OBAVEZNO POPUNITI VRIJEME RJEŠAVANJA ISPITA: OD DO .

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MATEMATIKA 3: KOLOKVIJ 1: Trajanje 120 minuta. Zabranjen je razgovor sa drugim studentima. Na klupama je dozvoljen samo pisaći pribor, kalkulator, indeks ili iksica i prazni papiri koji nose ime studenta. Sav ostali pribor, formule, uređaji, bilješke i nepotpisane prazne papire zabranjeno je koristiti i trebaju ostati u torbi ili pohranjeni kod nastavnika (elektronički uređaji trebaju biti isključeni) tokom cijelog trajanja ispita. Studenti koji primijete zabranjene predmete dužni su ih prijaviti nastavniku. Nije dozvoljeno međusobno posuđivanje pribora tijekom trajanja ispita. Povreda ovih pravila može za posljedicu imati udaljavanje s ispita. ZADATKE RIJEŠAVATE JEDNOSTRANO NA PREDLOŠKU KOJI MOŽETE DOBITI OD NASTAVNIKA. oooo

65

Broj ↓ bodova

Dana su vam 4 zadatka u prvom kolokviju, a biti će zadana još 4 u drugom kolokviju. Svaki zadatak nosi 25 bodova. Slika i točno postavljanje integrala u odgovarajućim koordinatama nosi 20 bodova, a uspješno integriranje još 5. Ukupno trebate sakupiti najmanje 100 bodova u dva kolokvija, od čega u prvom kolokviju potpuno točno treba biti riješen najmanje jedan zadatak, a isto tako i u drugom kolokviju.

1. Krug radijusa $r = 2$ sa središtem u točki $T(2,2)$ označen je sa $K(T,2)$. Izračunati $\displaystyle\iint_{K(T,2)} x\,\mathrm{d}x\mathrm{d}y$ 25
2. Odrediti volumen područja koji je omeđen plohama $z = y^2$, $x = 10$, $z = -\frac{x}{3}$ i $z = 9$. 25
3. Odrediti volumen područja koje odgovara nejednadžbama $x^2 + y^2 + z^2 \leq 9$ (kugla) i $x^2 + y^2 \geq z^2$ (stožac). ~~0~~
4. Zadano je područje X u koordinatnom sustavu na slici ispod (pravokutnik dimenzija 10x15, a iznad polukrug radijusa 5). Zadana je sila $f(x,y) = 5 + \frac{y}{5}$. Izračunati $\displaystyle\iint_{X} f(x,y)\,\mathrm{d}x\mathrm{d}y$ 15

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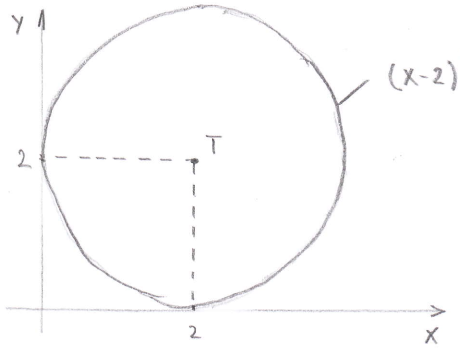
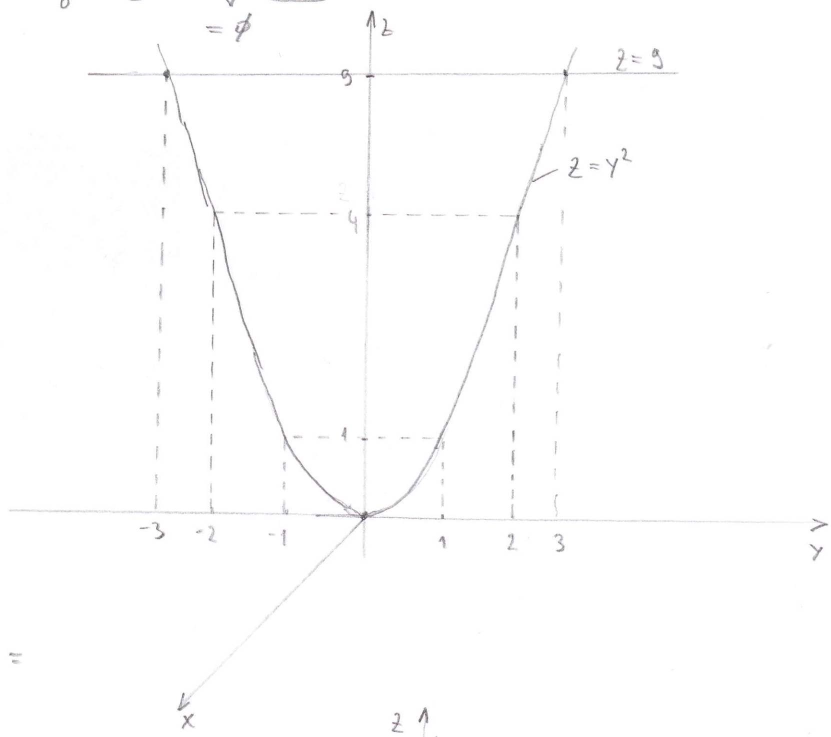


