MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** **Ime i prezime:** **Broj indeksa:** **Vrijeme početka:** RJEŠENJE 3	Popunjava nastavnik Broj ↓ bodova

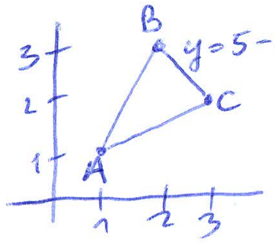
1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y) = y - x$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_K (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20
3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z) = (2x+1,\ 3y-z,\ z-y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20
5. Izračunaj volumen dijela prostora odozdo omeđenog paraboloidom $z = x^2 + y^2$, a odozgo ravninom $z = 5$. 20

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

(1)



AB: $(y-1)\cdot 1 = (x-1)\cdot 2$

$y = 2x-1$

AC: $(y-1)\cdot 2 = (x-1)\cdot 1$

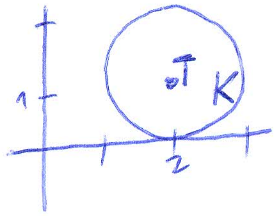
$y = \frac{x}{2} + \frac{1}{2}$

BC: $y = 5-x$

$$\iint\limits_T (y-x)\,dxdy = \int_1^2 \int_{\frac{x}{2}+\frac{1}{2}}^{2x-1} y-x\,dxdy + \int_2^3 \int_{\frac{x}{2}+\frac{1}{2}}^{5-x} y-x\,dxdy$$

$$= \ldots = 0$$

(2)



$$\iint\limits_K 3-2y\,dxdy = \left\{\begin{matrix} x = 2 + r\cos\varphi \\ y = 1 + r\sin\varphi \end{matrix}\right\} = \int_0^{2\pi}\int_0^1 (3-2-2r\sin\varphi)\,r\,dr\,d\varphi =$$

$$= \underbrace{\int_0^{2\pi}\int_0^1 r\,dr\,d\varphi}_{=\pi} - 2\underbrace{\int_0^{2\pi}\int_0^1 r^2\sin\varphi\,dr\,d\varphi}_{=0} = \pi$$

③ KRIVULJNI INTEGRAL JE NEOVISAN O PUTU AKO JE VEKTORSKA FUNKCIJA POTENCIJALNO POLJE. TRAŽIM f SKALARNU FUNKCIJU TAKO DA

$$\partial_x f = 2x+1 \Rightarrow f(x,y,z) = x^2 + x + \text{const}(y,z) \Rightarrow \partial_y f = \partial_y \text{const}(y,z)$$

$$\partial_y f = 3y - z \qquad \Rightarrow \text{const}(y,z) = \frac{3}{2}y^2 - zy + C(z) \Rightarrow \partial_z f = -y + C'(z)$$

$$\partial_z f = z - y \qquad \Rightarrow \qquad C(z) = \frac{z^2}{2}$$

$$f(x,y,z) = x^2 + x + \frac{3}{2}y^2 - zy + \frac{z^2}{2}$$

POSTOJI TRAŽENA FUNKCIJA PA SVAKI KRIVULJNI INTEGRAL FUNKCIJE g OVISI SAMO O POČETNOJ I ZAVRŠNOJ TOČKI!

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④ $w = \begin{bmatrix} 0 \\ 0 \\ 3-2y \end{bmatrix}$ $\quad \operatorname{div} w = 0$ $\quad$ PO TEOREMU O DIVERGENCIJI $\quad \iint_{\partial K} 3-2y \, dx\,dy = \iiint_K 0 \, dx\,dy\,dz = 0$

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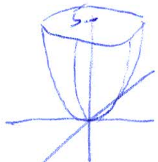
⑤ $z = x^2+y^2$

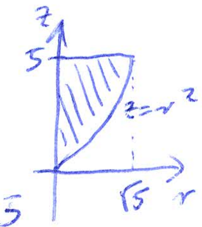
$z = 5$



$z \in [0,5]$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{z}]$



$$V = \int_0^{2\pi}\int_0^{5}\int_0^{\sqrt{z}} 1\cdot r \, dr\, dz\, d\varphi = \cancel{2}\pi \int_0^5 \frac{z}{\cancel{2}}\, dz = \frac{25\pi}{2}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: RIKO KOLEGA **BROJ INDEKSA:** 55849-2008

VRIJEME POČETKA: 08:00 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y) = y - x$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_K (3-2y)\,\mathrm{d}x\mathrm{d}y$. ~~20~~ 15
3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z) = (2x+1,\ 3y - z,\ z - y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20
5. Izračunaj volumen dijela prostora odozdo omeđenog paraboloidom $z = x^2 + y^2$, a odozgo ravninom $z = 5$. 20

