MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓

IME I PREZIME: Ivan Rubelj BROJ INDEKSA: 17-2-0085-2011

VRIJEME POČETKA: 08:45 VRIJEME ZAVRŠETKA:

POPUNJAVA NASTAVNIK
Broj ↓ bodova
27

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. — 20

2. Riješi sustav Gaussovom metodom: — 20

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = (\sqrt{x^2 - 5x + 1} - x)$. — 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{9-x^2}$. — 20(graf)

5. Odrediti domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x)$. — 2+5+4+9

Ukupno:

1.)

$$\frac{|z|}{2(z+i)} = 3i \qquad |z| = \sqrt{x^2+y^2}$$

$$\frac{\sqrt{x^2+y^2}}{2z+2i} = 3i$$

$$\sqrt{x^2+y^2} = 3i \cdot (2z+2i)$$

$$\sqrt{x^2+y^2} = 6zi + 6i^2$$

$$\sqrt{x^2+y^2} = 6zi - 6 \quad /^2$$

$$x^2+y^2 = (6zi)^2 - 2 \cdot 6zi \cdot 6 + 6^2$$

$$x^2+y^2 = 36z^2 i^2 - 72zi + 36$$

$$x^2+y^2 = -36z^2 - 72zi + 36$$

$$x^2+y^2 = -36 \cdot z^2 - 72 \cdot (x+yi)i + 36$$

3) $g(x)=\left(\sqrt{x^2-5x+1}-x\right)$

1) $x^2-5x+1 \geq 0$

$x^2-5x+1=0$

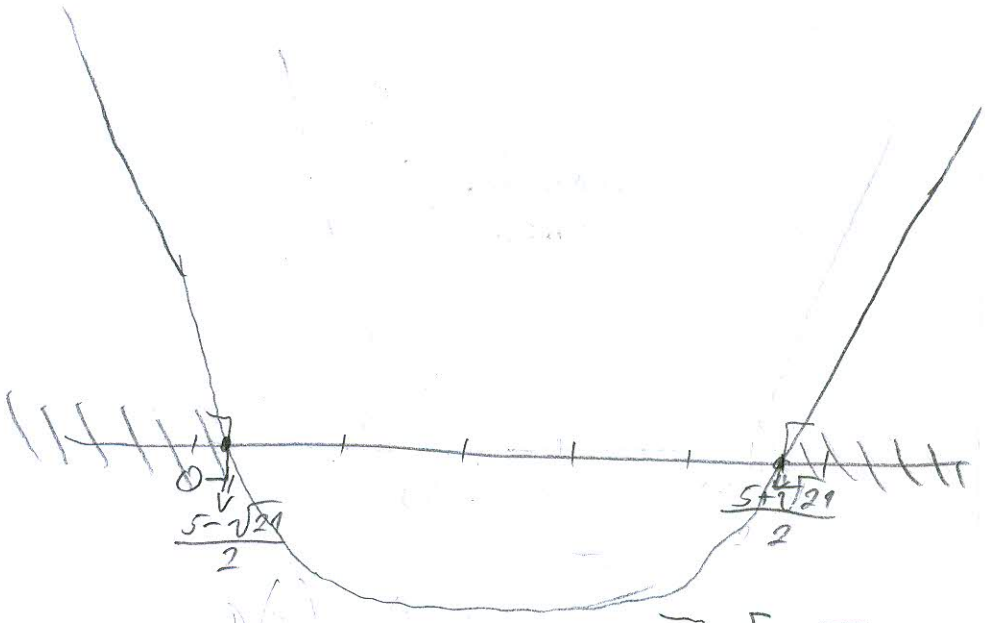
$a=1,\ b=-5,\ c=1$

$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x_{1,2}=\frac{5\pm\sqrt{25-4\cdot 1\cdot 1}}{2}$

$x_{1,2}=\frac{5\pm\sqrt{21}}{2}$

$x_1=\frac{5+\sqrt{21}}{2}\qquad x_2=\frac{5-\sqrt{21}}{2}$



$D(g)=\left\langle -\infty, \frac{5-\sqrt{21}}{2}\right] \cup \left[\frac{5+\sqrt{21}}{2}, +\infty\right\rangle$

5

ASIMPTOTE

1) $\lim_{x\to\frac{5+\sqrt{21}}{2}}\left[\sqrt{\frac{5+\sqrt{21}}{2}-5\cdot\left(\frac{5+\sqrt{2}}{2}\right)+1}-\left(\frac{5+\sqrt{2}}{2}\right)\right]$

$\lim_{x\to\frac{5+\sqrt{21}}{2}}$ = neki broj

Nema vertikalne asimptote

(5.) IME I PREZIME: Ivan Rubelj BROJ INDEKSA: 17-2-0085-2011

$h(x) = \cos(4x)$.

① $D(h) = \mathbb{R}$ ✓ (jer možemo izračunati cos od bilo kojeg realnog broja) (2)

② Funkcija je periodična, jer je trigonometrijska. ✓

③ $h(-x) = h(x)$

$h(-x) = \cos(-4x)$ => funkcija nije parna

$-h(x) = -[\cos(4x)]$

$= -\cos(4x)$ => funkcija nije neparna

$h'(h) = (\cos(4x))'$

$h'(h) = -\sin(4x)$

$h''(h) = [-\sin(4x)]''$

$h'(h) =$

---

(3) $h'(x) = \cos' \cdot (4x)'$

$h'(x) = (\cos x)' \cdot (4x) + (\cos x) \cdot (4x)'$

$h'(x) = -\sin x \cdot (4x) + (\cos x) \cdot 4$

$h'(x) = -4x \sin x + \cos x$

$h''(x) = (-4x \sin x)' + (\cos x)'$

$h''(x) = [(-4)' \cdot \sin x + (-4) \cdot (\sin x)'] + (-\sin x)$

$h''(x) = -4\cos x - \sin x$

2.)

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right]$$

$1R \cdot (-1) + 3R$

$2R \cdot 2 + 1R$
$2R \cdot (-5) + 3R$
$2R \cdot 7 + 3R$

$3R \leftrightarrow 4R$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -4 \end{array}\right]$$

$3R \cdot (-1) + 1R$
$3R \cdot 1 + 2R$
$3R \cdot 4 + 4R$

Sustav je nemoguć.

