}{x^4}$ $\quad f'''(1) = -6$

$f(x) \approx 1 - (x-1) + \frac{2}{2}(x-1)^2 - \frac{6}{6}(x-1)^2$
$\approx 1 - (x-1) + (x-1)^2 - (x-1)^2$

$\frac{-70+15+180}{6}$

$\frac{125}{6}$

$-\left(\frac{x^3}{3}\Big|_{-2}^{3}\right) + \frac{x^2}{2}\Big|_{-2}^{3} + 6x\Big|_{-2}^{3}$

$= -\left(\frac{27}{3} + \frac{8}{3}\right) + \frac{1}{2}(9-4) + 6(3+2)$
$= -\left(\frac{35}{3}\right) + \frac{1}{2}\cdot 5 + 30$
$= -\frac{35}{3} + \frac{5}{2} + 30$

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IME I PREZIME: NIKOLA KNEŽEVIĆ **BROJ INDEKSA:** 17-1-0002-2010

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

20

1. Riješiti integrale:

   (a) $\int\limits_0^{\pi} \sin^3 x \, \mathrm{d}x.$; 10

   (b) $\int\limits_0^{1} \frac{x^2}{x^2-2} \, \mathrm{d}x.$ 15

2. Izračunati nepravi integral $\int\limits_1^{\infty} \frac{1}{x^2} \, \mathrm{d}x.$ 5

3. Izračunati površinu lika omeđenog omeđenog parabolom $y = 9 - x^2$ i pravcem $y = 3 - x$. 20

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$\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1}, \ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln \|x\| + C$	$\int \sinh x \, \mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x \, \mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\| x + \sqrt{x^2 \pm a^2} \right\| + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \tanh x \, \mathrm{d}x = \ln \|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \coth x \, \mathrm{d}x = \ln \|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \tan x \, \mathrm{d}x = -\ln \|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x \, \mathrm{d}x = \ln \|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} \, \mathrm{d}x = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha \ (\alpha \neq 0)$	$\alpha x^{\alpha - 1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

IME I PREZIME: NIKOLA KNEŽEVIĆ **BROJ INDEKSA:** 17-1-0002-2010

3

$y = 9 - x^2$

$y = 3 - x$

$9 - x^2 = 3 - x$

$-x^2 + x + 9 - 3 = 0$

$x^2 - x - 9 + 3 = 0$

$x^2 - x - 6 = 0$

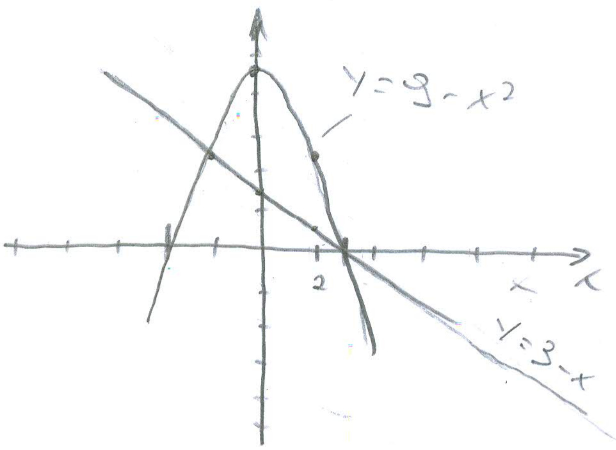
$x_{12} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{12} = \dfrac{1 \pm \sqrt{1 + 24}}{2}$

$x_{12} = \dfrac{1 \pm 5}{2}$

$x_1 = 3$

$x_2 = -2$



$y = 9 - x^2$

x	-2	0	2	
y	5	9	5	

$y = 3 - x$

x	-2	0	2
y	5	3	1

$$P = \int_{-2}^{3} 9 - x^2 - (3 - x)\,dx = \int_{-2}^{3} 9 - x^2 - 3 + x\,dx =$$

$$= 9\int_{-2}^{3} dx - \int_{-2}^{3} x^2 dx - 3\int_{-2}^{3} dx + \int_{-2}^{3} x\,dx = 9x\Big|_{-2}^{3} - \frac{x^3}{3}\Big|_{-2}^{3} - 3x\Big|_{-2}^{3} + \frac{x^2}{2}\Big|_{-2}^{3}$$

$$= 9(3+2) - \frac{3^3 - (-2)^3}{3} - 3(3+2) + \frac{3^2 - (-2)^2}{2}$$

$$= 45 - \frac{35}{3} - 15 + \frac{5}{2} = \frac{270 - 70 - 90 + 15}{6}$$

$$= \frac{125}{6}$$ ✓

20

⑤

$f(x,y) = x^3 + x^2 + y^2 + y^2$

$3x^2 = \cancel{3' \cdot x^3} + 3 \cdot x^{2'}$

$3 \cdot 2x = 6x$

$f_x = \frac{\partial f}{\partial x} = 3x^2 + 2x$

$\frac{\partial^2 f}{\partial x^2} = 6x + 2$

$\frac{\partial^3 f}{\partial x^3} = 6$

$f_y = \frac{\partial f}{\partial y} = 3y^2 + 2y$

$\frac{\partial^2 f}{\partial y^2} = 6y + 2$

$\frac{\partial^3 f}{\partial y^3} = 6$

$\frac{\partial^2 f}{\partial x \partial y} = 0$

$$\Delta = \begin{vmatrix} \frac{\partial^3 f}{\partial x^3}(T_0) & \frac{\partial^2 f}{\partial x \partial y}(T_0) \\ \frac{\partial^2 f}{\partial x \partial y}(T_0) & \frac{\partial^3 f}{\partial y^3}(T_0) \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix} = 36$$