Ukupno: 35

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

2. $r = 1 \quad T(2,1)$ $\quad (x-2)^2 + (y-1)^2 = r^2 \quad (x-2)^2 + (y-1)^2 = 1$

$\iint\limits_K (3-2y)\,dx\,dy$

$x = r\cos\varphi + 2 \quad y = r\sin\varphi + 1 \quad dx\,dy = r\,dr\,d\varphi$

$\varphi \in [0, 2\pi] \quad r \in [0,1]$

15

$$\boxed{\int_0^{2\pi}\int_0^1 (3 - 2\cdot(r\sin\varphi + 1))r\,dr\,d\varphi} = \boxed{\int_0^{2\pi} d\varphi \int_0^1 (3 - 2r^2\sin\varphi + 2r)\,dr} = \int_0^{2\pi} d\varphi \left(3r - 2\cdot\frac{r^3}{3}\sin\varphi + 2\cdot\frac{r^2}{2}\right)\Big|_0^1 =$$

$$= \int_0^{2\pi} d\varphi\left(3 - \frac{2}{3}\sin\varphi + 1\right) = \int_0^{2\pi} d\varphi\left(4 - \frac{2}{3}\sin\varphi\right) = 4\int_0^{2\pi} d\varphi - \frac{2}{3}\int_0^{2\pi}\sin\varphi\,d\varphi =$$

$$= 4\cdot\varphi\Big|_0^{2\pi} - \frac{2}{3}\cdot\sin\varphi\Big|_0^{2\pi} = 4\cdot 2\pi - (4\cdot 0) - \cancel{\frac{2}{3}\cdot\underbrace{\sin 2\pi}_{0}} - \cancel{\left(-\frac{2}{3}\cdot\underbrace{\sin 0}_{0}\right)} =$$

$$= 8\pi$$

5. $z = x^2 + y^2 \qquad z = 5$

$x = r\cos\varphi$

$y = r\sin\varphi$

$dxdy = r\,dr\,d\varphi$

$x^2 + y^2 = 5$

$r^2 = 5$

$r = \sqrt{5}$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{5}]$

$z \in [5, x^2 + y^2]$

$z \in [5, (r\cos\varphi)^2 + (r\sin\varphi)^2]$

$z \in [5, r^2\cos^2\varphi + r^2\sin^2\varphi]$

$z \in [5, r^2\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1}]$

$z \in [5, r^2]$

$$\int_0^{2\pi}\int_0^{\sqrt{5}}\int_5^{r^2} d\varphi\, r\,dr\, dz = \int_0^{2\pi} d\varphi \int_0^{\sqrt{5}} r\,dr\, z\Big|_5^{r^2} = \int_0^{2\pi} d\varphi \int_0^{\sqrt{5}} \left((r \cdot r^2 - (r \cdot 5)\right) dr = \int_0^{2\pi} d\varphi \int_0^{\sqrt{5}} (r^3 - 5r)\, dr$$

$$= \int_0^{2\pi} d\varphi \left(\frac{r^4}{4} - 5\frac{r^2}{2}\right)\Big|_0^{\sqrt{5}} = \int_0^{2\pi} d\varphi \left(\frac{25}{4} - \frac{25}{2}\right) = \int_0^{2\pi} -\frac{25}{4}\, d\varphi = -\frac{25}{4}\int_0^{2\pi} d\varphi =$$

$$= -\frac{25}{4}\varphi\Big|_0^{2\pi} = \frac{25}{\cancel{4}} \cdot \cancel{2}\pi = \frac{25\pi}{2}$$

✓ 20

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇓⇓ **Ime i prezime:** MARINO PRENDŽA **Broj indeksa:** 57659 **Vrijeme početka:** 07:55 **Vrijeme završetka:**	Popunjava nastavnik Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y) = y - x$. Odrediti $\iint_T f(x,y)\mathrm{d}x\mathrm{d}y$. ~~20~~ 15
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint_K (3-2y)\,\mathrm{d}x\mathrm{d}y$. ~~20~~
3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z) = (2x+1,\ 3y-z,\ z-y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20
5. Izračunaj volumen dijela prostora odozdo omeđenog paraboloidom $z = x^2+y^2$, a odozgo ravninom $z = 5$. ~~20~~ 15

Ukupno: 30

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
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2. KRUG

$r = 1$

$T(2,1)$

$\iint_K (3-2y)\,dxdy$

$r \in [0,1]$

$\varphi \in [0, 2\pi]$

0

$$\int_0^{2\pi} d\varphi \int_0^1 (3 - 2r\sin\varphi)\, r\,dr = \int_0^{2\pi} d\varphi \int_0^1 3r - 2r^2\sin\varphi\, dr =$$

