

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME:

BROJ INDEKSA:

VRIJEME

POČETKA: **RJEŠENJE 3**

- Zadan trokut T sa vrhovima: $A(1, 1)$, $B(2, 3)$ i $C(3, 2)$ i funkcija $f(x, y) = y - x$. Odrediti $\iint_T f(x, y) dx dy$. 20
- Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2, 1)$. Izračunati $\iint_K (3 - 2y) dx dy$. 20
- Provjeriti da li je krivoljni integral u vektorskom polju $g(x, y, z) = (2x + 1, 3y - z, z - y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
- Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (3 - 2y) dx dy$. 20
- Izračunaj volumen dijela prostora odozdo omeđenog paraboloidom $z = x^2 + y^2$, a odozgo ravninom $z = 5$. 20

Tablica integrala

Ukupno:

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x \sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

①

$AB: (y-1) \cdot 2 = (x-1) \cdot 1$
 $y = 2x - 1$

$AC: (y-1) \cdot 2 = (x-1) \cdot 1$
 $y = \frac{x}{2} + \frac{1}{2}$

$BC: y = 5 - x$

$\iint_T (y-x) dx dy = \int_{1/2}^{22x-1} \int_{x/2+1/2}^{5-x} (y-x) dx dy = \dots = 0$

②

$\iint_K (3-2y) dx dy = \int_0^{2\pi} \int_0^1 (3-2(1+r \sin \varphi)) r dr d\varphi = \int_0^{2\pi} (3-2-2r \sin \varphi) r dr d\varphi = \int_0^{2\pi} (r - 2r \sin \varphi) dr d\varphi = \pi - 0 = \pi$

③ KRIVULJNI INTEGRAL JE NEOVISAN O PUTU AKO JE VEKTORSKA FUNKCIJA POTENCIJALNO POLJE. TRAJEŠI f SKALARNU FUNKCIJU TAKO DA

$$\partial_x f = 2x + 1 \Rightarrow f(x, y, z) = x^2 + x + \text{const}(y, z) \Rightarrow \partial_y f = \partial_y \text{const}(y, z)$$

$$\partial_y f = 3y - z$$

$$\Rightarrow \text{const}(y, z) = \frac{3}{2}y^2 - zy + C(z) \Rightarrow \partial_z f = -y + C'(z)$$

$$\partial_z f = z - y$$

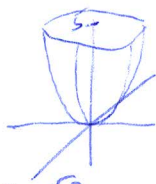
$$\Rightarrow C(z) = \frac{z^2}{2}$$

POSTOJI TRAJEŠNA FUNKCIJA PA SVAKI KRIVULJNI INTEGRAL FUNKCIJE g OVISI SAMO O POČETNOJ I ZAVRŠNOJ TOČKI!

$$f(x, y, z) = x^2 + x + \frac{3}{2}y^2 - zy + \frac{z^2}{2}$$

④ $w = \begin{bmatrix} 0 \\ 0 \\ 3-2y \end{bmatrix}$ $\text{div } w = 0$ PO TEOREMU O DIVERGENCIJI $\iint_{\partial K} (3-2y) dx dy = \iiint_K 0 dx dy dz = 0$

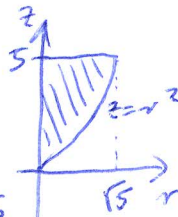
⑤ $z = x^2 + y^2$
 $z = 5$



$$z \in [0, 5]$$

$$\varphi \in [0, 2\pi]$$

$$r \in [0, \sqrt{z}]$$



$$V = \int_0^{2\pi} \int_0^5 \int_0^{\sqrt{z}} r dr dz d\varphi = 2\pi \int_0^5 \frac{z}{2} dz = \frac{25\pi}{2}$$