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -4 & 8 & -8 \\ 0 & 0 & 2 & -4 & 2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 2 & -4 & 2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 & -2 \end{array}\right]$$

$3R : (-4)$

$2R \cdot (-1) + 1R$
$3R \cdot 1 + 2R$
$3R \cdot (-2) + 4R$

IME I PREZIME: Ivan Kubelj

BROJ INDEKSA: 17-2-0085-2011

4. $f(x) = \frac{5-x}{9-x^2}$

① DOMENA

$9 - x^2 \neq 0$

$-x^2 \neq -9$

$x^2 \neq 9$

$x \neq \pm 3$

$D(f) = \mathbb{R} \setminus \{-3, 3\}$

② NULTOČKE

$5 - x = 0$

$-x = -5$

$x = 5$

$N(5, 0)$

③ PERIODIČNOST

Funkcija $f(x) = \frac{5-x}{9-x^2}$ nije periodična jer nije trigonometrijska.

④ PARNOST, NEPARNOST

$f(-x) = fx$

$f(-x) = \frac{5+x}{9-x^2} \Rightarrow$ Funkcija $f(x) = \frac{5-x}{9-x^2}$ nije parna funkcija.

$-f(x) = -\frac{5+x}{9+x^2} \Rightarrow$ Funkcija $f(x) = \frac{5-x}{9-x^2}$ nije neparna funkcija.

⑤ ASIMPTOTE

1) $\lim_{x \to 3} \frac{5-x}{9-x^2} = \lim_{x \to 3} \frac{5-3}{9-3} = \frac{2}{0} = \frac{1}{2}$

$\lim_{x \to (-3)} \frac{5-x}{9-x^2} = \lim_{x \to (-3)} \frac{5+3}{9+3} = \frac{8}{24} = \frac{3}{4}$

$x = 3$, $x = -3$ } Vertikalne asimptote.

2) $\lim_{x \to \pm\infty} \frac{5-x}{9-x^2} \Big/ :x^2 = \lim_{x \to \infty} \frac{\frac{5}{x^2} - \frac{x}{x^2}}{\frac{9}{x^2} - \frac{x^2}{x^2}} \Rightarrow 0$

$= \lim_{x \to \pm\infty} \frac{0}{-1}$

$= \lim_{x \to \pm\infty} = 0$

$y = 0$ } Horizontalna asimptota.

S obzirom da funkcija $f(x) = \frac{5-x}{9-x^2}$ ima horizontalnu asimptotu $y = 0$, ta funkcija stoga nema kosu asimptotu.