X

$$= \int_0^{2\pi} d\varphi \left(3\cdot\frac{r^2}{2} - 2\frac{r^3}{3}\sin\varphi\right)\Big|_0^1 = \int_0^{2\pi} d\varphi\left(3\cdot\frac{1}{2} - 2\cdot\frac{1}{3}\sin\varphi\right) =$$

$$= \int_0^{2\pi} \frac{3}{2} - \frac{2}{3}\sin\varphi\, d\varphi = \frac{3}{2}\int_0^{2\pi} d\varphi - \frac{2}{3}\int_0^{2\pi}\sin\varphi\, d\varphi =$$

$$= \frac{3}{2}\cdot 2\pi + \frac{2}{3}\cos\varphi\Big|_0^{2\pi} = 3\pi + \frac{2}{3}\cos 2\pi - \left(\frac{2}{3}\cos 0\right) =$$

$$= 3\pi + \frac{2}{3} - \frac{2}{3} = 3\pi$$

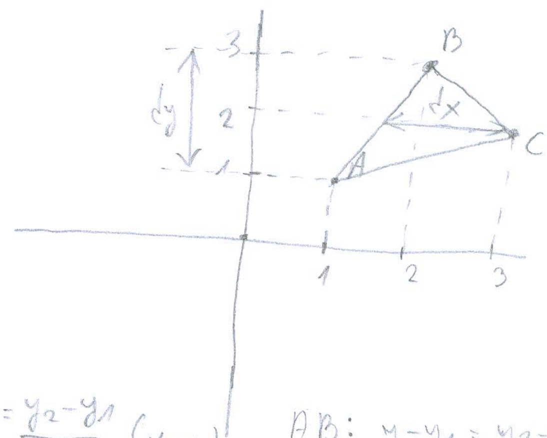
1. TROKUT

$A(1,1)$

$B(2,3)$

$C(3,2)$

$\underline{f(x,y) = y - x}$



$AC: \; y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 1 = \dfrac{2-1}{3-1}(x-1)$

$y - 1 = \dfrac{1}{2}(x-1)$

$y - 1 = \dfrac{1}{2}x - \dfrac{1}{2} \;/\cdot 2$

$2y - 2 = x - 1$

$-x = -2y + 2 - 1$

$x = 2y - 2 + 1$

$x = 2y - 1$

$BC: \; y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 3 = \dfrac{2-3}{3-2}(x-2)$

$y - 3 = \dfrac{-1}{1}(x-2)$

$y - 3 = -x + 2$

$x = -y + 3 + 2$

$x = -y + 5$

$AB: \; y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 1 = \dfrac{3-1}{2-1}(x-1)$

$y - 1 = \dfrac{2}{1}(x-1)$

$y - 1 = 2x - 2 \;/:2$

$\dfrac{1}{2}y - \dfrac{1}{2} = x - 1$

$-x = -\dfrac{1}{2}y + \dfrac{1}{2} - 1$

$x = \dfrac{1}{2}y - \dfrac{1}{2} + 1$

$x = \dfrac{1}{2}y + \dfrac{1}{2}$

$\underline{15}$

$$\int_1^2 \int_{\frac{1}{2}y+\frac{1}{2}}^{2y-1} (y-x)\,dy\,dx + \int_2^3 \int_{\frac{1}{2}y+\frac{1}{2}}^{-y+5} (y-x)\,dy\,dx = \int_1^2 dy \left(yx - \frac{x^2}{2}\right)\Bigg|_{\frac{1}{2}y+\frac{1}{2}}^{2y-1} + \int_2^3 dy \left(yx - \frac{x^2}{2}\right)\Bigg|_{\frac{1}{2}y+\frac{1}{2}}^{-y+5} =$$

$$= \int_1^2 dy \left(y\cdot(2y-1) - \frac{(2y-1)^2}{2} - \left(y\cdot\left(\frac{1}{2}y+\frac{1}{2}\right) - \frac{\left(\frac{1}{2}y+\frac{1}{2}\right)^2}{2}\right)\right) +$$

$$+ \int_2^3 dy \left(y\cdot(-y+5) - \frac{(-y+5)^2}{2}\right) - \left(y\cdot\left(\frac{1}{2}y+\frac{1}{2}\right) + \frac{\left(\frac{1}{2}y+\frac{1}{2}\right)^2}{2}\right) =$$

$$= \int_1^2 dy \left(2y^2 - y - \frac{2y^2 - 4y + 1}{2} - \frac{1}{2}y^2 - \frac{1}{2}y + \frac{\frac{1}{4}y^2 + \frac{1}{2}y + \frac{1}{4}}{2}\right) +$$

$$+ \int_2^3 dy \left(-y^2 + 5y - \frac{y^2 + 10y + 25}{2} - \frac{1}{2}y^2 - \frac{1}{2}y + \frac{\frac{1}{2}y^2 + \frac{1}{2}y + \frac{1}{4}}{2}\right)$$