$\Delta > 0$; $\frac{\partial^3 f}{\partial x^3} > 0$ – TOČKA TO JE MINIMUM FUNKCIJE

②

$$\int_1^\infty \frac{1}{x^2}\,dx = \int_1^\infty x^{-2}\,dx = \int_1^\infty \frac{x^{-1}}{-1}\,dx = \int_1^\infty \frac{-1}{x}\,dx$$

$$= -\int_1^\infty \frac{dx}{x} = -\ln|x|\Big|_1^\infty$$

$$\lim = -\ln(\infty) + \ln(1) = |-\infty + 0| = -\infty$$

Ime i prezime: NIKOLA KNEŽEVIĆ **Broj indeksa:** 17-1-0002-2010

1 a

$$\int_0^{\pi} \sin^3 x\,dx = \left| t = \tan\frac{x}{2} \right|'$$

$$\int_0^{\pi} = t.$$

$2\sin x \cos x$

$\sin x = t \,|'$

$\cos x\,dx = dt$

$dx = \frac{dt}{\cos x}$

$$\int_0^{\pi} \sin^3 x\,dx = \left| \begin{array}{l} \sin^3 x = t \,|' \\ 3\sin x^2 \cos x\,dx = dt \\ dx = \frac{dt}{3\sin^2 x \cos x} \end{array} \right|$$

$t^2 \cdot \frac{dt}{\cos x}$

$u\,dv = u \cdot v - \int v \cdot du$

$$\int_0^{\pi} \sin^3 x\,dx = \left| \begin{array}{l} \sin^3 x = u \,|' \\ 3\sin^2 x \cos x\,dx = du \end{array} \right|$$

$dx = dv$

$x = v$

$$\sin^3 x \cdot x - \int x \cdot 3\sin^2 x \cos x\,dx$$

$$\sin^3 x \cdot x - 3\int x \sin^2 x \cos x\,dx \left| \begin{array}{l} \cos x = t \,|' \\ -\sin x\,dx = dt \\ dx = \frac{dt}{-\sin x} \end{array} \right|$$

$$\sin^3 x \cdot x - 3\int x \sin^2 x\, t\, \frac{dt}{-\sin x}$$

$$\sin^3 x \cdot x - 3\int x \sin x\, t\, dt$$

1.) b)

$$\int_0^1 \frac{x^2}{x^2-2}\,dx = \int_0^1 \frac{x^2-2+2}{x^2-2}\,dx$$

$$= \int_0^1 \frac{\cancel{x^2-2}}{\cancel{x^2-2}}\,dx + \int_0^1 \frac{2}{x^2-2}\,dx$$

$$= \int_0^1 dx + 2\int_0^1 \frac{dx}{x^2-2} = \int_0^1 dx + 2\int_0^1 \frac{dx}{x^2-(\sqrt{2})^2}$$

$$= x\Big|_0^1 + 2\left(\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}+x}{\sqrt{2}-x}\right|\right)\Bigg|_0^1$$

$$= 1-0+2\left[\left(\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|\right)-\left(\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}+0}{\sqrt{2}-0}\right|\right)\right]$$

$$= 1+2\left[\left(\frac{1}{2\sqrt{2}}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)\right)-\underbrace{\left(\frac{1}{2\sqrt{2}}\underbrace{\ln(1)}_{0}\right)}_{0}\right]$$

$$= 1+2\left[\frac{1}{2\sqrt{2}}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)\right]$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: FRANE JORDAN **BROJ INDEKSA:** 55161

VRIJEME POČETKA: 08:10 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

15

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$\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x \, \mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x \, \mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \tanh x \, \mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \coth x \, \mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x \, \mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \, \mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

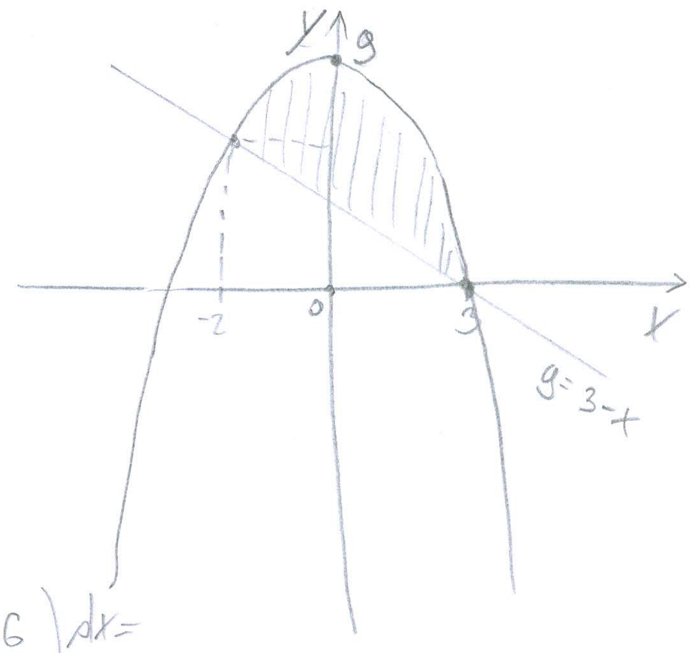
f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

