

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

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BROJ INDEKSA: *17-2-0085-2011*

VRIJEME POČETKA: *08:45*

VRIJEME ZAVRŠETKA:

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

27

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. 20

2. Riješi sustav Gaussovom metodom: 20

$$\begin{array}{rccccrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 & = & 0 \\ & & x_2 & - & x_3 & + & x_4 & = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 & = & 7 \\ & & - & 7x_2 & + & 3x_3 & + & x_4 & = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = (\sqrt{x^2 - 5x + 1} - x)$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{9-x^2}$. 20(graf)

5. Odrediti domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x)$. 2+5+4+9

Ukupno:

1.

$$\frac{|z|}{2(z+i)} = 3i$$

$$|z| = \sqrt{x^2 + y^2}$$

$$\frac{\sqrt{x^2 + y^2}}{2z + 2i} = 3i$$

$$\sqrt{x^2 + y^2} = 3i \cdot (2z + 2i)$$

$$\sqrt{x^2 + y^2} = 6zi + 6i^2$$

$$\sqrt{x^2 + y^2} = 6zi - 6$$

$$x^2 + y^2 = (6zi)^2 - 2 \cdot 6zi \cdot 6 + 6^2$$

$$x^2 + y^2 = 36z^2 i^2 - 72zi + 36$$

$$x^2 + y^2 = -36z^2 - 72zi + 36$$

$$x^2 + y^2 = -36 \cdot \frac{1}{2} - 72 \cdot (x+yi)i + 36$$

limp ...

$$3) g(x) = (\sqrt{x^2 - 5x + 1} - x)$$

$$1) x^2 - 5x + 1 \geq 0$$

$$x^2 - 5x + 1 = 0$$

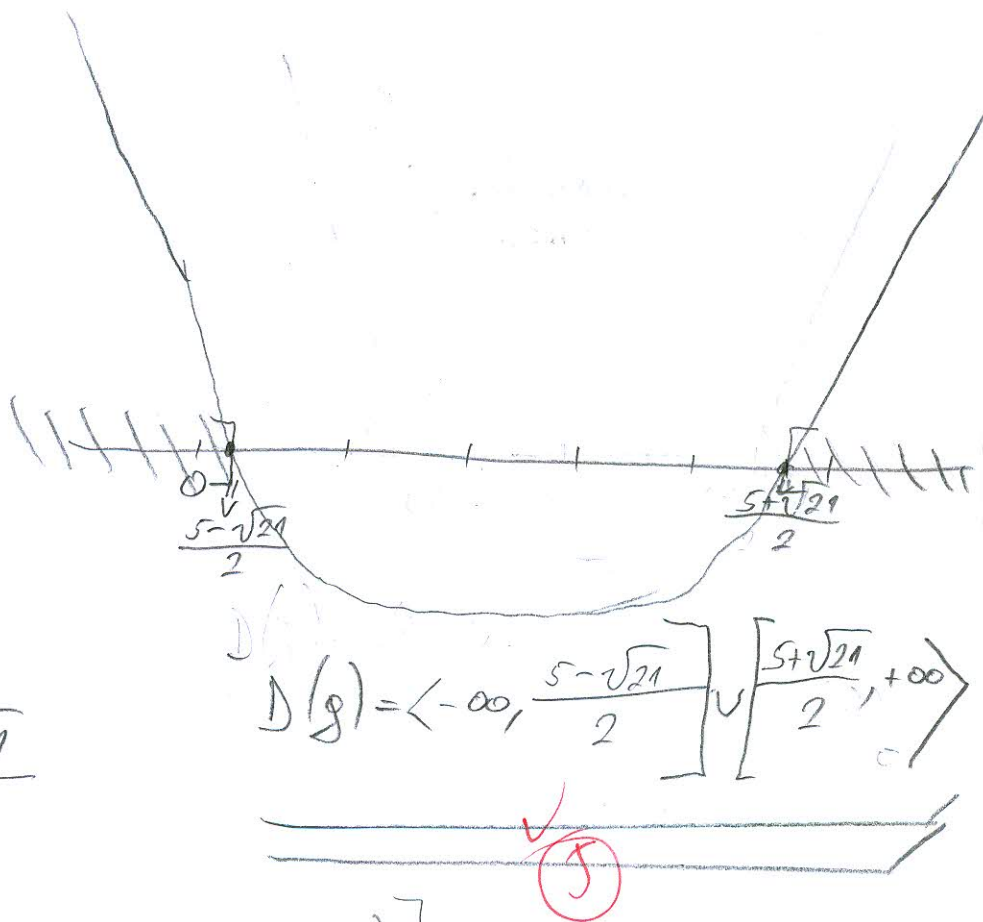
$$a=1, b=-5, c=1$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot 1}}{2}$$

