MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Filip Bačinić **BROJ INDEKSA:** 17-2-0168-2012

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,2)$, $B(2,0)$ i $C(4,4)$. 15

2. Zadano je $f(x) = \dfrac{2x^2+x+2}{x^2-1}$. Odrediti $\int_{2}^{3} f(x)\,\mathrm{d}x$. 15

3. Grafički prikazati funkciju $f(x,y) = \ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. 15

4. Istražiti domenu i ekstreme funkcije $f(x,y) = xy - x^3 - y^2$. ~~20~~ 8

5. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1,1,z_0)$. 15

6. Riješiti diferencijalnu jednadžbu: $9y'' - 6y' + y = xe^{-x}$ uz početne uvjete $y(0)=0$ i $y'(0)=1$. Provjeri rješenje. ~~20~~

$$y'' + 4y = 4, \quad y(0) = 0, \; y'(0) = 2$$

POGREŠKOM ZADANA 2 PROBLEMA

Ukupno: 53

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

④ $f(x,y) = xy - x^3 - y^2$ $\quad$ $\mathcal{D}f \in R$ ~~x~~ $\mathcal{D}(f) = \mathbb{R}^2$

$\partial_x f = y - 3x^2$ ✓ $\quad$ $\partial_y f = x - 2y$ ✓

$\partial_{xx} f = -6x$ ✓ $\quad$ $\partial_{yy} f = -2$ ✓

$\partial_{xy} f = 1 = \partial_{yx} f$ ✓

$\Delta = \begin{vmatrix} -6x & 1 \\ 1 & -2 \end{vmatrix}$ $\quad$ $A = -6x$

$y - 3x^2 = 0$

$x - 2y = 0 \quad x = 2y$

$y - 3(2y)^2 = 0$

$y - 3(4y^2) = 0 \quad y - 12y^2 = 0$

$y(1 - 12y) = 0$

$\boxed{y = 0}$ $\quad$ $1 - 12y = 0$

$-12y = -1 \quad / : -12$

$\downarrow$

$x = 0$ $\quad$ $\boxed{y = \frac{1}{12}} \to \boxed{x = \frac{1}{6}}$

$T_1(0,0)$ $\quad A = 0$ ✓

$\Delta = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = -1$

SEDLASTA

$T_2\left(0, \frac{1}{12}\right)$ $\quad A = 0$

$\Delta = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = -1$

SEDLASTA

$T_3\left(\frac{1}{6}, 0\right)$ $\quad A = -1$

$\Delta = \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} = 1$

LOK. MAKS.

$T_4\left(\frac{1}{6}, \frac{1}{12}\right)$ $\quad A = -1$

$\Delta = \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} = 1$ ✓

LOK. MAKSIMUM

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⑤ $z = \arctan\left(\frac{y}{x}\right)$ $\quad M(\overset{x_0}{1}, \overset{y_0}{1}, z_0)$

$z_0 = x_0^2 + y_0^2$

$z_0 = \arctan(1) = \frac{\pi}{4} \qquad M\left(1, 1, \frac{\pi}{4}\right)$

$$f_x' = \frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{y'x - y \cdot x'}{x^2} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{-y}{x^2}$$

$$f_x'(T) = \frac{1}{1+\left(\frac{1}{1}\right)^2} \cdot \frac{-1}{1^2} = \frac{1}{2} \cdot -1 = -\frac{1}{2}$$

$$f_y' = \frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{y'x - yx'}{x^2} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{x}{x^{\cancel{2}}} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x}$$

$$f_y(T) = \frac{1}{1+\left(\frac{1}{1}\right)^2} \cdot \frac{1}{1} = \frac{1}{2}$$