(6.) EKSTREMI

$$f(x)=\frac{5-x}{9-x^2}$$

$$f'(x)=\frac{(5-x)'\cdot(9-x^2)-(5-x)\cdot(9-x^2)'}{(9-x^2)^2}$$

$$f'(x)=\frac{-1(9-x^2)-(5-x)\cdot(-2x)}{(9-x^2)^2}$$

$$f'(x)=\frac{-9+x^2-(-10x+2x^2)}{(9-x^2)^2}$$

$$f'(x)=\frac{-9+x^2+10x-2x^2}{(9-x^2)^2}$$

$$f'(x)=\frac{-x^2+10x-9}{(9-x)^2}$$

$$-x^2+10x-9=0$$

$a=1,\ b=10,\ c=-9$

$$x_{1,2}=\frac{-10\pm\sqrt{100-4\cdot(-1)\cdot(-9)}}{-2}$$

$$x_{1,2}=\frac{-10\pm\sqrt{64}}{-2}$$

$$x_1=\frac{-10+8}{-2}\qquad x_2=\frac{-10-8}{-2}$$

$x_1=1$ $\quad$ $x_2=9$

$$f(x)=\frac{5-x}{9-x^2}$$

$$f(x)=\frac{5-1}{9-1}=\frac{4}{8}=\frac{1}{2}$$

$E_1\left(1,\frac{1}{2}\right)$

$E_2\left(9,\frac{1}{18}\right)$

$$f(x)=\frac{5-x}{9-x^2}$$

$$f(x)=\frac{5-9}{9-9^2}$$

$$f(x)=\frac{-4}{-72}$$

$$f(x)=\frac{4}{72\ 18}$$

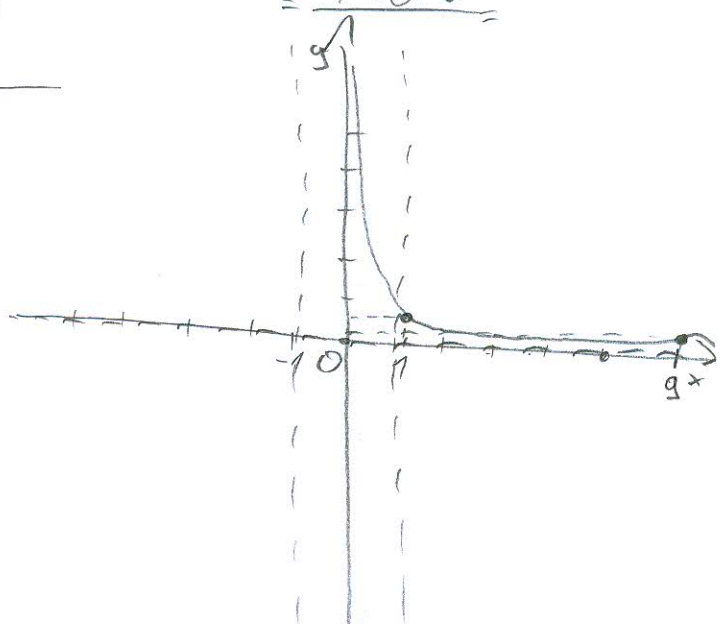
$$f(x)=\frac{1}{18}$$

(7.) EKSTREMNI RAST I PAD

	$-\infty$		-1		1		9		$+\infty$
$f'(x)$		$-$		$-$		$+$		$-$	
$f(x)$		↘		↘		↗		↘	

$$f'(x)=\frac{-x^2+10x-9}{(9-x)^2}$$

$$=\frac{-4-20-9}{}=$$



IME I PREZIME: Ivan Rubelj

BROJ INDEKSA: 17-2-0085-2011

$$\frac{|z|}{2(z+i)} = 3;$$

$$\frac{\sqrt{x^2+y^2}}{2\cdot(z+i)} = 3; \quad / \cdot 2(z+i)$$

$$\sqrt{x^2+y^2} = 3i\cdot(2z+2i)$$

$$\sqrt{x^2+y^2} = 6zi - 6 \quad /^2$$

$$x^2+y^2 = (6zi)^2 - 2\cdot 6zi\cdot 6 + 6^2$$

$$x^2+y^2 = -36z^2 - 72zi + 36$$

$$x^2+y^2 = -36z^2 - 72\cdot(x+yi)\cdot i + 36$$

$$x^2+y^2 = -36z^2 - (72x - 72yi)\cdot i + 36$$

$$x^2+y^2 = -36z^2 - 72xi + 72yi^2 + 36$$

$$x^2+y^2 = -36z^2 - 72xi - 72yi + 36$$

$$\therefore z = \pm 6$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓

IME I PREZIME: Ivan Colić **BROJ INDEKSA:** 17-2-0152-2011

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova
40

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. 20

2. Riješi sustav Gaussovom metodom: 20

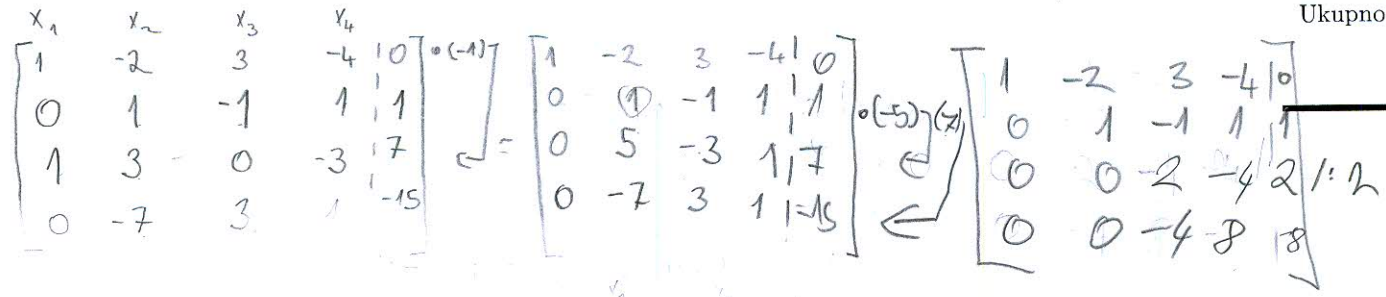
$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

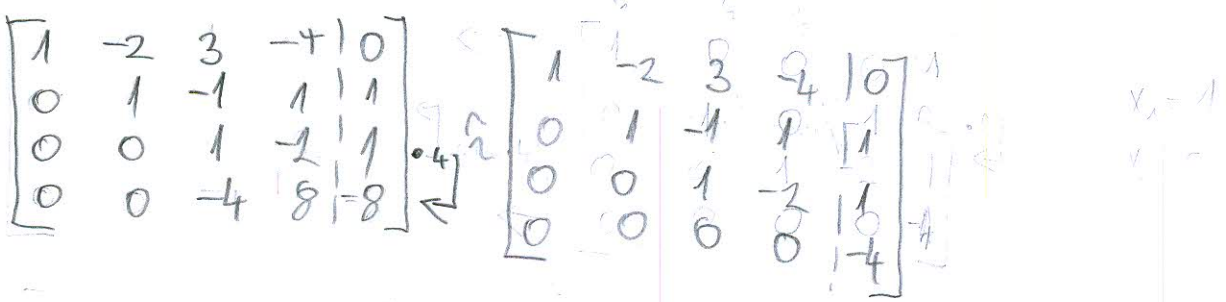
3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{x^2 - 5x + 1} - x\right)$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{9-x^2}$. 20(graf)

5. Odrediti domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x)$. 2+5+4+9

Ukupno:





20

Ro Sustav je nemoguć, nema rješenja

$\sqrt{x^2-5x+1} - x)$

$x_1 = \frac{5-\sqrt{21}}{2}$

$x_2 = \frac{5+\sqrt{21}}{2} = 4{,}8$

$x^2 - 5x + 1 \geq 0$

$x_{1,2} = \frac{-b \mp \sqrt{b^2-4ac}}{2a}$

$= \frac{5 \mp \sqrt{25-4 \cdot 1 \cdot 1}}{2}$

$= \frac{5 \mp \sqrt{21}}{2}$

$Df = \left\langle -\infty, \frac{5-\sqrt{21}}{2} \right] \cup \left[\frac{5+\sqrt{21}}{2}, +\infty \right\rangle$

5

$\lim_{x \to \frac{5-\sqrt{21}}{2}^-} (\sqrt{x^2-5x+1} - x) = +\infty$ //

$\lim_{x \to \frac{5-\sqrt{21}}{2}} (\sqrt{x^2-5x+1} - x) = \emptyset$

$\lim_{x \to \frac{5-\sqrt{21}}{2}^+} (\sqrt{x^2-5x+1} - x) = \emptyset$

$\lim_{x \to \frac{5+\sqrt{21}}{2}^-} (\sqrt{x^2-5x+1} - x) = -\infty$ //

Pravaci $x = \frac{5-\sqrt{21}}{2}$ i $x = \frac{5+\sqrt{21}}{2}$ su asimtote funkcije

horizontalna

$\lim_{x \to \infty} (\sqrt{x^2-5x+1} - x) /:x = \lim_{x \to \infty} \sqrt{\frac{x^2}{x^2} + \frac{5x}{x^2} + 1} - \frac{x}{x^2}$

$= \sqrt{1} - 0 = 1$

Vertikalna asimptota nema

$y = 1$

Kose asimptote nema

IME I PREZIME: Ivna Colić BROJ INDEKSA: 17-2-0152-2011

$g(x) = \frac{5-x}{5-x^2}$

$5 - x^2 \neq 0$

$-x^2 \neq -5$

$x^2 \neq 5$

$x_{1,2} = \frac{\mp\sqrt{-4 \cdot 5 \cdot 1}}{2}$

$x_{1,2} = \frac{\sqrt{36}}{2}$

$x_1 = -5$

$x_2 = 3$

$D(f) = R \setminus \{-3, 3\}$ ✓

Parnost

$f(-x) = \frac{5+x}{5-x}$

Nije ni parna ni neparna

Nul točka

Na x osi

$0 = \frac{5-x}{5-x^2} \mid 5-x^2$

$0 = 5 - x$

$x = 5$ ✓

y. os

$y = \frac{5-0}{5-0^2}$

$y = \frac{5}{5}$

H. A.

$\lim_{x \to -5} \frac{5-x}{5-x^2} = -\infty$

$\lim_{x \to -3^+} \frac{5-x}{5-x^2} = +\infty$

$\lim_{x \to 3^-} \frac{5-x}{5-x^2} = +\infty$

$\lim_{x \to 3^+} \frac{5-x}{5-x^2} = -\infty$

V. A.

$\lim_{x \to \infty} \frac{5-x}{5-x^2} \cdot \frac{x^2}{x^2} = \frac{0-0}{0-1} = \infty$

V. A. Nema

K. A.

$y = kx + l$

$k \Rightarrow \lim_{x \to \infty} \frac{\frac{5-x}{5-x^2}}{x} = \lim_{x \to \infty} \frac{(5-x)x}{5-x^2} = \lim_{x \to \infty} \frac{5x - x^2 / :x^2}{5 - x^2 / :x^2}$