Ime i prezime: MARINO PRENDŽA **Broj indeksa:** 57659

$$\int_1^2 dy\left(2y^2-y-\frac{2y^2-4y+1-y^2-y+\frac{1}{4}y^2+\frac{1}{2}y+\frac{1}{4}}{2}\right)+$$

$$+\int_2^3 dy\left(-y^2+5y-\frac{\cancel{y^2}+10y+25-\cancel{y^2}-y+\frac{1}{4}y^2+\frac{1}{2}y+\frac{1}{4}}{2}\right)=$$

$$=\int_1^2 dy\left(2y^2-y-\frac{\frac{5}{4}y^2-\frac{9}{2}y+\frac{5}{4}}{2}\right)+\int_2^3 dy\left(-y^2+5y-\frac{\frac{1}{4}y^2+\frac{19}{2}y+\frac{101}{4}}{2}\right)\Big/\cdot 2$$

$$=\int_1^2\left(4y^2-2y-\frac{5}{4}y^2-\frac{9}{2}y+\frac{5}{4}\right)dy+\int_2^3\left(-2y^2+10y-\frac{1}{4}y^2+\frac{19}{2}y+\frac{101}{4}\right)dy=$$

$$=\int_1^2\left(\frac{11}{4}y^2-\frac{13}{2}y+\frac{5}{4}\right)dy+\int_2^3\left(-\frac{9}{4}y^2+\frac{39}{2}y+\frac{101}{4}\right)dy=$$

$$=\frac{11}{4}\frac{y^3}{3}-\frac{13}{2}\cdot\frac{y^2}{2}+\frac{5}{4}y\Big|_1^2-\frac{9}{4}\cdot\frac{y^3}{3}+\frac{39}{2}\cdot\frac{y^2}{2}+\frac{101}{4}y\Big|_2^3=$$

$$=\frac{11}{4}\cdot\frac{8}{3}-\frac{13}{2}\cdot\frac{4}{2}+\frac{5}{4}\cdot 2-\left(\frac{11}{4}\cdot\frac{1}{3}-\frac{13}{2}\cdot\frac{1}{2}+\frac{5}{4}\right)-\frac{9}{4}\cdot 9+\frac{39}{2}\cdot\frac{9}{2}+\frac{101}{4}\cdot 3-\left(-\frac{9}{4}\cdot\frac{8}{3}+\frac{39}{2}\cdot\frac{4}{2}+\frac{101}{4}2\right.$$

$$=\frac{22}{3}-\frac{13}{4}+\frac{5}{2}-\frac{11}{12}+\frac{13}{4}-\frac{5}{4}-\frac{18}{4}+\frac{351}{4}+\frac{303}{4}+6-\frac{39}{4}-\frac{202}{4}=$$

$$=\frac{1349}{12}=112,41$$

5. $z = x^2 + y^2$ $\quad z = 5$

$r^2 = z \,/\sqrt{\ }$

$r = \sqrt{z}$

$r \in [0, \sqrt{z}]$

$z \in [0, 5]$

$\varphi \in [0, 2\pi]$

15

$$V = \int_0^{2\pi} d\varphi \int_0^5 dz \int_0^{\sqrt{z}} r\,dr =$$

$$= \int_0^{2\pi} d\varphi \int_0^5 dz \left(\frac{r^2}{2}\right)\Big|_0^{\sqrt{z}} = \int_0^{2\pi} d\varphi \int_0^5 \frac{(\sqrt{z})^2}{2}\,dz =$$

$$= \int_0^{2\pi} d\varphi \, \frac{1}{2}\int_0^5 z\,dz = \int_0^{2\pi} d\varphi \cdot \left(\frac{1}{2}\cdot\frac{z^2}{2}\right)\Big|_0^5 =$$

$$= \frac{5}{2}\int_0^{2\pi} d\varphi = \frac{5}{2}\cdot 2\pi = 5\pi$$

4. KOCKA

$a = 2$

$T(0,0,0)$

$$\iint_{\partial K} (3 - 2y)\,dx\,dy$$

$\frac{\partial}{x} = -2$

$\frac{\partial}{y} = 0$

NE

$t \in [0, 2\pi]$

$$\int_0^{2\pi} 0 - (-2)\,dt = 2\int_0^{2\pi} dt = 2 \cdot 2\pi = 4\pi$$

NE

$\cancel{0}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: NIKOLA MILUTIN **BROJ INDEKSA:** 58150	
VRIJEME POČETKA: 08:05 **VRIJEME ZAVRŠETKA:**	

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y) = y - x$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20 ~~20~~

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_K (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z) = (2x+1,\ 3y-z,\ z-y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20