(2) $\int_1^{\infty} \frac{1}{x^2}\,dx = \left| \begin{array}{l} t = \frac{-1}{x} \\ dt = \frac{1}{x^2} \end{array} \; ; \begin{array}{l} \lim\limits_{x\to 1} -\frac{1}{x} = -1 \\ \lim\limits_{x\to \infty} -\frac{1}{x} = 0 \end{array} \right|$

$$\int_{-1}^{0} dt = t\Big|_{-1}^{0} = 0+1 = 1$$

(3.) pr $y = 9 - x^2$

pr $y = 3 - x$



$9 - x^2 = 3 - x$

$x^2 - x - 6 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2}$

$x_1 = -2 \quad x_2 = 3$

$$P = \int_{-2}^{3} \left(9 - x^2 - (3 - x)\right)dx = \int_{-2}^{3} \left(-x^2 + x + 6\right)dx =$$

$$= \left(\frac{-x^3}{3} + \frac{x^2}{2} + 6\right)\Big|_{x=-2}^{3} =$$

$$= -9 + \frac{9}{2} + 18 - \left(\frac{8}{3} + 2 - 12\right) =$$

$$= -9 + \frac{9}{2} + 18 - \frac{8}{3} + 12 = 21 - \frac{11}{6}$$

Ime i prezime: FRANE JORDAN Broj indeksa: 55161

(5) $f(x,y) = x^3 + x^2 + y^3 - y^2$

$f'_x = 3x^2 + 2x = 0 \Rightarrow x_1 = 0 \quad x_2 = -\frac{2}{3}$

$f'_y = 3y^2 + 2y = 0 \Rightarrow y_1 = 0 \quad y_2 = -\frac{2}{3}$

$f''_{xx} = 6x$

$f''_{yy} = 6y$

$f''_{xy} = 0$

$\Delta_T = \left| f''_{xx} \cdot f''_{yy} - (f''_{xy})^2 \right|_T$

$\boxed{\Delta_T = 36 \cdot xy \big|_T > 0}$ uvjet

$T_1(0,0)$, $T_2\left(0, -\frac{2}{3}\right)$, $T_3\left(-\frac{2}{3}, 0\right)$, $T_4\left(-\frac{2}{3}, -\frac{2}{3}\right)$

samo točka $T_4\left(-\frac{2}{3}, -\frac{2}{3}\right)$ zadovoljava uvjet i pošto je $f''_{xx} = -4 < 0$ lokalni maximum.

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: STIPE DUŠEVIĆ **BROJ INDEKSA:** 17-2-0051-2010

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješiti integrale:

   (a) $\int\limits_0^{\pi} \sin^3 x \,\mathrm{d}x.$; — 10

   (b) $\int\limits_0^{1} \frac{x^2}{x^2-2} \,\mathrm{d}x.$ — 15

2. Izračunati nepravi integral $\int\limits_1^{\infty} \frac{1}{x^2} \,\mathrm{d}x.$ — 5

3. Izračunati površinu lika omeđenog omeđenog parabolom $y = 9 - x^2$ i pravcem $y = 3 - x$. — 20

4. Riješiti diferencijalnu jednadžbu: $y'' + y' - 2y = x^2$. Uvrstiti izračunato rješenje u jednadžbu i provjeriti da li je zadovoljena. — 15+5

5. Istražiti ekstreme funkcije $f(x, y) = x^3 + x^2 + y^3 + y^2$. — 20

6. Razviti funkciju $f(x) = \frac{1}{x}$ u Taylorov red po potencijama od $x - 1$. Izračunati barem prva 4 člana. — 10

Ukupno: 25

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x \,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x \,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \tanh x \,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \coth x \,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
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$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

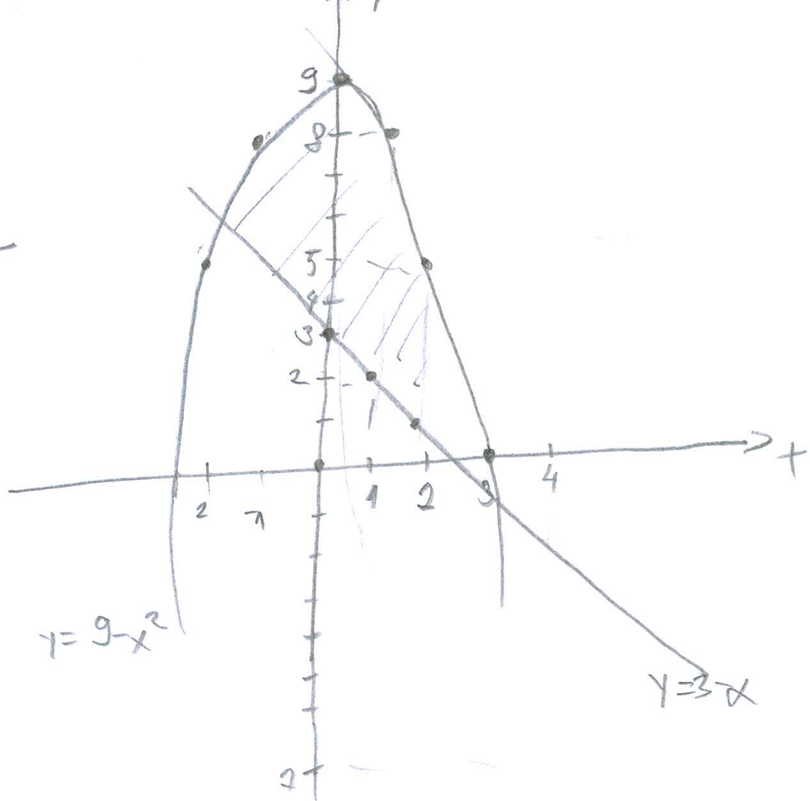
f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