$= \lim_{x \to \infty} \frac{\frac{5}{x^2} - \frac{x^2}{x^2}}{\frac{5}{x^2} - \frac{x^2}{x^2}} = \frac{-1}{-1} = 1$ $\quad k = 1$

$l = \lim_{x \to \infty} \left(\frac{5-x}{5-x^2} - x\right) = \lim_{x \to \infty} \frac{(5-x) - x(5-x^2)}{5-x^2}$

$\lim_{x \to \infty} \frac{5 - x - 5x + x^3}{5-x^2} = \lim_{x \to \infty} \frac{5 - 10x + x^3 / :x^3}{5 - x^2 / :x^3} = \frac{0 - 0 + 1}{0 - 0} = \frac{1}{0} = +\infty$

Kose asimtote nema

5

$h(x) = \cos(4x)$

$D(h) = \langle -\infty, +\infty \rangle$ ✓ 2

$$g'(x) = \frac{(5-x)'(9-x^2) - (5-x)\cdot(9-x^2)'}{(9-x^2)^2} = \frac{9-x^2 - [(5-x)2x]}{(9-x^2)^2} = \frac{9-x^2-(10x-2x^2)}{(9-x^2)^2}$$

$$= \frac{9-x^2-10x+2x^2}{(9-x^2)^2} = \frac{x^2-10x+9}{(9-x^2)^2}$$

$$\frac{x^2-10x+9}{(9-x^2)^2} = 0$$

$$x^2-10x+9=0$$

$$x_{1,2} = \frac{10 \mp \sqrt{100-4\cdot 1\cdot 9}}{2}$$

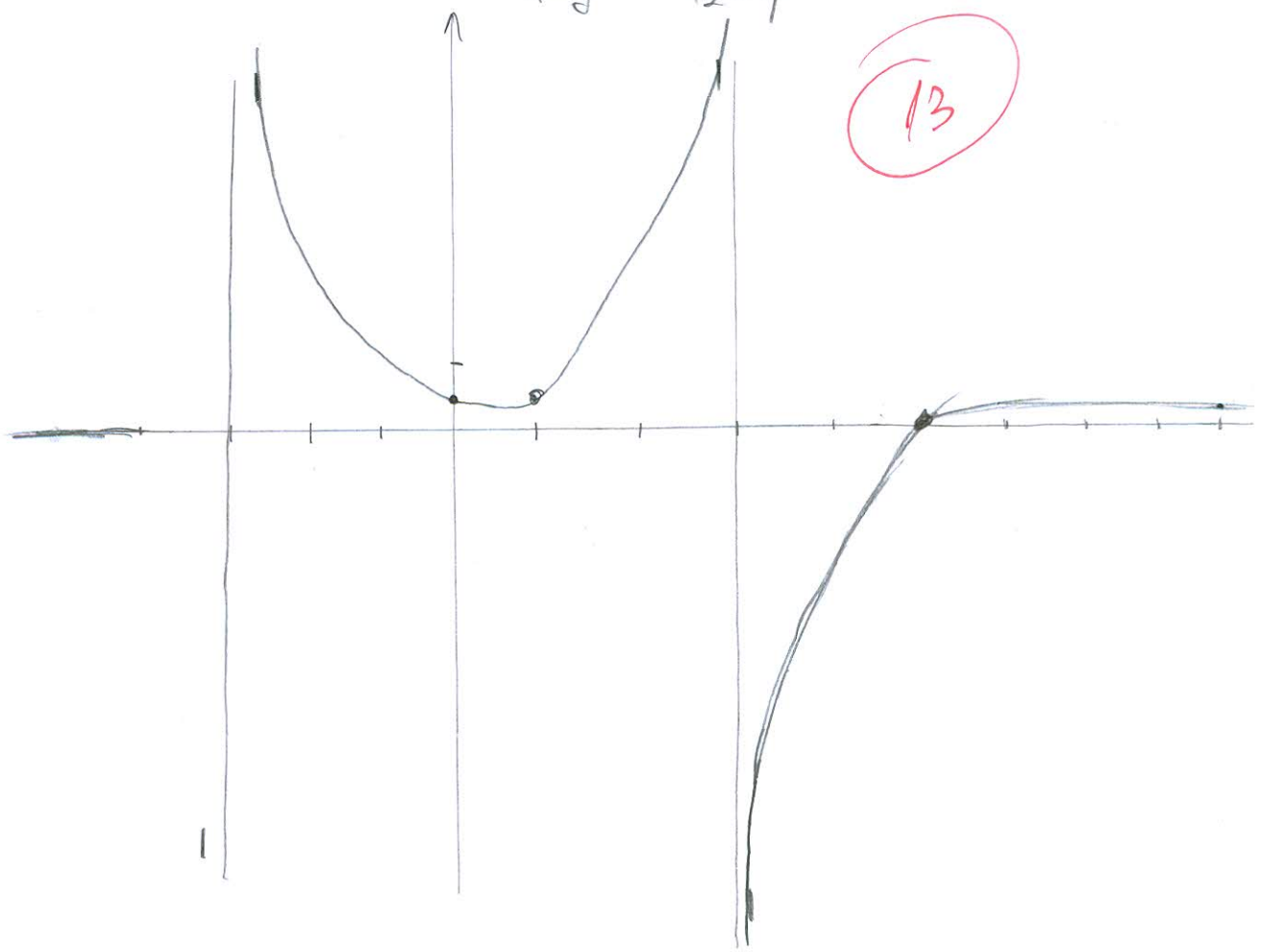
$$x_{1,2} = \frac{10 \mp 8}{2}$$

$x_1 = 9 \qquad x_2 = 1$

$$f(9) = \frac{5-9}{9-9^2} = 0{,}05 = \frac{1}{18}$$

$$f(1) = \frac{5-1}{5-1^2} = \frac{4}{8} = \frac{1}{2}$$

13



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: ANDRO KLARIN **BROJ INDEKSA:** 17-02-0089-2011

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova
2

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. — 20

2. Riješi sustav Gaussovom metodom: — 20

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

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4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{9-x^2}$. — 20(graf)

5. Odrediti domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x)$. — 2+5+4+9

Ukupno:

2.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ -1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \quad | :3$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 1/3 & 1 & 0 & -1 & 7/3 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right]$$

(3) $g(x) = \left| \sqrt{x^2 - 5x + 1} - x \right|$

V.A. → NEMA

$x^2 - 5x + 1 \geq 0$

$a = 1 > 0$

$x^2 - 5x + 1 = 0$

$x_{1,2} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 1}}{2}$

$x_1 = \frac{5 - \sqrt{25 - 4}}{2}$

$x_1 = \frac{5 - \sqrt{21}}{2} = 0{,}21$

$x_2 = \frac{5 + \sqrt{21}}{2}$

$x_2 = 4{,}93$

$D_f = x \in \langle -\infty, 0{,}21] \cup [4{,}93, +\infty \rangle$

H.A.