4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

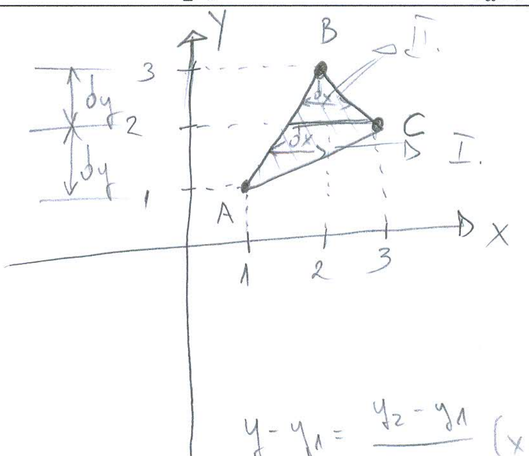
5. Izračunaj volumen dijela prostora odozdo omeđenog paraboloidom $z = x^2 + y^2$, a odozgo ravninom $z = 5$. ~~20~~

Ukupno: 20

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

(1.) $A(1,1)$
$B(2,3)$
$C(3,2)$



$\iint\limits_T f(x,y)\,dx\,dy \Rightarrow \iint\limits_T (y-x)\,dx\,dy$

~~$\iint xy\,dx\,dy$~~

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$AC: y - 1 = \frac{2-1}{3-1}(x-1)$

$y - 1 = \frac{1}{2}x - \frac{1}{2} \quad / \cdot 2$

$2y - 2 = x - 1$

$-x = -1 + 2 - 2y \quad / \cdot (-1)$

$x = 2y - 1$

$BC: y - 3 = \frac{2-3}{3-2}(x-2)$

$y - 3 = -x + 2$

$x = 5 - y$

$AB: y - 1 = \frac{3-1}{2-1}(x-1)$

$y - 1 = 2x - 2$

$-2x = -y - 2 + 1$

$-2x = -y - 1 \quad / : (-2)$

$x = \frac{1}{2}y + \frac{1}{2}$

$\ldots x\,dy$

$- \iint x\,dx\,dy$

$\ldots dx - \int_2^3 dy \int_{\frac12 y+\frac12}^{5-y} x\,dx$

$\int_1^2 \int_0^0 (y-x)\,dx\,dy + \int_2^3 \int_0^0 (y-x)\,dx\,dy$

$\ldots y-1-\frac12 y-\frac12\Big)dy - \int_2^3 \Big(5-y-\frac12 y-\frac12\Big)dy$

$\ldots \frac32 y-\frac32\Big)dy - \int_2^3 \Big(-\frac32 y+\frac92\Big)dy$

$\ldots \frac32 y^2-\frac32 y\Big)dy - \int_2^3 \Big(-\frac32 y+\frac92\Big)dy$

$\ldots \frac32\cdot\frac{y^3}{3}-\frac32\cdot\frac{y^2}{2}\Big)\Big|_1^2 - \Big(-\frac32\,\frac{y^2}{2}+\frac92 y\Big)\Big|_2^3$

$\ldots 1-\Big(-\frac14\Big)\Big) - \Big(\frac{27}{4}-(6)\Big)$

$\frac54-\frac{27}{4}+6$

$\frac12$

PREZIME: NIKOLA MILUTIN

BROJ INDEKSA: 58150

1 $\quad T(2,1)$

$$dxdy = r\,dr\,d\varphi$$
$$x = r\cos\varphi + 2$$
$$y = r\sin\varphi + 1$$

$r\,[0,1]$

$\varphi \in [0, 2\pi]$

$$(3-2y)\,dxdy$$

$$(3-2(r\sin\varphi+1))\,r\,dr\,d\varphi$$

$$(3-2r\sin\varphi-2)\,r\,dr\,d\varphi$$

$$\int_0^1 (1-2r\sin\varphi)\,r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^1 r\,dr\,d\varphi - 2\int_0^{2\pi}\int_0^1 r^2\sin\varphi\,dr\,d\varphi$$

$$\int_0^{2\pi} d\varphi \int_0^1 r\,dr - 2\int_0^{2\pi}\sin\varphi\,d\varphi\int_0^1 r^2\,dr$$

$$\int_0^{2\pi} d\varphi \left(\frac{r^2}{2}\right)\Big|_0^1 - 2\int_0^{2\pi}\sin\varphi\,d\varphi\left(\frac{r^3}{3}\right)\Big|_0^1$$

$$\frac{1}{2}\int_0^{2\pi} d\varphi - 2\cdot\frac{1}{3}\int_0^{2\pi}\sin\varphi\,d\varphi$$

20

$$\frac{1}{2}(\varphi)\Big|_0^{2\pi} + \frac{2}{3}(\cos\varphi)\Big|_0^{2\pi} \Rightarrow \pi + \cancel{\frac{2}{3}(\cos 2\pi - \cos 0)}$$