) $y = 9 - x^2$

$y = 3 - x$

x	0	1	2	3	-1	-2
y	9	8	5	0	8	5

x	0	1	2
y	3	2	1



$9 - x^2 = 3 - x$

$9 - x^2 - 3 + x = 0$

$-x^2 + x + 6 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{1 + 24}}{-2}$

$x_{1,2} = \frac{-1 \pm 5}{-2}$

$x_1 = 3$

$x_2 = -2$

$$P = \int_{-2}^{3} \left(9 - x^2 - (3 - x)\right) dx = \int_{-2}^{3} (9 - x^2 - 3 + x)\, dx =$$

$$= \int_{-2}^{3} (-x^2 + x + 6)\, dx =$$

$$= -\int_{-2}^{3} x^2\, dx + \int_{-2}^{3} x\, dx + \int_{-2}^{3} 6\, dx =$$

$$= -\frac{x^3}{3}\Big|_{-2}^{3} + \Big|_{-2}^{3} \frac{x^2}{2} + 6x \Big|_{-2}^{3} =$$

15

$$= -\frac{27}{3} - \left(-\frac{8}{3}\right) + \frac{9}{2} - \frac{4}{2} + 18 + 12 =$$

$$= -\frac{27}{3} + \frac{8}{3} + \frac{9}{2} - \frac{4}{2} + 18 + 12 =$$

$$= \frac{-54 + 16 + 27 - 12 + 108 + 72}{6} = \frac{157}{6}$$

Ime i prezime: STIPE DUŠEVIĆ **Broj indeksa:** A-2-0051-2010

5) $f(x,y)=x^3+x^2+y^3+y^2$

$f'(x,y)_x=3x^2+2x$

$f(x,y)_y=3y^2+2y$

$3x^2+2x=0$ $\quad$ $3y^2+2y=0$

$x(3x+2)=0$ $\quad$ $y(3y+2)=0$

$x_1=0$ $\quad$ $y_1=0$

$3x+2=0$ $\quad$ $3y+2=0$

$3x=-2$ $\quad$ $3y=-2$

$x_2=-\frac{2}{3}$ $\quad$ $y_2=-\frac{2}{3}$

$A(0,0)$

$B\left(-\frac{2}{3},-\frac{2}{3}\right)$

$f''(x,y)_{xx}=6x+2$

$f''(x,y)_{xy}=0$

$f''(x,y)_{yx}=0$

$f''(x,y)_{yy}=6y+2$

$$\Delta A=\begin{vmatrix}6x+2 & 0\\ 0 & 6y+2\end{vmatrix}=\begin{vmatrix}\overset{>0}{2} & 0\\ 0 & 2\end{vmatrix}=4>0$$

$$\Delta B\begin{vmatrix}6x+2 & 0\\ 0 & 6y+2\end{vmatrix}=\begin{vmatrix}\overset{<0}{-4} & 0\\ 0 & -4\end{vmatrix}=16>0$$

$A(0,0)\Rightarrow$ minimum funkcije

$B\left(-\frac{2}{3},-\frac{2}{3}\right)\Rightarrow$ maksimum funkcije

~~MAX~~

10

$$y'' + y' - 2y = x^2$$

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{9}}{2}$$

$$\lambda_{1,2} = \frac{-1 \pm 3}{2}$$

$$\lambda_1 = -2$$

$$\lambda_2 = 1$$

$$y(x) = c_1 \cdot e^{\lambda_1 \cdot x} + c_2 e^{\lambda_2 \cdot x}$$

$$\boxed{y = c_1 \cdot e^{-2x} + c_2 \cdot e^{x}}$$

$$y' = c_1' \cdot e^{-2x} + c_1 \cdot (-2e^{-2x}) + c_2' \cdot e^{x} + c_2 \cdot e^{x}$$

$$y'' = c_1'' \cdot e^{-2x} + c_1' \cdot (-2e^{-2x}) + c_1' \cdot (-2e^{-2x}) + c_1'' \cdot e^{-2x} + c_2'' \cdot e^{x} + c_2' \cdot e^{x} + c_2' \cdot e^{x} + c_2 \cdot e^{x}$$

$$y'' = 2(c_1'' \cdot e^{-2x}) + 2(c_1' \cdot (-2e^{-2x})) + 2(c_2' \cdot e^{x}) + c_2'' \cdot e^{x} + c_2 \cdot e^{x}$$

$$2(c_1'' \cdot e^{-2x}) + 2(c_1' \cdot (-2e^{-2x})) + 2(c_2' \cdot e^{x}) + c_2'' \cdot e^{x} + c_2 \cdot e^{x}$$

$$+ c_1' \cdot e^{-2x} + c_1 \cdot (-2e^{-2x}) + c_2' \cdot e^{x} + c_2 \cdot e^{x} + c_1 \cdot e^{-2x} + c_2 \cdot e^{x} = x^2$$