$\lim_{x \to \pm\infty} \left| \sqrt{x^2 - 5x + 1} - x \right| \left| : x \right. = \lim_{x \to \pm\infty} \frac{\sqrt{\frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{1}{x^2}} - 1}{\frac{x}{x}}$

$\lim_{x \to \pm\infty} \sqrt{0}$

$y = 0$ H.A.

$y = kx + l$

$\lim_{x \to \infty} \frac{\sqrt{x^2 - 5x + 1} - x}{x} \left| : x \right.$

$\lim_{x \to \infty} \frac{\sqrt{\frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{1}{x^2}} - \frac{x}{x}}{1}$

$\lim_{x \to \infty} \frac{\sqrt{1} - 1}{1} = \frac{0}{1} = 0$

IME I PREZIME: ANDRO KLARIN BROJ INDEKSA: 17-02-0083-2011

4.

$f(x) = \frac{5-x}{3-x^2}$

$3 - x^2 \neq 0$

$-x^2 \neq -3 \mid \cdot(-1)$

$x^2 = 3 \mid \sqrt{\ }$ $D = \mathbb{R} \setminus \{-3, 3\}$

$x = \pm 3$

$5 - x = 0$

$-x = -5 \mid \cdot(-1)$

$x = 5$

$(5, 0)$

$f(0) = \frac{5-0}{3-0^2}$

$f(0) = \frac{5}{3}$

$(0, \frac{5}{3})$

ASIMPTOTE

V.A.

$\lim_{x \to -3} \frac{5-x}{3-x^2} = \frac{5-(-3)}{3-(-3)^2} = \frac{5+3}{0} = \frac{8}{0} = \infty$ $x = -3$ V.A.

$x = 3$

$\lim_{x \to 3} \frac{5-x}{3-x^2} = \frac{5-3}{3-3^2} = \frac{2}{0} = \infty$

H.A.

$\lim_{x \to \pm\infty} \frac{5-x \mid \cdot x^2}{3-x^2 \mid \cdot x^2} = \lim_{x \to -\infty} \frac{\frac{5}{x^2} - \frac{x}{x^2}}{\frac{3}{x^2} - \frac{x^2}{x^2}} = \frac{2}{-1} = 0$ H.A. $y = 0$

K.A. NEMA

$$f(x) = \frac{5-x}{3-x^2}$$

$$f'(x) = \frac{-1(3-x^2) - (5-x)\cdot(-2x)}{[(3-x^2)]^2}$$

$$f'(x) = \frac{-3+x^2-(-10x+2x^2)}{[(3-x)^2]^2}$$

$$D(f) = \frac{-3+x^2+10x-2x^2}{[3-x^2]^2}$$

$$f'(x) = \frac{-x^2+10x-3}{(3-x)^4}$$

$$-x^2+10x-3=0$$

$$x_{1,2} = \frac{-10 \pm \sqrt{100-4\cdot(-1)\cdot(-3)}}{-2}$$

$$x_1 = 1$$

$$x_2 = 3$$

	$-\infty$		-3		1		3		9		$+\infty$
$f'(x)$		$-$		$-$		$+$		$+$		$+$	
$f(x)$		↘		↘		↗		↗		↗	

$$m$$

$$m\left(1, \frac{1}{2}\right)$$

$$f(-x) = f(x)$$

$$f(-x) = \frac{5-(-x)}{3-(-x)^2} = \frac{5+x}{3-x^2}$$

$$\frac{5-1}{3-1^2} = \frac{4}{2}$$

IME I PREZIME: ANDRO KLARIN BROJ INDEKSA: 19-02-0089-2011

5.

$h(x) = \cos(4x)$

$D = R$ ✓ (2)

$h(x) = \cos(4x)$

$h'(x) = -\sin 4x \cdot 4$

$h'(x) = -4 \sin 4x$

$f(-x) = f(x)$

$f(-x) = \cos(-4x)$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓

IME I PREZIME: IVAN BABIĆ **BROJ INDEKSA:** 17-2-0135-2014

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova
20

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. — 20
2. Riješi sustav Gaussovom metodom: — 20

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = (\sqrt{x^2 - 5x + 1} - x)$. — 5+15
4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{9-x^2}$. — 20(graf)
5. Odrediti domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x)$. — 2+5+4+9

Ukupno:

2.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{array}{l} \\ \\ \text{I}\cdot(-1)+\text{III} \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{array}{l} \text{II}\cdot 2+\text{I} \\ \\ \text{II}\cdot(-5)+\text{III} \\ \text{II}\cdot 7+\text{IV} \end{array} \sim$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \begin{array}{l} \\ \\ :2 \\ :4 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 2 & -2 \end{array}\right] \begin{array}{l} \\ \\ \\ \text{III}+\text{IV} \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

(22) ✓

SUSTAV NEMA RJEŠENJA!

4. $f(x) = \frac{5-x}{9-x^2}$

$D_f \mathbb{R} \setminus \{3\}$

$9 - x^2 \neq 0$

$x \neq 3$

NUL TOČKE

$f(x) = \frac{5-x}{9-x^2}$

$5 - x = 0$

$-x = -5$

$x = 5$

$f(5) = \frac{5-5}{9-5^2}$

$f(5) = \frac{0}{9-25}$

$f(5) = \frac{0}{-16}$

$f(5) = 0$

$(5, 0)$

STACIONARNE TOČKE, RAST I PAD

$f'(x) = \frac{(5-x)'(9-x^2) - (5-x)(9-x^2)'}{(9-x^2)^2}$

$f'(x) = \frac{-1(9-x^2) - (5-x)(-2x)}{(9-x^2)^2}$

$f'(x) = \frac{(-9+x^2) - (-10x+2x^2)}{(9-x^2)^2}$

$f'(x) = \frac{-9+x^2+10x-2x^2}{(9-x^2)^2}$

$f'(x) = \frac{-9+10x-x^2}{(9-x^2)^2}$

$f'(x) = \frac{-9+10x-x^2}{81-18x^2+x^4}$

$f'(x) = \frac{-1+10x-x^2}{9-18x^2+x^4}$

$-1 + 10x - x^2 = 0$

$10x - x^2 = 1$

$x(10 - x) = 1$

$x_1 = 1$ $\quad 10 - x = 1$

$x_2 = 9$

	$\langle -\infty, 1\rangle$	$\langle 1, 3\rangle$	$\langle 3, 9\rangle$	$\langle 9, \infty\rangle$
f'	− ↘	− ↘	+ ↗	− ↘