$= \pi$ → rješenje

⑤ $z = \underbrace{x^2 + y^2}_{r^2}$ $z = 5$

$z = r^2 \,/\sqrt{\;}$

$\sqrt{z} = r$

$\int_0^{2\pi} \int_0^{\sqrt{z}} \int_0^{5} \underline{\underline{r}} \; d\varphi \, dr \, dz$

$\varphi \in [0, 2\pi]$

$z \in [0, 5]$

$r \in [0, \sqrt{z}]$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊	Popunjava nastavnik
Ime i prezime: ANĐELO UGRINIĆ **Broj indeksa:** 55581-2008	Broj ↓ bodova
Vrijeme početka: 08 : 40 **Vrijeme završetka:**	

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y) = y - x$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_K (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z) = (2x+1,\ 3y-z,\ z-y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20

4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

5. Izračunaj volumen dijela prostora odozdo omeđenog paraboloidom $z = x^2 + y^2$, a odozgo ravninom $z = 5$. 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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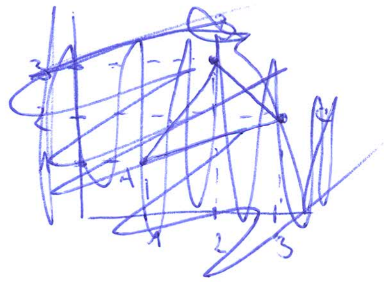
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

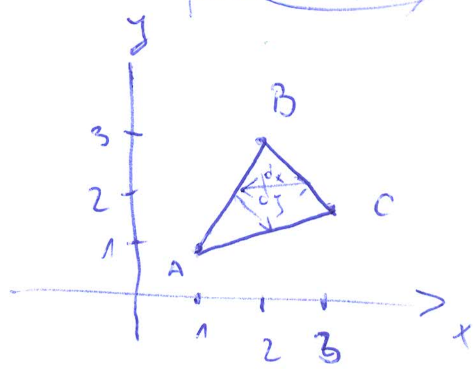
IME I PREZIME: ANĐELO UGRINIĆ   BROJ INDEKSA: 55581 - 2008

①

$\iint (y-x)\,dx\,dy = \ldots$



~~$x \in (1, 3)$~~



② $r = 1 \qquad T(2,1)$

$\iint_K \underbrace{(3-2y)}_{P}\,dx\,dy$

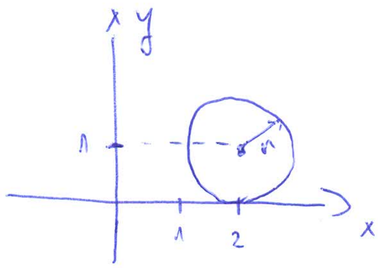
$\iint$

$Q = 0 \qquad \frac{\partial P}{\partial y} = 2$ NE

$2\iint dx\,dy =$

$2\int_0^{2\pi} d\varphi \int_0^1 r\,dr = 2\int_0^{2\pi} d\varphi \left(\frac{r^2}{2}\right)\Big|_0^1 =$

$= \cancel{2}\cdot\frac{1}{\cancel{2}}\,\varphi\Big|_0^{2\pi} = 2\pi = 6.28.$

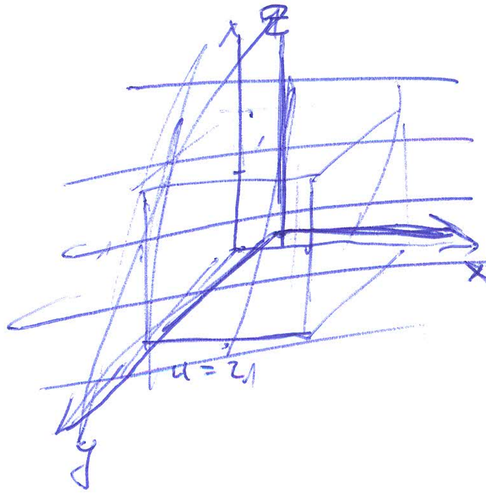


$r \in [0, 1]$

$\varphi \in [0, 2\pi]$

4) a f R

$\iint (3 - 2y)\,dx\,dy$

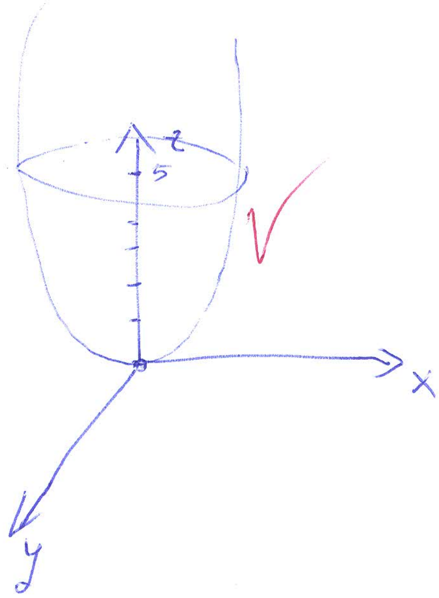


5)