Ime i prezime: STIPE DUŠEVIĆ **Broj indeksa:** 17-2-0051-2010

1) a) $\int_0^{\pi} \sin^3 x dx = \left| \begin{array}{c} \sin x = t \,/' \\ \cos x dx = dt \\ dx = \frac{dt}{\cos x} \end{array} \right| = \int_0^{\pi} t^3 \cdot \frac{dt}{\cos x}$

b) $\int_0^1 \frac{x^2}{x^2-2} dx =$

$\frac{x^2}{x^2-2} = \frac{A}{x-2} + \frac{Bx+C}{x^2-2}$ / inazivnik

$x^2 = A(x^2-2) + Bx + C$

$x^2 = Ax^2 - 2A + Bx + C$

$1 = A \quad 0 = Bx \quad 0 = -2A + C$

$A = 1 \quad B = 0 \quad C = 2A$

$C = 2$

$\Rightarrow \int_0^1 \frac{dx}{x-2} + \int_0^1 \frac{2dx}{x^2-2} =$

$= \ln|x-2| \Big|_0^1 + 2 \ln|x^2-2| \Big|_0^1$

6) $f(x) = \frac{1}{x}$

$x - 1$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊	Popunjava nastavnik
Ime i prezime: ZLATKO LALIĆ **Broj indeksa:** 57676	Broj ↓ bodova
Vrijeme početka: 09:55 **Vrijeme završetka:**	

1. Riješiti integrale:

   (a) $\displaystyle\int_0^\pi \sin^3 x\,\mathrm{d}x.$; — 10

   (b) $\displaystyle\int_0^1 \frac{x^2}{x^2-2}\,\mathrm{d}x.$ — 15

2. Izračunati nepravi integral $\displaystyle\int_1^\infty \frac{1}{x^2}\,\mathrm{d}x.$ — 5

3. Izračunati površinu lika omeđenog omeđenog parabolom $y = 9 - x^2$ i pravcem $y = 3 - x$. — 20

4. Riješiti diferencijalnu jednadžbu: $y'' + y' - 2y = x^2$. Uvrstiti izračunato rješenje u jednadžbu i provjeriti da li je zadovoljena. — 15+5

5. Istražiti ekstreme funkcije $f(x,y) = x^3 + x^2 + y^3 + y^2$. — 20

6. Razviti funkciju $f(x) = \dfrac{1}{x}$ u Taylorov red po potencijama od $x-1$. Izračunati barem prva 4 člana. — 10

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

$z_x = 3x^2 + 2x$

$z_y = 3y^2 + 2y$

$3x^2 + 2x = 0$
$3y^2 + 2y = 0$

$3x^2 + 2x = 0$

$x(3x+2) = 0$

$x_1 = 0 \qquad 3x + 2 = 0$

$3x = -2 \quad | :3$

$x_2 = -\frac{2}{3}$

$3y^2 + 2y = 0$

$y(3y+2) = 0$

$y_1 = 0 \qquad 3y + 2 = 0$

$3y = -2 \quad | :3$

$y_2 = -\frac{2}{3}$

$T_1(0,0)$

$T_2\left(-\frac{2}{3}, -\frac{2}{3}\right)$

$z_{xx} = 6x + 2$

$z_{xy} = 0$

$z_{yy} = 6y + 2$

$\Delta = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4 > 0 \ \exists$

$z_{xx} < 0 \Rightarrow \max$

$z_{max}\left(-\frac{2}{3}, -\frac{2}{3}\right) = \left(-\frac{2}{3}\right)^3 + \left(-\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^3 + \left(-\frac{2}{3}\right)^2$
$= \frac{8}{27}$

$z_{xx}\left(-\frac{2}{3}, -\frac{2}{3}\right) = 6 \cdot \left(-\frac{2}{3}\right) + 2$
$= -\frac{12}{3} + 2$
$= -4 + 2$
$= -2$

$z_{xy}\left(-\frac{2}{3}, -\frac{2}{3}\right) = 0$

$z_{yy}\left(-\frac{2}{3}, -\frac{2}{3}\right) = 6 \cdot \left(-\frac{2}{3}\right) + 2$
$= -2$