$$x_{1,2} = \frac{5 \pm \sqrt{21}}{2}$$

$$x_1 = \frac{5 + \sqrt{21}}{2} \quad x_2 = \frac{5 - \sqrt{21}}{2}$$



ASIMPTOTE

$$1) \lim_{x \rightarrow \frac{5 + \sqrt{21}}{2}} \left[\sqrt{\frac{5 + \sqrt{21}}{2} - 5 \cdot \left(\frac{5 + \sqrt{21}}{2}\right) + 1} - \left(\frac{5 + \sqrt{21}}{2}\right) \right]$$

$$\lim_{x \rightarrow \frac{5 + \sqrt{21}}{2}} = \text{nekli broj}$$

Nema vertikalne asimptote

5.

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$$h(x) = \cos(4x)$$

① $D(h) = \mathbb{R}$ ✓ (jer možemo izračunati cos od bilo kojeg realnog broja)

② Funkcija je periodična, jer je trigonometrijska. ✓

③ ~~$h(-x) = h(x)$
 $h(-x) = \cos(-4x) \Rightarrow$ funkcija nije parna
 $-h(x) = -[\cos(4x)]$
 $= -\cos(4x) \Rightarrow$ funkcija nije neparna~~

~~$$h'(h) = (\cos(4x))'$$~~

~~$$h'(h) = -\sin(4x)$$~~

~~$$h''(h) = [-\sin(4x)]''$$~~

~~$$h''(h) =$$~~

③ $h'(x) = \cos' \cdot (4x)'$

$$h'(x) = (\cos x)' \cdot (4x) + (\cos x) \cdot (4x)'$$

$$h'(x) = -\sin x \cdot (4x) + (\cos x) \cdot 1$$

$$h'(x) = -4x \sin x + \cos x$$

$$h''(x) = (-4x \sin x)' + (\cos x)'$$

$$h''(x) = [(-4)' \cdot \sin x + (-4) \cdot (\sin x)'] + (-\sin x)$$

$$h''(x) = 4 \cos x - \sin x$$

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2.)

$$\begin{bmatrix} 1 & -2 & 3 & -4 & | & 0 \\ 0 & 1 & -1 & 1 & | & 1 \\ 1 & 3 & 0 & -3 & | & 7 \\ 0 & -7 & 3 & 1 & | & -15 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 & -4 & | & 0 \\ 0 & 1 & -1 & 1 & | & 1 \\ 0 & 5 & -3 & 1 & | & 7 \\ 0 & -7 & 3 & 1 & | & -15 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & -2 & | & 2 \\ 0 & 1 & -1 & 1 & | & 1 \\ 0 & 0 & 2 & -4 & | & 2 \\ 0 & 0 & -4 & 8 & | & -8 \end{bmatrix}$$

$$1R - (-1) + 3R$$

$$2R \cdot 2 + 1R$$

$$2R \cdot (-5) + 3R$$

$$2R \cdot 7 + 3R$$

$$3R \leftrightarrow 4R$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -2 & | & 2 \\ 0 & 1 & -1 & 1 & | & 1 \\ 0 & 0 & 1 & -2 & | & 1 \\ 0 & 0 & -4 & 8 & | & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & -1 & | & 2 \\ 0 & 0 & 1 & -2 & | & 1 \\ 0 & 0 & 0 & 0 & | & -4 \end{bmatrix}$$

$$3R \cdot (-1) + 1R$$

$$3R \cdot 1 + 2R$$

$$3R \cdot 4 + 4R$$

Sustav je nemoguć.