$z = x^2 + y^2 \qquad z = 5$

$z = r^2 \rightarrow r^2 = 5$

$r = \sqrt{5}$



$x = r\cos\varphi$

$y = r\sin\varphi$

$r \in [0, \sqrt{5}]$

$\varphi \in [0, 2\pi]$

$$\int_0^{2\pi}\int_0^{\sqrt{5}} r^2\, r\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{5}} r^3\,dr\,d\varphi =$$

$$\int_0^{2\pi} d\varphi \int_0^{\sqrt{5}} r^3\,dr = \int_0^{2\pi} d\varphi \left(\frac{r^4}{4}\right)\Bigg|_0^{\sqrt{5}} =$$

$$= \frac{(\sqrt{5})^4}{4}\int_0^{2\pi} d\varphi = \frac{25}{4}\,\varphi \Bigg|_0^{2\pi}$$

$$= \frac{25}{4_2}\cdot \overset{1}{2}\pi = \frac{25}{2}\pi$$ [redacted]

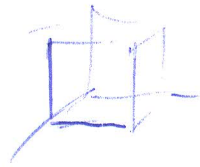
$= 39.27.$

Ime i prezime: ANĐELO UGRINIĆ Broj indeksa: 55581-2008

(3) $g(x,y,z) = (2x+1,\ 3y-2,\ z-y)$

4

$$\underset{\partial K}{\iint} \underbrace{(3-2y)}_{P} \cdot dx\,dy \qquad a = 2$$



$x \in (-1, 1)$

$y \in (-1; 1)$

$Q = 0$

$$\frac{\partial P}{\partial y} = 2$$

$$2 \underset{\partial K}{\iint} dx\,dy = 2\int_{-1}^{1} dy \int_{-1}^{1} dx = 2\int_{-1}^{1} dy \; x\Big|_{-1}^{1} = 2\int_{-1}^{1} dy \; 1+1 =$$

$$\cancel{\int_{-1}^{1}} \quad 4 \cdot y\Big|_{-1}^{1} = 4 \cdot (1+1) = 4 \cdot 2 = 8.$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: NINO MIKULANDRA **BROJ INDEKSA:** 57645	
VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**	

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y)=y-x$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. — 20
2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_K (3-2y)\,\mathrm{d}x\mathrm{d}y$. — 20
3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z)=(2x+1,\ 3y-z,\ z-y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? — 20
4. Neka je K kocka stranice duljine $a=2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. — 20
5. Izračunaj volumen dijela prostora odozdo omeđenog paraboloidom $z=x^2+y^2$, a odozgo ravninom $z=5$. — 20

Ukupno:

Tablica integrala

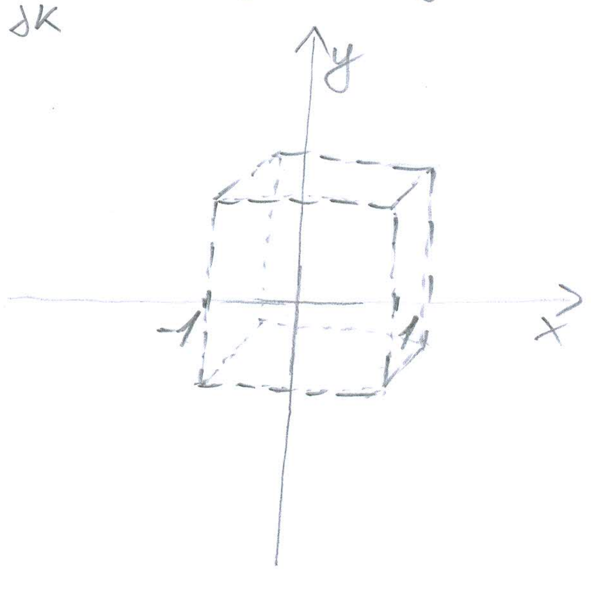
$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