$x = r\cos\varphi + 2$

$y = r\sin\varphi + 2$

$$\int_0^{2\pi}\int_0^{2} (r\cos\varphi+2)\,r\,dr\,d\varphi = \int_0^{2\pi}\int_0^{2} (r^2\cos\varphi+2r)\,dr\,d\varphi = \int_0^{2\pi}\left(\cos\varphi\cdot\frac{r^3}{3}\Big|_0^2 + 2\frac{r^2}{2}\Big|_0^2\right)d\varphi =$$

$$= \int_0^{2\pi}\left(\cos\varphi\cdot\frac{8}{3} + 4\right)d\varphi = \frac{8}{3}\sin\varphi\Big|_0^{2\pi} + 4\varphi\Big|_0^{2\pi} = \underbrace{\frac{8}{3}(\sin 2\pi - \sin\phi)}_{=\phi} + 4(2\pi - 0) = 8\pi$$



② $z = y^2$, $x = 10$, $z = -\frac{x}{3}$, $z = 9$

$y^2 = 9$

$y = \pm\sqrt{9} = \pm 3$

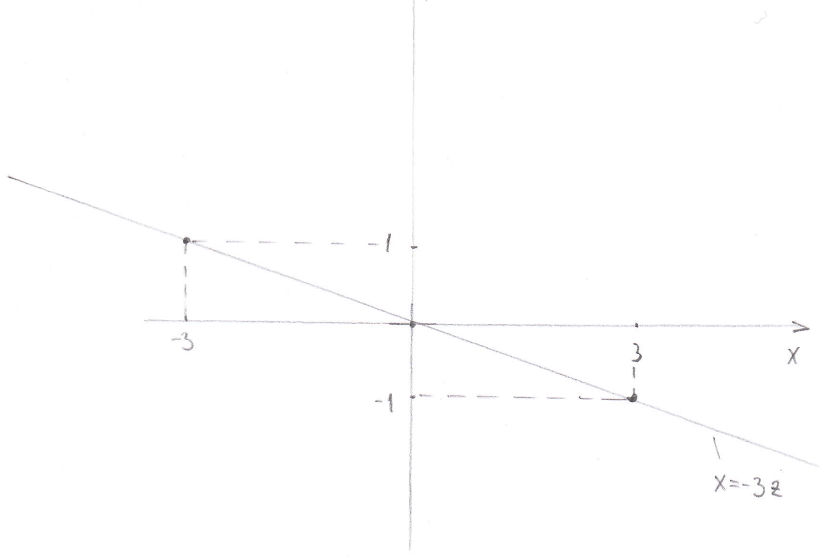
$$V = \int_{-3}^{3}\int_{y^2}^{9}\int_{-3z}^{10} dx\,dz\,dy =$$

$$= \int_{-3}^{3}\int_{y^2}^{9} (10+3z)\,dz\,dy = \int_{-3}^{3}\left(10z\Big|_{y^2}^{9} + 3\frac{z^2}{2}\Big|_{y^2}^{9}\right)dy =$$

$$= \int_{-3}^{3}\left[10(9-y^2) + 3\cdot\left(\frac{81}{2} - \frac{y^4}{2}\right)\right]dy =$$

$$= \int_{-3}^{3}\left(90 - 10y^2 + \frac{243}{2} - \frac{3}{2}y^4\right)dy =$$

$$= \int_{-3}^{3}\left(\frac{423}{2} - 10y^2 - \frac{3}{2}y^4\right)dy =$$



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(2) NASTAVAK:

$$V = \frac{423}{2} y \Big|_{-3}^{3} - 10 \frac{y^3}{3} \Big|_{-3}^{3} - \frac{3}{2} \frac{y^5}{5} \Big|_{-3}^{3} = \frac{423}{\cancel{2}} \overset{3}{(3+3)} - 10\left(\frac{27}{3} + \frac{27}{3}\right) - \frac{3}{2}\left(\frac{243}{5} + \frac{243}{5}\right) =$$

$$= 1269 - 180 - \frac{729}{5} = 1089 - \frac{729}{5}$$

✓

25

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(3) $x^2+y^2+z^2 \leq 9 \ \dots$ (1)