$z_{xx}(0,0) = 2$

$z_{xy}(0,0) = 0$

$z_{yy}(0,0) = 2$

$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 > 0 \ \exists$

$z_{xx} > 0 \Rightarrow \min$

$z_{min}(0,0) = 0^3 + 0^2 + 0^3 + 0^2$
$= 0$

Ime i prezime: ZLATKO LALIĆ Broj indeksa: 57676

4. $y'' + y' - 2y = x^2$

$r^2 + r - 2 = 0$

$$r_{12} = \frac{-1 \pm \sqrt{1+4\cdot1\cdot(-2)}}{2} = \frac{-1 \pm \sqrt{9}}{2}$$

$$= \frac{-1 \pm 3}{2}$$

$$= \frac{-1+3}{2}$$

$$r_1 = \frac{2}{2} = 1$$

$$r_2 = \frac{-4}{2} = -2$$

$r_1 \neq r_2$

$y_H = C_1 e^{r_1 x} + C_2 e^{r_2 x}$

$y_H = C_1 e^{x} + C_2 e^{-2x}$

1. b) $\int_0^1 \frac{x^2}{x^2-2}\,dx$

$$\int \frac{x^2}{x^2-2}\,dx$$

$$(x^2):(x^2-2)=1$$

$$\underline{-x^2+2}$$

$$+2$$

$$\int 1+\frac{2}{x^2-2}\,dx =$$

$$=\int 1\,dx+\int\frac{2}{x^2-2}\,dx$$

$$= x+2\int\frac{dx}{x^2-2}$$

$$= x+2\ln|x^2-2|+C$$

$$=$$

Ime i prezime: ZLATKO LALIĆ **Broj indeksa:** 57676

~~1. a)~~ $\cancel{\int_0^{\pi} \sin^3 x \, dx = \int \sin^3 x \, dx = \left| \begin{array}{l} \sin 3 = t \\ dx = dt \end{array} \right| = \int \sin^t x \, dt}$

$$P = \int_{-2}^{3} (3-x) - (9-x^2) = \int_{-2}^{3} [3 - x - 9 - x^2]$$

$$= \int_{-2}^{3} (-x^2 - x - 6)\, dx$$

$$= \left[-\frac{x^3}{3} - \frac{x^2}{2} \right]_{-2}^{3}$$

$$P = \left(-\frac{3^3}{3} - \frac{3^2}{2} \right) - \left(-\frac{-2^3}{3} - \frac{-2^2}{2} \right) =$$

$$= \cancel{-\frac{27}{3}} = \left(-\frac{27}{3} - \frac{9}{2} \right) - \left(\frac{-8}{3} - \frac{+4}{2} \right)$$

$$= -\frac{27}{3} - \frac{9}{2} + \frac{8}{3} - \frac{4}{2}$$

$$= -\frac{19}{3} - \frac{13}{2} = -\frac{77}{6} \approx -12.833$$

4. ~~[illegible]~~

~~$f(x,y) = x^0 \cdot Ax + B$~~

3. $y = 9 - x^2 \qquad y = 3 - x$

x	y
0	3
1	2
-1	4

$9 - x^2 = 0$

$-x^2 = -9 \quad | \sqrt{\ }$

$x = \pm 3$

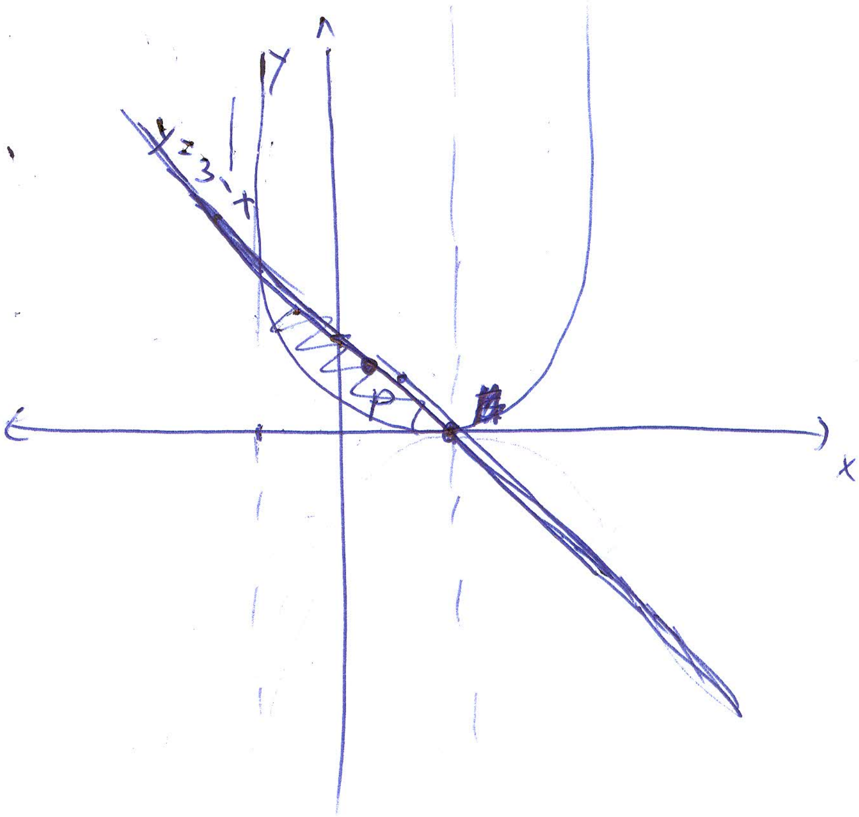
~~[illegible]~~ $y_{(3)} = 0$

$9 - 3^2 = 0$

$0 = 0$

~~[illegible]~~

~~$(3, 0)$~~



$9 - x^2 = 0$

$3 - x = 0$

$9 - x^2 = 3 - x$

$-x^2 + x + 6$

$x_{1,2} = \dfrac{-1 \pm \sqrt{1 - 4 \cdot -1 \cdot 6}}{-2}$

$= \dfrac{-1 \pm \sqrt{25}}{-2} = \dfrac{-1 \pm 5}{-2} =$

~~$= -1 \pm$~~

$x_1 = \dfrac{-1+5}{-2} = \dfrac{+4}{-2} = -2$

$x_2 = \dfrac{-1-5}{-2} = \dfrac{-6}{-2} = 3$

Ime i prezime: ZLATKO LALIĆ Broj indeksa: 57676

$$\int_0^{\pi} \sin^3 x \, dx = \int \sin^3 x \, dx = \left| \begin{matrix} x = t \\ dx = dt \end{matrix} \right|$$