$$z - z_0 = f_x'(T)(x - x_0) + f_y'(T)(y - y_0)$$

$$z - \frac{\pi}{4} = -\frac{1}{2}(x-1) + \frac{1}{2}(y+1)$$

$$z - \frac{\pi}{4} = -\frac{x}{2} + \frac{1}{2} + \frac{y}{2} + \frac{1}{2}$$

$$z - \frac{\pi}{4} = -\frac{x}{2} + 1 + \frac{y}{2}$$

$$\boxed{\frac{x}{2} - \frac{y}{2} + z - \frac{\pi}{4} - 1 = 0}$$

$$\frac{x - x_0}{f_x(T)} = \frac{y - y_0}{f_y(T)} = \frac{z - z_0}{-1}$$

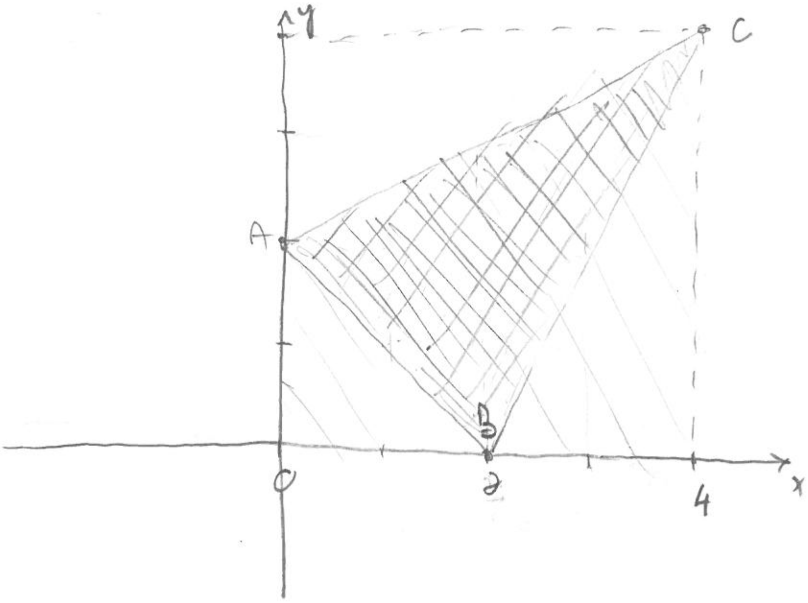
$$\frac{x-1}{-\frac{1}{2}} = \frac{y-1}{\frac{1}{2}} = \frac{z - \frac{\pi}{4}}{-1}$$

$$\boxed{-2x + 2 = 2y - 2 = -z + \frac{\pi}{4}}$$

normala

① $A(0,2)$ $B(2,0)$ $C(4,4)$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$



$\overline{AB} \Rightarrow y-2=\frac{0-2}{2-0}(x-0)$

$\Rightarrow y-2=\frac{-2}{2}x$

$\Rightarrow y=-x+2$

$\overline{AC} \Rightarrow y-2=\frac{4-2}{4-0}(x-0)$

$\Rightarrow y-2=\frac{1}{2}x$

$\Rightarrow y=\frac{1}{2}x+2$

$\overline{BC} \Rightarrow y-0=\frac{4-0}{4-2}(x-2)$

$\Rightarrow y=2(x-2)$

$y=2x-4$

$$P=\int_0^4 \frac{1}{2}x+2\,dx-\int_0^2 -x+2\,dx-\int_2^4 2x-4\,dx$$

$$=\int_0^4 \frac{1}{2}x\,dx+\int_0^4 2\,dx-\int_0^2 -x\,dx-\int_0^2 2\,dx-\int_2^4 2x\,dx+\int_2^4 4\,dx$$

$$=\frac{1}{2}\int_0^4 x\,dx+2\int_0^4 dx+\int_0^2 x\,dx-2\int_0^2 dx-2\int_2^4 x\,dx+4\int_2^4 dx$$

$$=\frac{1}{2}\left(\frac{x^2}{2}\Big|_0^4\right)+2\left(x\Big|_0^4\right)+\left(\frac{x^2}{2}\Big|_0^2\right)-2\left(x\Big|_0^2\right)-2\left(\frac{x^2}{2}\Big|_2^4\right)+4\left(x\Big|_2^4\right)$$