$x^2+y^2 \geq z^2 \ \dots$ (2) $\Rightarrow r^2 = z^2$

$r = \pm z$

$z = \pm r$

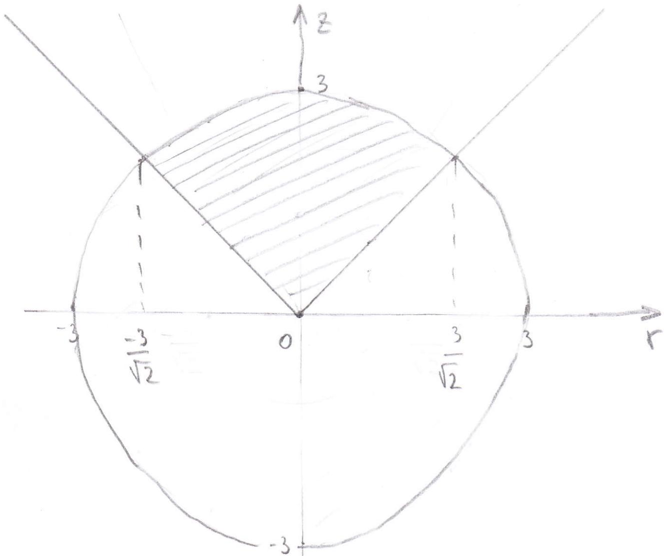
$x = r\cos\varphi$

$y = r\sin\varphi$

$$\underbrace{r^2\cos^2\varphi + r^2\sin^2\varphi}_{=r^2} + z^2 = 9$$

$r^2 + z^2 = 9$

$z = \pm\sqrt{9-r^2}$



iz (1): $z^2 = 9 - x^2 - y^2$

$9 - x^2 - y^2 = x^2 + y^2$

$2x^2 + 2y^2 = 9$

$x^2 + y^2 = \frac{9}{2}$

$r^2 = \frac{9}{2} \Rightarrow r = \pm\frac{3}{\sqrt{2}}$

NAZNAČENO POGREŠNO PODRUČJE NA SLICI
ZA OVO ŠTO JE NAZNAČENO INTEGRACIJA
JE TREBALA OBUHVAĆATI: $\varphi \in [0, 2\pi]$
$r \in [0, \frac{3}{\sqrt{2}}]$
$z \in [r, \sqrt{9-r^2}]$

$$V = \int_0^{2\pi}\int_0^{\frac{3}{\sqrt{2}}}\int_0^{\sqrt{9-r^2}} r \, dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\frac{3}{\sqrt{2}}} r\sqrt{9-r^2}\, dr\,d\varphi = \left\{\begin{array}{l} t = 9-r^2 \\ dt = -2r\,dr \\ dr = \frac{-dt}{2r} \end{array}\right\} = \int_0^{2\pi}\int_0^{\frac{3}{\sqrt{2}}} \cancel{r}\cdot\sqrt{t}\,\frac{-dt}{2\cancel{r}} =$$

X

$$= \int_0^{2\pi} \frac{-1}{2}\int_0^{\frac{3}{\sqrt{2}}} \sqrt{t}\,dt = \int_0^{2\pi} \frac{-1}{\cancel{2}}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{\cancel{2}}}\Bigg|_0^{\frac{3}{\sqrt{2}}} = \int_0^{2\pi} -\frac{1}{3}(9-r^2)\sqrt{9-r^2}\,\Big|_0^{\frac{3}{\sqrt{2}}} d\varphi = \int\left[\left(\frac{-1}{3}\left(9-\frac{9}{2}\right)\sqrt{9-\frac{9}{2}}\right) - \left(\frac{-1}{3}(9)\sqrt{9}\right)\right] d\varphi$$

$$= \int_0^{2\pi}\left(\frac{-3}{2}\cdot\frac{3}{\sqrt{2}} + 9\right)d\varphi = 9\varphi\Big|_0^{2\pi} - \frac{9}{2}\sqrt{2}\,\varphi\Big|_0^{2\pi} = 9(2\pi - 0) - \frac{9\sqrt{2}}{\cancel{2}}(\cancel{2}\pi - 0) = 18\pi - 9\sqrt{2}\,\pi$$