$f'(0) = -\frac{1}{9}$

$f'(2) = -\frac{15}{42}$

$f'(5) = \frac{3}{23}$

$f'(10) = -\frac{1}{87}$

INFLEKCIJSKA, KONKAVNOST I KONVEKSNOST

$f''(x) \quad \frac{10-2x(9-18x^2+x^4) - (-1+10x-x^2)(-36x+4x^3)}{(9-18x^2+x^4)^2}$

	$\langle -\infty, 0\rangle$	$\langle 0, \infty\rangle$

5. $h(x) = \cos(4x)$

$h'(x) = -\sin(4x) \cdot x$

$h''(x) = -\cos(4 \cdot) \cdot 1$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓

IME I PREZIME: KRISTINA TISSAUER **BROJ INDEKSA:**

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova
48

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. — 20

2. Riješi sustav Gaussovom metodom: — 20

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{x^2 - 5x + 1} - x\right)$. — 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{9-x^2}$. — 20(graf)

5. Odrediti domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x)$. — 2+5+4+9

Ukupno:

---

(4) $f(x) = \dfrac{5-x}{9-x^2}$

(1) DOMENA

$9 - x^2 \neq 0$

$-x^2 + 9 \neq 0$

$-x^2 = -9 \;/\cdot(-)$

$x^2 = 9 \;/\sqrt{\ }$

$x_1 = 3$

$x_2 = -3$

$\forall x \in \mathbb{R} \setminus \{3, -3\}$

(2) PERIODIČNOST

Nije periodična.

(3) PARNOST I NEPARNOST

$f(-x) = \dfrac{5-(-x)}{9-(-x)^2} = \dfrac{5+x}{9-x}$ Funkcija nije parna ni neparna.

(4) NUL TOČKE

$5 - x = 0$

$-x = -5 \;/\cdot(-)$

$x = 5$

$N(5, 0)$

⑤ ASIMPTOTE

- **VERTIKALNA ASIMPTOTA**

$$\lim_{x\to 3} f(x)\ \frac{5-x}{9-x^2} = \lim_{x\to 3}\ \frac{5-3}{9-(3)^2} = \frac{2}{0} = \infty \ //$$

$$\lim_{x\to -3} f(x)\ \frac{5-x}{9-x^2} = \lim_{x\to -3}\ \frac{5-(-3)}{9-(-3)^2} = \frac{5+3}{9-9} = \frac{8}{0} = \infty \ //$$

$$\boxed{x_1 = 3} \quad \boxed{x_2 = -3}$$

- **HORIZONTACNA ASIMPTOTA**

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{5-x \mid :x^2}{9-x^2 \mid :x^2} = \frac{\frac{5}{x^2} - \frac{x}{x^2}}{\frac{9}{x^2} - 1} = \frac{0}{1} = 0$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{5-(-x)}{9-(-x^2)} = \frac{5+x \mid :x^2}{9-x^2 \mid :x^2} = \frac{\frac{5}{x^2} + \frac{x}{x^2}}{\frac{9}{x^2} - \frac{x^2}{x^2}} = \frac{0}{1} = 0$$

$$y = 0$$

Ako ima horizontalne nema kose.

⑥ STACIONARNE TOČKE

$$f(x) = \frac{5-x}{9-x^2}$$

$$f'(x) = \frac{(5-x)'\cdot(9-x^2) - (5-x)\cdot(9-x^2)'}{(9-x^2)^2}$$

$$f'(x) = \frac{(9-x^2) - (5-x)\cdot(-2x)}{(9-x^2)^2}$$

$$f'(x) = \frac{(9-x^2) - (-10x+2x^2)}{(9-x^2)^2}$$

$$f'(x) = \frac{9-x^2+10x+2x^2}{(9-x^2)^2} = \frac{x^2+10x+9}{(9-x^2)^2}$$

IME I PREZIME: KRISTINA TISSAUER BROJ INDEKSA:

③ $g(x) = (\sqrt{x^2-5x+1} - x)$

1) DOMENA

$x^2-5x+1 \geq 0$

$x_{1/2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \geq 0$

$x_{1/2} = \frac{5 \pm \sqrt{25-4}}{2}$

$x_{1/2} = \frac{5 \pm \sqrt{21}}{2}$

$x_1 = 4.8$

$x_2 = 0,2$

$D(x) \in R \setminus \{0,2, 4.8\}$

② ASIMPTOTE

VERTIKALNA ASIMPTOTA

$\lim_{x \to 0.2} g(x) = \sqrt{0.2^2 - 5 \cdot 0.2 + 0.2} = \sqrt{0.8}$

Nema V.A.

horizontalna

$\lim_{x \to \infty} f(x) = \sqrt{x^2-5x+1} \,|:x^2 = \frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{1}{x^2} = 1$

$\lim_{x \to -\infty} f(x) = \sqrt{-x^2+5x+1} : x^2 = -\frac{x^2}{x^2} + \frac{5x}{x^2} + \frac{1}{x^2} = -1$

$y_1 = 1 \quad | \quad y_2 = -1$

Nema kose!

② $x_1 - 2x_2 + 3x_3 - 4x_4 = 0$

$x_2 - x_3 + x_4 = 1$

$x_1 + 3x_2 - 3x_4 = 7$

$-7x_2 + 3x_3 + x_4 = -15$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{matrix} \\ \\ \\ \downarrow \cdot(-1) \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & -2 & 3 & -4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{matrix} \uparrow + \\ \cdot(7) \\ \sim \end{matrix}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \begin{matrix} \\ \\ \\ \cdot(7) \\ \downarrow \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 5 & -3 & 1 & 7 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \begin{matrix} \\ \\ \\ \cdot(-5) \\ \downarrow + \end{matrix}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & -4 & 8 & -8 \end{array}\right] \begin{matrix} \\ \\ \\ \cdot 2 \\ \downarrow + \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -2 & 2 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & -4 & 2 \\ 0 & 0 & 0 & 0 & -4 \end{array}\right] \checkmark$$

(2b)

Sustav nema rješenja!

IME I PREZIME: KRISTINA TISSAUER BROJ INDEKSA:

(1) $\frac{|z|}{2(z+i)} = 3i$ odredi sve z.