$$=\frac{1}{2}\left(\frac{4^2}{2}-\frac{0^2}{2}\right)+2(4-0)+\left(\frac{2^2}{2}-\frac{0^2}{2}\right)-2(2-0)-2\left(\frac{4^2}{2}-\frac{2^2}{2}\right)+4(4-2)$$

$$=\left(\frac{1}{2}\cdot\frac{16}{2}\right)+8+2-4-2(8-2)+8=4+8+2-4-12+8=6$$ ✓

(2) $f(x) = \frac{2x^2+x+2}{x^2-1}$ $\int_2^3 f(x)\,dx = ?$

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$$\int_2^3 \frac{2x^2+x+2}{x^2-1}dx = \int_2^3 2 + \frac{x+4}{x^2-1}\,dx =$$

$$2x^2+x+2 : x^2-1 = 2$$
$$\underline{-2x^2+2}$$
$$x+4$$

$$= \int_2^3 2 + \frac{x}{x^2-1} + \frac{4}{x^2-1}\,dx =$$

$$= 2\int_2^3 dx + \overbrace{\int_2^3 \frac{x}{x^2-1}dx}^{I_1} + 4\int_2^3 \frac{1}{x^2-1}dx =$$

$$I_1 = \int \frac{x}{x^2-1} = \left| \begin{matrix} x^2-1=t \\ 2x\,dx = dt \quad x\,dx = \frac{dt}{2} \end{matrix} \right|$$
$$= \int \frac{dt}{2t} = \frac{1}{2}\int\frac{dt}{t} = \frac{1}{2}\ln|t|$$
$$= \frac{1}{2}\ln|x^2-1|$$

$$= 2\left(x\Big|_2^3\right) + \frac{1}{2}\ln|x^2-1|\Big|_2^3 + 4\left(\frac{1}{2\cdot1}\ln\left|\frac{x-1}{x+1}\right|\Big|_2^3\right) =$$

$$= 2(3-2) + \frac{1}{2}\ln|3^2-1| - \frac{1}{2}\ln|2^2-1| + 4\left(\frac{1}{2}\ln\left|\frac{3-1}{3+1}\right| - \frac{1}{2}\ln\left|\frac{2-1}{2+1}\right|\right)$$

$$= 2 + \frac{1}{2}\ln|8| - \frac{1}{2}\ln|3| + 2\ln\left|\frac{1}{2}\right| - 2\ln\left|\frac{1}{3}\right|$$

$$= 2 + \frac{3}{2}\ln 2 - \frac{1}{2}\ln 3 - 2\ln 2 + 2\ln 3$$

$$= 2 - \frac{1}{2}\ln 2 + \frac{3}{2}\ln 3 \quad ✓$$

(6) $9y'' - 6y' + y = xe^{-x}$ $\quad y(0) = 0$, $y'(0) = 1$

Filip Bačinić

$k = x$

$b = -1$

$9r^2 - 6r + 1 = 0$ $\quad r_{1,2} = \frac{6 \pm \sqrt{36 - 4 \cdot 9 \cdot 1}}{18} = \frac{6 \pm 0}{18}$ $\quad r_{1,2} = \frac{1}{3}$

$\eta = \frac{kx^2 e^{bx}}{P''(b)}$

$P(b) = 9b^2 - 6b + 1$

$P'(b) = 18b - 6$

$P''(b) = 18$

$$\eta = \frac{x \cdot x^2 e^{-x}}{18} = \boxed{\frac{x^3 e^{-x}}{18}}$$ X

Ø

$y'' + 4y = 4$ $\quad y = y_c + \eta$ $\quad r = 0$, $n = 0$ $\quad y(0) = 0$, $y'(0) = 2$