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(4)

$$\int_{-5}^{5}\int_{0}^{15}\left(5+\frac{y}{5}\right)dx\,dy+\int_{0}^{2\pi}\int_{0}^{5}\left(5+\frac{r\sin\varphi+15}{5}\right)r\,dr\,d\varphi=$$

$$=\int_{-5}^{5}\left(5(15-0)+\frac{1}{5}\frac{225}{2}\right)dx+\int_{0}^{2\pi}\int_{0}^{5}\left(5r+\frac{1}{5}r^2\sin\varphi+3r\right)dr\,d\varphi=$$

$$=\int_{-5}^{5}\left(75+\frac{45}{2}\right)dx+\int_{0}^{2\pi}\left(8\frac{r^2}{2}\Big|_0^5+\frac{1}{5}\frac{r^3}{3}\Big|_0^5\sin\varphi\right)d\varphi=$$

$$=\frac{195}{2}x\Big|_{-5}^{5}+\int_{0}^{2\pi}\left(100+\frac{25}{3}\sin\varphi\right)d\varphi=\frac{195}{2}(5+5)+100\cdot 2\pi+\frac{25}{3}\left[-\cos\varphi\right]_0^{2\pi}=$$

$$=975+200\pi=$$

ZA OVO BI DOBILI 10 BODOVA

ZA DRUGI DIO BI DOBILI 20 BODOVA

OBZIROM DA NISTE NAZNAČILI KOJE RJEŠENJE JE TOČNO ČINI MI SE POŠTENO BODOVATI OVAJ ZADATAK SA $\frac{10+20}{2}=15$ BODOVA.

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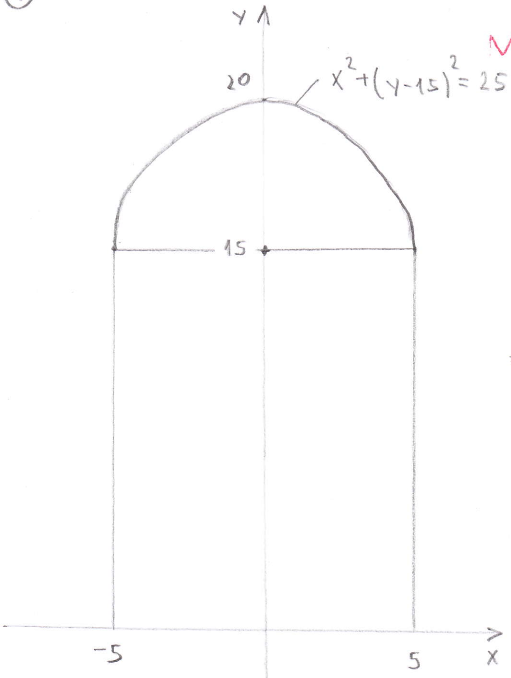

$(y-15)^2=25-x^2$

$y-15=\pm\sqrt{25-x^2}$

$y=\sqrt{25-x^2}+15$

$$\int_{-5}^{5}\int_{0}^{\sqrt{25-x^2}+15}\left(5+\frac{y}{5}\right)dx\,dy=\int_{-5}^{5}\left(5y\Big|_{0}^{\sqrt{25-x^2}+15}+\frac{1}{5}\frac{y^2}{2}\Big|_{0}^{\sqrt{25-x^2}+15}\right)dx=$$

$$=\int_{-5}^{5}\left[5\left(\sqrt{25-x^2}+15\right)+\frac{1}{5}\frac{\left(\sqrt{25-x^2}+15\right)^2}{2}\right]dx=$$

$$=\int_{-5}^{5}\left(5\sqrt{25-x^2}+75+\frac{(25-x^2)+30\sqrt{25-x^2}+225}{10}\right)dx=$$

$$=\int_{-5}^{5}\frac{50\sqrt{25-x^2}+750+25-x^2+30\sqrt{25-x^2}+225}{10}dx=$$

$750+225=975$ -

$+25=1000$ ✓

$$=\frac{1}{10}\int_{-5}^{5}\left(80\sqrt{25-x^2}-x^2+1000\right)dx=8\int_{-5}^{5}\sqrt{25-x^2}\,dx-\frac{1}{10}\int_{-5}^{5}x^2dx+100\int_{-5}^{5}dx=$$

$$=8\left[\overbrace{\cancel{\frac{x}{2}\sqrt{25-x^2}}}^{=\emptyset}+\frac{25}{2}\arcsin\frac{x}{5}\right]_{-5}^{5}-\frac{1}{10}\cdot\frac{x^3}{3}\Big|_{-5}^{5}+100x\Big|_{-5}^{5}=$$

$$=8\left[\frac{25}{2}\arcsin 1-\frac{25}{2}\underbrace{\arcsin(-1)}_{=-\frac{\pi}{2}}\right]-\frac{1}{10}\left(\frac{125}{3}+\frac{125}{3}\right)+100(5+5)=$$

$$=\overset{2}{\cancel{8}}\left(\frac{25}{2}\cdot\frac{\pi}{\cancel{2}}-\frac{25}{2}\cdot\frac{3\pi}{2}\right)-\frac{25}{3}+1000=50\pi-150\pi-\frac{25}{3}+1000=$$

$$=1000-\frac{25}{3}-100\pi=\frac{2975}{3}-100\pi$$

+

ZA OVO BI DOBILI 20 BODOVA