$$= \int \sin^3 t \, dt$$

$$= -\cos^3 t$$

$$= -\cos^3 x$$

$$\left(-\cos^3 x\right)\Big|_0^{\pi} = -\cos^3 \pi + \cos^3 0$$

$0/7$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓

IME I PREZIME: Adriano Viprotnik **BROJ INDEKSA:** 17-2-0138-2011

VRIJEME POČETKA: 08:00 **VRIJEME ZAVRŠETKA:** 09:50

POPUNJAVA NASTAVNIK
Broj ↓ bodova

10

1. Riješiti integrale:

   (a) $\int\limits_0^{\pi} \sin^3 x \, \mathrm{d}x.$; — 10

   (b) $\int\limits_0^{1} \frac{x^2}{x^2-2} \, \mathrm{d}x.$ — 15

2. Izračunati nepravi integral $\int\limits_1^{\infty} \frac{1}{x^2} \, \mathrm{d}x.$ — 5

3. Izračunati površinu lika omeđenog omeđenog parabolom $y = 9 - x^2$ i pravcem $y = 3 - x$. — 20

4. Riješiti diferencijalnu jednadžbu: $y'' + y' - 2y = x^2$. Uvrstiti izračunato rješenje u jednadžbu i provjeriti da li je zadovoljena. — 15+5

5. Istražiti ekstreme funkcije $f(x,y) = x^3 + x^2 + y^3 + y^2$. — 20

6. Razviti funkciju $f(x) = \frac{1}{x}$ u Taylorov red po potencijama od $x-1$. Izračunati barem prva 4 člana. — 10

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x \, \mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x \, \mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \tanh x \, \mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \coth x \, \mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x \, \mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \, \mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$x^\alpha\ (\alpha \neq 0)$	$\alpha x^{\alpha-1}$
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{-1}{\sin^2 x}$
$\sinh x$	$\cosh x$

f	$\frac{\mathrm{d}f}{\mathrm{d}x}$
$\cosh x$	$\sinh x$
$\tanh x$	$\frac{1}{\cosh^2 x}$
$\coth x$	$\frac{-1}{\sinh^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1}$	$\frac{1}{\sqrt{x^2-1}}$

3.)