$r^2 + 4 = 0$

$r^2 = -4$ $\quad r = \sqrt{-4}$

$r_1 = 2i$ $\quad r_2 = -2i$

$$y_0 = e^{\overset{0}{a}x}[C_1 \cos \overset{4}{b}x + C_2 \sin bx]$$

$\eta = a_0$ $\quad 0 + 4a_0 = 4$

$\eta' = 0$ $\quad a_0 = 1$

$\eta'' = 0$

$$y = C_1 \underset{\downarrow 2}{\cos 4x} + C_2 \underset{\downarrow 2}{\sin 4x} + 1$$

$0 = C_1 \cos 0 + C_2 \sin 0 + 1$

$0 = C_1 + 1$ $\quad C_1 = -1$

$y' = C_1(-\sin 4x \cdot 4) + C_2(\cos 4x \cdot 4)$

$y' = -C_1 \cdot -4 \sin 4x + C_2 \cdot 4 \cos 4x$

$2 = C_2 \cdot 4$ $\quad C_2 = \frac{1}{2}$

Ø

$$y = -\cos 4x + \frac{1}{2} \sin 4x + 1$$

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IME I PREZIME: KRISTIAN JOZIĆ

BROJ INDEKSA: 17-1-0012-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 58

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$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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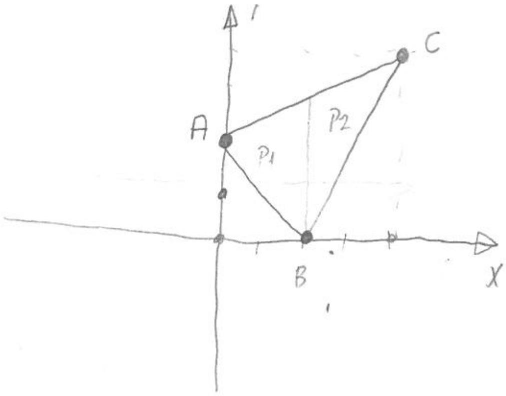
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1. $A(0,2),\ B(2,0),\ C(4,4)$ Sozić



$AC = (y-y_1)(x_2-x_1) = (y_2-y_1)(x-x_1)$

$AC = (y-2)(4-0) = (4-2)(x-0)$

$= (y-2)4 = 2(x-0)$

$= 4y-8 = 2x$

$= 4y = 2x+8 \ /:4$

$AC = \boxed{y = \frac{1}{2}x+2}$

$AB = (y-2)(2-0) = (0-2)(x-0)$

$= (y-2)2 = -2(x-0)$

$= 2y-4 = -2x$

$= 2y = -2x+4 \ /:2$

$AB = \boxed{y = -x+2}$

$BC = (y-0)(4-2) = (4-0)(x-2)$

$= (y-0)2 = 4(x-2)$

$2y = 4x-8 \ /:2$

$BC = \boxed{y = 2x-4}$

Nastavak iza

$$P_1 = \int_0^2 A_C - A_B$$

$$= \int_0^2 \frac{1}{2}x + 2 - (-x+2)\,dx = \int_0^2 \frac{1}{2}x + 2 + x - 2\,dx$$

$$= \frac{1}{2}\int_0^2 x\,dx + 2\int_0^2 dx + \int_0^2 x\,dx - 2\int_0^2 dx = \frac{1}{2}\,\frac{x^2}{2}\Big|_0^2 + 2x\Big|_0^2 + \frac{1}{2}x^2\Big|_0^2 - 2x\Big|_0^2$$

$$= \frac{1}{4}\left(2^2 - 0^2\right) + 2\cdot(2-0) + \frac{1}{2}\left(2^2 - 0^2\right) - 2\cdot(2-0)$$