4) K kocka ; $a = 2$

$\iint\limits_{\partial K} (3-2y)\, dx\, dy$

$\varphi \in [0, 2\pi]$

$z \in [-2, 2]$



$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

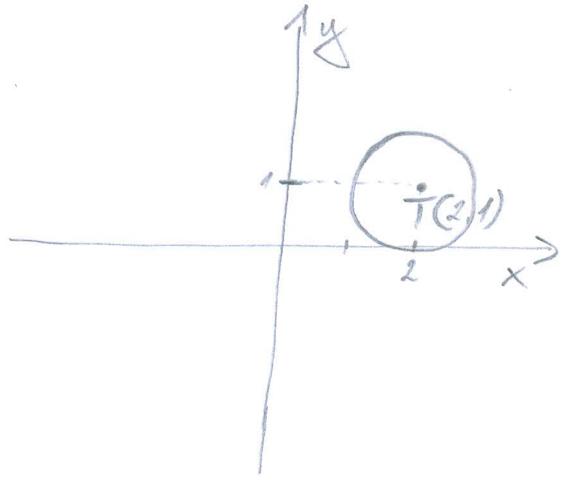
→

IME I PREZIME: NINO MIKULANDRA **BROJ INDEKSA:** 57645

2.) K krug ; $r = 1$

$T(\overset{x}{2}, \overset{y}{1})$

$$\iint_K (3-2y)\,dx\,dy$$



$x = r\cos\varphi$

$y = r\sin\varphi$

$\varphi \in [0, \pi]$

$z \in [-2, 2]$

$$\iint_K (3-2y)\,dx\,dy$$

$$= \int_0^{\pi} dx \int_{-2}^{2} (3-2y)\,dy =$$

$$= \int_0^{\pi} (3y - 2y^2)\Big|_{-2}^{2} dx =$$

$$= \int_0^{\pi} \left[(6y - 4y^2) - (-6y - 4y^2)\right] dx =$$

$$= \int_0^{\pi} \left[6y - 4y^2 + 6y + 4y^2)\right] dx =$$

$$= (12y)\Big|_0^{\pi} dx = (12\pi - 0)\,dx$$

$$= 12\pi$$

$\emptyset$

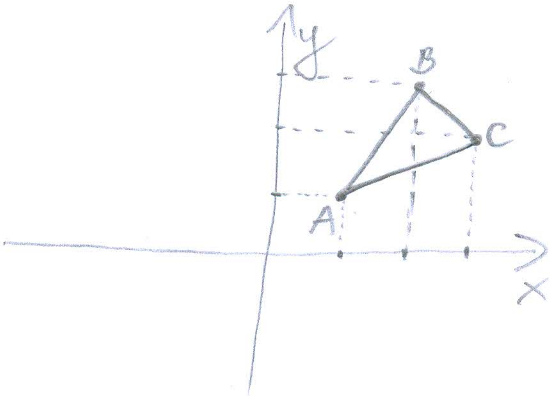
IME I PREZIME: NINO MIKULANDRA BROJ INDEKSA: 57645

1.) trokut T ; funkcija $(x,y) = y - x$

$A(1,1)$

$B(2,3)$

$C(3,2)$



$$\iint_T f(x,y)\,dx\,dy$$

$$= \int_0^1 dx \int_{2x-2}^{\frac{1}{2}x-\frac{1}{2}} (y-x)\,dy =$$

$$= \int_0^1 (y^2 - xy)\Big|_{2x-2}^{\frac{1}{2}x-\frac{1}{2}} dx =$$

$$= \int_0^1 \left[\left(\frac{1}{2}x - \frac{1}{2}\right)^2 - x \cdot \frac{1}{2}x - \frac{1}{2}\right] -$$

$$\left[y(2x-2) + x(2x-2)\right] dx =$$

$$= \int_0^1 \left[\frac{1}{4}x + \frac{1}{4} - \frac{1}{4}x^2 + \frac{1}{4}\right] -$$

$$\left[2xy + 2 - 2x^2 - 2x\right] dx$$

$$= \int_0^1 \left[\frac{1}{4}x - \frac{1}{4}x^2 + \frac{1}{2} - 4xy - 4x^3\right] dx$$

$A(\overset{x}{1}, \overset{y}{1})$

$B(2,3)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{3-1}{2-1}(x-1)$$

$$y - 1 = \frac{2}{1}(x-1)$$

$$y - 1 = 2x - 2$$

$A(\overset{x}{1}, \overset{y}{1})$

$C(3,2)$

$$y - 1 = \frac{2-1}{3-1}(x-1)$$

$$y - 1 = \frac{1}{2}x - \frac{1}{2}$$

Ime i prezime: NINO MIKULANDRA **Broj indeksa:** 57645

4)

$$\iint_{\partial K} (3-2y)\,dx\,dy$$

$$= \int_0^{2\pi} dx \int_{-2}^{2} (3-2y)\,dy =$$

$$= 2\pi \int_{-2}^{2} (3y - 2y^2)\,dx =$$

$=$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: LUKA MARĐETKO **BROJ INDEKSA:** 55821-2008

VRIJEME POČETKA: 8:35 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y) = y - x$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_K (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20 15

3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z) = (2x+1,\ 3y-z,\ z-y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20

4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20

5. Izračunaj volumen dijela prostora odozdo omeđenog paraboloidom $z = x^2 + y^2$, a odozgo ravninom $z = 5$. 20

Ukupno: 15

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$