$z = x + y \cdot i$

$\frac{|x+yi|}{2(x+yi+i)} = 3i$

$\frac{\sqrt{x+yi}}{2(x+yi+i)} = \frac{3i}{i}$

$\sqrt{x^2+y^2} = \frac{3i}{2(x+yi+i)}$

IME I PREZIME: KRISTINA TISSAUER

BROJ INDEKSA:

nastavak (4.)

[redacted]

(6) nastavak

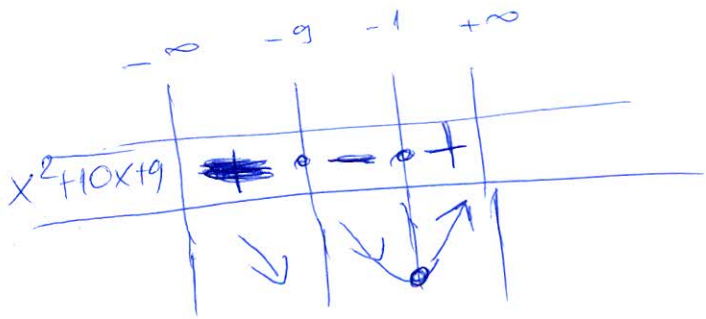
$x^2+10x+9=0$

$x_{1/2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1/2} = \frac{-10 \pm \sqrt{100-36}}{2}$

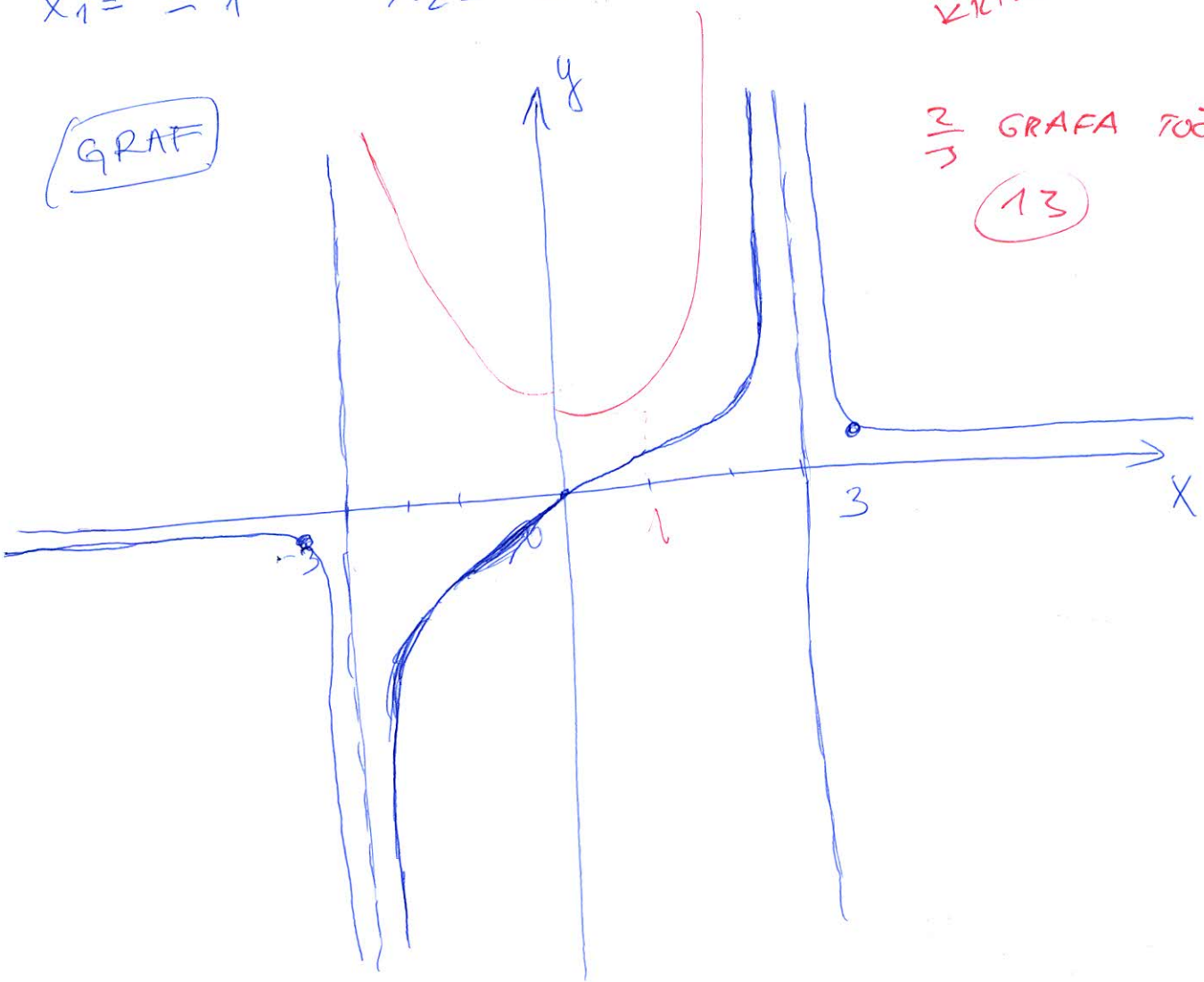
$x_{1/2} = \frac{-10 \pm \sqrt{64}}{2}$

$x_1 = -1 \qquad x_2 = -9$



min (-1, 0)

GRAF



JEDINA KRIVA SLIKA

3 GRAFA TOČNO

(13)

(5) $h(x) = \cos(4x)$

(1) DOMENA

$x \in \mathbb{R}$ ✓ (2)

(2) PERIODIČNOST

15

(3) PARNOST / (NE)PARNOST

$f(x) = \cos(+4x) = \cos(-4x) = \cos(4x)$ Funkcija je (4) parna

(4) 1 DERIVACIJA

$h(x) = \cos(4x)$

$h'(x) = -\sin(4x) \cdot (4x)'$

$h'(x) = -\sin(4x) \cdot 4$

$h'(x) = -4\sin(4x)$ //

(3)

$h''(x) = -4\sin(4x)$

$h''(x) = -4\cos(4x) \cdot (4x)'$

$h''(x) = -4\cos 4x \cdot 4$

$h''(x) = -16\cos 4x$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓

IME I PREZIME: TINO BRAJKOVIĆ **BROJ INDEKSA:** 17-2-0100-2011

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

0

1. Odrediti kompleksne brojeve z koji zadovoljava jednadžbu $\frac{|z|}{2(z+i)} = 3i$. 20

2. Riješi sustav Gaussovom metodom: 20

$$\begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & - & 4x_4 = & 0 \\ & & x_2 & - & x_3 & + & x_4 = & 1 \\ x_1 & + & 3x_2 & & & - & 3x_4 = & 7 \\ & - & 7x_2 & + & 3x_3 & + & x_4 = & -15 \end{array}$$