~~$$\begin{bmatrix} 1 & 0 & 1 & -2 & | & 2 \\ 0 & 1 & -1 & 1 & | & 1 \\ 0 & 0 & -4 & 8 & | & -8 \\ 0 & 0 & 2 & -4 & | & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & -2 & | & 2 \\ 0 & 1 & -1 & 1 & | & 1 \\ 0 & 0 & 1 & -2 & | & 2 \\ 0 & 0 & 2 & -4 & | & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & -1 & | & 3 \\ 0 & 0 & 1 & -2 & | & 2 \\ 0 & 0 & 0 & 0 & | & -2 \end{bmatrix}$$~~

~~$$3R \cdot (-4)$$~~

~~$$2R \cdot (-1) + 1R$$~~

~~$$3R \cdot 1 + 2R$$~~

~~$$3R \cdot (-2) + 4R$$~~

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4.) $f(x) = \frac{5-x}{9-x^2}$

1) DOMENA

$$9-x^2 \neq 0$$

$$-x^2 \neq -9$$

$$x^2 \neq 9$$

$$x \neq \pm 3$$

$$D(f) = \mathbb{R} \setminus \{-3, 3\}$$

2) NULTOČKE

$$5-x=0$$

$$-x=-5$$

$$x=5$$

$$N(5, 0)$$

3) PERIODIČNOST

Funkcija $f(x) = \frac{5-x}{9-x^2}$ nije periodična jer nije trigonometrijska.

4) PARNOST, NEPARNOST

$$f(-x) = f(x)$$

$$f(-x) = \frac{5+x}{9-x^2}$$

$$-f(x) = -\frac{5-x}{9-x^2}$$

\Rightarrow Funkcija $f(x) = \frac{5-x}{9-x^2}$ nije parna funkcija.

\Rightarrow Funkcija $f(x) = \frac{5-x}{9-x^2}$ nije neparna funkcija.

5) ASIMPTOTE

$$1) \lim_{x \rightarrow 1} \frac{5-x}{9-x^2} = \lim_{x \rightarrow 1} \frac{5-1}{9-1} = \frac{4^0}{8^0} = \frac{1}{2}$$

$x=1$ } Vertikalne
 $x=-1$ } asimptote.

$$\lim_{x \rightarrow (-1)} \frac{5-x}{9-x^2} = \lim_{x \rightarrow (-1)} \frac{5+1}{9+1} = \frac{6^0}{8^0} = \frac{3}{4}$$

$$2) \lim_{x \rightarrow \pm\infty} \frac{5-x}{9-x^2} \begin{array}{l} \cdot : x^2 \\ \cdot : x^2 \end{array} = \lim_{x \rightarrow \pm\infty} \frac{\frac{5}{x^2} - \frac{x}{x^2}}{\frac{9}{x^2} - \frac{x^2}{x^2}} = \lim_{x \rightarrow \pm\infty} \frac{\frac{5}{x^2} - \frac{1}{x}}{\frac{9}{x^2} - 1}$$

$$= \lim_{x \rightarrow \pm\infty} \frac{0}{-1}$$

$$= \lim_{x \rightarrow \pm\infty} 0 = 0$$

$y=0$ } Horizontalna asimptota.

S obzirom da funkcija $f(x) = \frac{5-x}{9-x^2}$ ima horizontalnu asimptotu $y=0$, ta funkcija stoga nema kosu asimptotu.