$$= \frac{1}{4}\cdot 4 + 4 + 2 - 4 = 3 \qquad \boxed{P_1 = 3}$$

$$P_2 = \int_2^4 AC - BC = \int_2^4 \frac{1}{2}x + 2 - (2x-4)\,dx$$

$$= \int_2^4 \frac{1}{2}x + 2 - 2x + 4\,dx = \frac{1}{2}\int_2^4 x\,dx + 2\int_2^4 dx - 2\int_2^4 x\,dx + 4\int_2^4 dx$$

$$= \frac{1}{2}\cdot\frac{x^2}{2}\Big|_2^4 + 2x\Big|_2^4 - \cancel{2}\,\frac{x^2}{\cancel{2}}\Big|_2^4 + 4x\Big|_2^4$$

$$= \frac{1}{4}\left(4^2 - 2^2\right) + 2(4-2) - \left(4^2 - 2^2\right) + 4(4-2)$$

$$= \frac{1}{4}\cdot 12 + 2\cdot 2 - 12 + 8 = \underline{\underline{3}}$$

$$P_{ukupno} = P_1 + P_2$$

$$P_{ukupno} = 3 + 3 = \boxed{6}$$

2. $F(x) = \int_3 \frac{2x^2+x+2}{x^2-1}\,dx$ $\quad 2x^2+x+2 : x^2-1 = 2$ $\quad$ SOZIĆ

$$\begin{array}{l} -2x^2-2 \\ \hline x+4 \end{array}$$

$$= \int_2^3 2\,dx + \int_2^3 \frac{x+4}{x^2-1}\,dx = 2\int_2^3 dx + \int_2^3 \frac{x\,dx}{x^2-1} + 4\int_2^3 \frac{dx}{x^2-1}$$

$$\int_2^3 \frac{x\,dx}{x^2-1} = \left[\begin{array}{c} x^2-1=t \\ 2x\,dx = dt \\ dx = \frac{1}{2}dt \end{array}\right] \quad \begin{array}{c|c|c} x & 2 & 3 \\ t & 3 & 8 \end{array}$$

$$= \int_3^8 \frac{\frac{1}{2}dt}{t} = \frac{1}{2}\int_3^8 \frac{dt}{t} = \frac{1}{2}\ln|t|\Big|_3^8 = \frac{1}{2}\ln|8| - \frac{1}{2}\ln|3|$$

$$= 2x\Big|_2^3 + \frac{1}{2}\ln|8| - \frac{1}{2}\ln|3| + 4\cdot\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\Big|_2^3$$

$$= 2\cdot 3 - 2\cdot 2 + \frac{1}{2}\ln|2^3| - \frac{1}{2}\ln|3| + 2\ln\left|\frac{3-1}{3+1}\right| - 2\ln\left|\frac{2-1}{2+1}\right|$$

$$= 2 + \frac{3}{2}\ln|2| - \frac{1}{2}\ln|3| + 2\ln\left|\frac{2}{4}\right| - 2\ln\left|\frac{1}{3}\right|$$

$$= 2 + \frac{3}{2}\ln|2| - \frac{1}{2}\ln|3| + 2\ln\left|\frac{1}{2}\right| - 2\ln|3^{-1}|$$

$$= 2 + \frac{3}{2}\ln|2| - \frac{1}{2}\ln|3| + 2\ln|2^{-1}| + 2\ln|3|$$

$$= 2 + \frac{3}{2}\ln|2| + \frac{3}{2}\ln|3| + 2\ln|2| - 2 - \frac{1}{2}\ln|2| + \frac{3}{2}\ln|3|$$

$$= 2 + \frac{3}{2}\ln|2| + \frac{3}{2}\ln|3| + 2\ln|2| = 2 - \frac{1}{2}\ln|2| + \frac{3}{2}\ln|3| \checkmark$$