3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{x^2 - 5x + 1} - x\right)$. 5+15

4. Ispitati tok i nacrtati graf funkcije: $f(x) = \frac{5-x}{9-x^2}$. 20(graf)

5. Odrediti domenu, periodičnost, (ne)parnost i drugu derivaciju funkcije: $h(x) = \cos(4x)$. 2+5+4+9

Ukupno:

4. $f(x) = \frac{5-x}{9-x^2}$

Domena
$x^2 - 9 \neq 0$ $D_f = \mathbb{R} \setminus \{\pm 3\}$
$x^2 \neq 9$
$x = \sqrt{9}$
$x = \pm 3$

Asimptote

V.A. $x = 3$ $x = -3$

$\lim_{x \to 3} \frac{5-x}{9-x^2} = \lim_{x \to 3} \frac{5-3}{9-9} = \frac{2}{0} = \infty$

$\lim_{x \to -3} \frac{5-x}{9-x^2} = \lim_{x \to -3} \frac{5+3}{9-9} = \frac{8}{0} = \infty$

H.A.

$\lim_{x \to \infty} \frac{5-x}{9-x^2} \,/ : x^2 = \lim_{x \to \infty} \frac{\frac{5}{x^2} - \frac{x}{x^2}}{\frac{9}{x^2} - \frac{x^2}{x^2}} = \frac{0}{1} = 0$

sjecište s osima

s osi y $x = 0$

$f(0) = \frac{5-0}{9-0} = \frac{5}{9}$ $S\left(0, \frac{5}{9}\right)$

s osi x

$5 - x = 0$ $N(5, 0)$
$-x = -5$
$x = 5$

Parnost

$f(x) = \frac{5-x}{9-x^2} = \frac{5-(-x)}{9-(-x)^2} = \frac{5+x}{9-x^2} \neq f(x)$ nije parna

$f(-x) = \frac{-(5-x)}{-(9+x^2)} \neq f(x)$ nije neparna

Derivacija

$f(x) = \frac{5-x}{9-x^2}$

$f'(x) = \frac{-1(9-x^2) - (5-x)\cdot 2x}{(9-x^2)^2} = \frac{-9+x^2-10x+2x^2}{(9-x^2)^2} = \frac{-9+3x^2-10x}{(9-x^2)^2}$

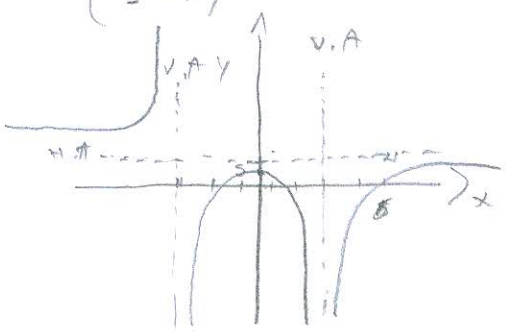
$-9+3x^2-10x = 0$

$x_{1,2} = \frac{10 \pm \sqrt{100-108}}{9}$

$y_{1,2} = \frac{10 \pm 2.82}{9} =$

$x_1 = 1.42$

$x_2 = 0.71$



5. $h(x) = \cos(4x)$

Domena

$4x \neq 0$

$x = -4$

$D = \mathbb{R} \setminus \{4\}$

Parnost $h(x) = \cos(4x)$

$h(-x) = \cos(-4x) \neq h(x)$ → nije parna

$= -(\cos(4x)) = -h(x)$ → neparna

Derivacija

$h(x) = \cos(4x)$

$h'(x) = (\cos)' \cdot 4x - \cos \cdot (4x)'$

$= \frac{1}{\sin x} \cdot 4x - \cos x$

$= \frac{4x}{\sin x} - \cos x$

$h''(x) = \frac{4 \cdot \sin x - 4x \cdot (-\cos x)}{\sin^2 x} + \sin x$

$= \frac{4\sin x + 4\cos x^2}{\sin^2 x} + \sin x$

IME I PREZIME: TINO BRAJKOVIĆ BROJ INDEKSA: 17-2-0100-2011

3. $g(x) = (\sqrt{x^2-5x+1} - x)$

Domena

$x^2 - 5x + 1 \geq 0$

$x_{1,2} = \frac{5 \pm \sqrt{25-4}}{2}$

$x_{1,2} = \frac{5 \pm \sqrt{21}}{2}$

Nula

2.

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 1 & 3 & 0 & -3 & 7 \\ 0 & -7 & 3 & 1 & -15 \end{array}\right] \left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & -7 & 3 & 1 & -15 \\ 1 & 3 & 0 & -3 & 7 \end{array}\right] \cdot 7 \left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -4 & 8 & -8 \\ 1 & 3 & 0 & -3 & 7 \end{array}\right] \cdot (-1)$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & -4 & 8 & -8 \\ 0 & 5 & -3 & -7 & 7 \end{array}\right] \cdot \left(-\frac{1}{4}\right) \left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 5 & -3 & -7 & 7 \end{array}\right] \cdot (-5) \left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 2 & -12 & 2 \end{array}\right] \cdot (-2)$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & -16 & -2 \end{array}\right] \cdot \left(-\frac{1}{16}\right) \left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 1 & \frac{1}{8} \end{array}\right] \cdot 2 \left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{8} \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & 0 & 1 & \frac{13}{4} \\ 0 & 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{8} \end{array}\right] \cdot (-1) \left[\begin{array}{cccc|c} 1 & -2 & 3 & 4 & 0 \\ 0 & 1 & 0 & 0 & \frac{25}{8} \\ 0 & 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{8} \end{array}\right] \cdot 2 \left[\begin{array}{cccc|c} 1 & 0 & 3 & 4 & \frac{50}{8} \\ 0 & 1 & 0 & 0 & \frac{25}{8} \\ 0 & 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{8} \end{array}\right] \cdot (-3)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & \frac{25}{8} \\ 0 & 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{8} \end{array}\right] \cdot (-4) \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & \frac{25}{8} \\ 0 & 0 & 1 & 0 & \frac{9}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{8} \end{array}\right] \quad \left[\begin{array}{c} 0 \\ 1 \\ 7 \\ -15 \end{array}\right] = \left[\begin{array}{c} -1 \\ \frac{25}{8} \\ \frac{9}{4} \\ \frac{1}{8} \end{array}\right]$$

1. $\frac{|z|}{2(z+i)} = 3i$

$\frac{x^2+y^2}{2(x+yi+i)} = 3i = \frac{x^2+y^2}{2x+2yi+2i} = 3i$