$y = 9 - x^2$

$y = 3 - x$

$9 - x^2 = 3 - x$

$-x^2 + x + 6 = 0 \;/\cdot(-1)$

$x^2 - x - 6 = 0$

$x = \frac{1 \pm \sqrt{1+24}}{2}$

$x = \frac{1 \pm 5}{2}$

$x_1 = -2 \quad x_2 = 3$

$y_1 = 3 - x_1$

$y_1 = 3 - (-2)$

$y_1 = 5$

$S_1(-2, 5)$

$y_2 = 3 - x_2$

$y_2 = 3 - 3$

$y_2 = 0$

$S_2 = (3, 0)$

$y = 3 - x$

x	0	1
$y=3-x$	3	2

$y = 9 - x^2$

$T\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$

$T\left(\frac{0}{-2}, \frac{-36-0}{-4}\right)$

$T(0, 9)$

$9 - x^2 = 0$

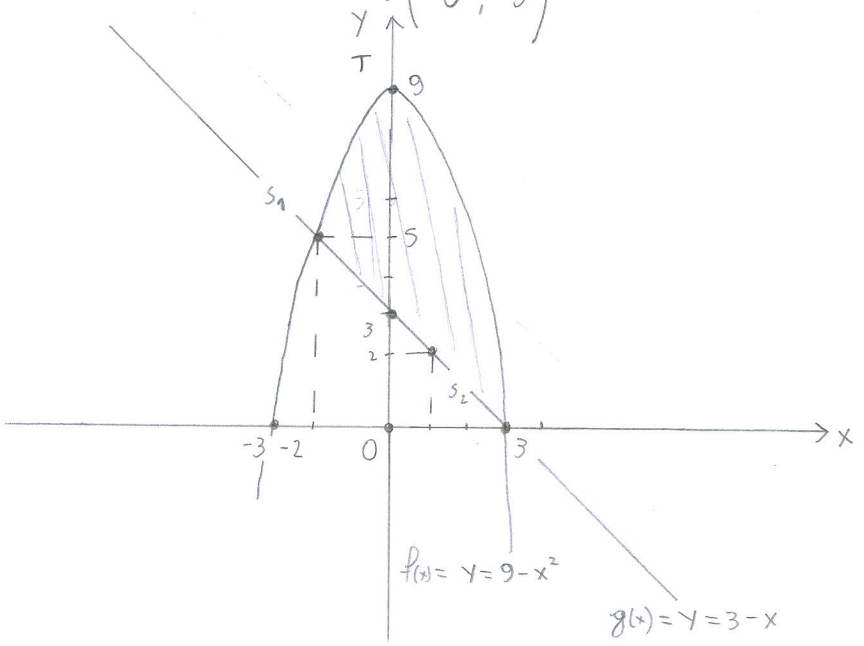
$-x^2 = -9 \;/\cdot(-1)$

$x^2 = 9 \;/\sqrt{\ }$

$x = \pm 3$

$x_1 = -3 \quad x_2 = 3$

x	-3	3
$y=9-x^2$	0	0



$P = \int_{-3}^{3} [f(x) - g(x)]\,dx = \int_{-3}^{3} [9 - x^2 - (3 - x)]\,dx = \int_{-3}^{3} (-x^2 + x + 6)\,dx$

$= -\int_{-3}^{3} x^2\,dx + \int_{-3}^{3} x\,dx + 6\int_{-3}^{3} dx = -\frac{x^3}{3}\Big|_{-3}^{3} + \frac{x^2}{2}\Big|_{-3}^{3} + 6x\Big|_{-3}^{3}$

$= -9 + \frac{9}{2} + 18 - \left(9 + \frac{9}{2} - 18\right) = -9 + \cancel{\frac{9}{2}} + 18 - 9 - \cancel{\frac{9}{2}} + 18 = -18 + 36 = 18$

5.) $f(x,y) = x^3 + x^2 + y^3 + y^2$

$\partial x f = 3x^2 + 2x$ $\qquad$ $\partial y f = 3y^2 + 2y$

$\partial xx f = 6x + 2$ $\qquad$ $\partial yy f = 6y + 2$

$\partial xy f = 0$ $\qquad$ $\partial yx f = 0$

$A = \partial xx f$

$A = 6x + 2$

$$\Delta = \begin{vmatrix} \partial xx f & \partial xy f \\ \partial yx f & \partial yy f \end{vmatrix}$$

10

$\partial x f$

$\partial y f$

$3x^2 + 2x = 0$

$3y^2 + 2y = 0$

$3x^2 + 2x = 0$

$x(3x+2) = 0$

$x_1 = 0 \qquad 3x + 2 = 0$

$3x = -2 \;/ : 3$

$x_2 = -\frac{2}{3}$

$3y^2 + 2y = 0$

$y(3y+2) = 0$

$y_1 = 0 \qquad 3y + 2 = 0$

$y_2 = -\frac{2}{3}$

$$\Delta_1 = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = -4 - 0 = 4$$

$A = 0 + 2 = 2$

$\left.\begin{array}{l} \Delta_1 = 4 < 0 \\ A = 2 > 0 \end{array}\right\}$ lokalni maximum

$f(0,0) = x^3 + x^2 + y^3 + y^2$

$f(0,0) = 0$

$f_M = 0$ u točki $T_1(0,0)$

$$\Delta_2 = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix} = 4 - 0 = 4$$

$\left.\begin{array}{l} \Delta_2 = 4 > 0 \\ A = -2 < 0 \end{array}\right\}$ lokalni minimum

$f(-\frac{2}{3}, -\frac{2}{3}) = x^3 + x^2 + y^3 + y^2$

$f(-\frac{2}{3}, -\frac{2}{3}) = -\frac{8}{27} + \frac{4}{9} - \frac{8}{27} + \frac{4}{9}$

$f(-\frac{2}{3}, -\frac{2}{3}) = \frac{8}{27}$

$f_m = \frac{8}{27}$ u točki $T_2(-\frac{2}{3}, -\frac{2}{3})$

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4.) $y'' + y' - 2y = x^2$

$y'' + y' - 2y = 0$

$\lambda^2 + \lambda - 2 = 0$

$\lambda = \frac{-1 \pm \sqrt{1+8}}{2}$

$\lambda = \frac{-1 \pm 3}{2}$

$\lambda_1 = -2 \quad \lambda_2 = 1$

$y_H = e^{-2x} C_1 + e^{x} C_2$

$r = 0 \quad r + n = 0 + 2 = 2$ polinom drugog stupnja

$n = 2$

$Y = a_2 x^2 + a_1 x + a_0$

Ime i prezime: Adriano Vipotnik **Broj indeksa:** 17-2-0138-2011

1.) b) $\int_0^1 \frac{x^2}{x^2-2}\,dx = \int_0^1 dx - \frac{1}{2}\int_0^1 x^2\,dx = x\Big|_0^1 - \frac{x^3}{6}\Big|_0^1$

$= 1 - \frac{1}{6} - (0-0) = \frac{5}{6} + C$

a) $\int_0^\pi \sin^3 x\,dx = \begin{bmatrix} x = t \\ dx = dt \end{bmatrix} = \int_0^\pi \sin^3 t\,dt = \cos^3 x\Big|_0^\pi$

$= \cos^3\pi - \cos^3 0$

2.) $\int_1^\infty \frac{1}{x^2}\,dx = \int_1^\infty x^{-2}\,dx = \lim_{a\to\infty}\int_1^a x^{-2}\,dx = \lim_{a\to\infty} -x\Big|_1^a$

$= -\infty - (-1) = -\infty + 1 = -\infty$