3. $\ln(x-y+2)$

$D_f = \{(x,y) \in \mathbb{R}$

$x - y + 2 > 0$

$x - y > -2$

$y < x + 2$ ✓

$a = \ln(x-y+2)$

$e^a = x - y + 2$

$y = x - 2 - e^a < x + 2$

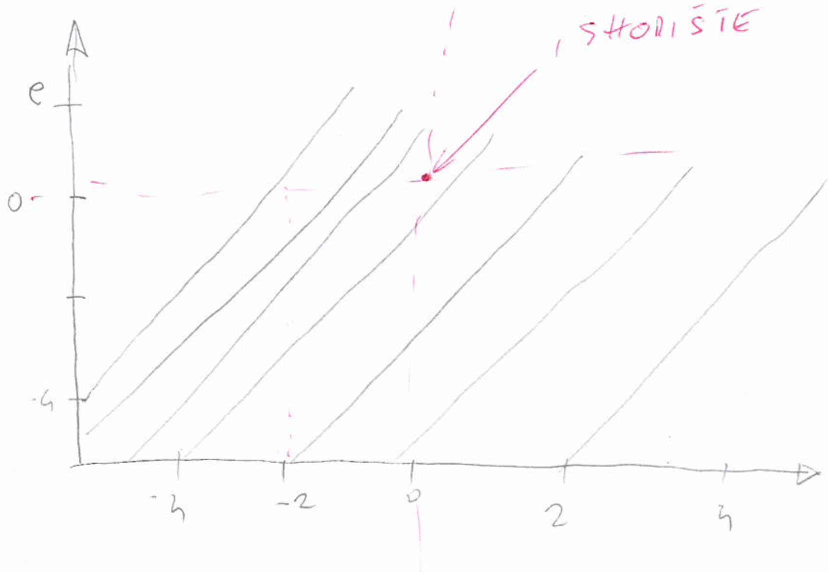
$f(x,y) = a \Rightarrow (x,y) = (x, x+2-e^a)$

$f(x, x+2-e^a) = \ln(x-(x+2-e^a)+2)$

$= \ln(x - x - 2 + e^a + 2)$

$= \ln e^a = a$

KODOMENA: svi realni brojevi ✓



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VRLO ŠLAMPAVA SLIKA RAZINSKIH KRIVULJA.

POTPUNO JEDNAKO KAO KREŠIĆ???

4. $f(x,y) = xy - x^3 - y^2$

Jozic

$\mathcal{D}(f) = \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

$\mathcal{D}\{x,y\} \setminus \mathbb{R}$ ✗

$\frac{\delta f}{\delta x} = y - 3x^2 \Rightarrow y - 3x^2 = 0$

$y = 3x^2$

$\frac{\delta f}{\delta y} = x - 2y \Rightarrow x - 2y = 0$

$x - 2 \cdot (3x^2) = 0$

$x - 6x^2 = 0$ $\boxed{x_1 = 0}$

$x(-6x + 1) = 0$

$-6x + 1 = 0$

$-6x = -1 \; / : -6$

$\boxed{x_2 = \frac{1}{6}}$

$y = 3 \cdot x^2$

$y = 3 \cdot 0^2 = 0$

$\boxed{y_1 = 0}$

$y_2 = 3x^2$

$= 3 \left(\frac{1}{6}\right)^2$

$= \frac{1}{12}$

$\boxed{y_2 = \frac{1}{12}}$

$T_1 = (0,0)$

$T_2 = \left(\frac{1}{6}, \frac{1}{12}\right)$

Za T_2

$\frac{\delta^2 f}{\delta x^2} = -6x = -6 \cdot \frac{1}{6} = -1$

$\frac{\delta f}{\delta y^2} = -2$

$$\Delta_2 = \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} = 2 - 1 = \boxed{1}$$

Točka $T_2\left(\frac{1}{6}, \frac{1}{12}\right)$ je maximum ✓

Za T_1

$\frac{\delta^2 f}{\delta x^2} = -6x \Rightarrow -6 \cdot 0 = 0$

$\frac{\delta^2 f}{\delta y^2} = -2$

$$\Delta_1 = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = \boxed{-1}$$

$\frac{\delta^2 f}{\delta x \delta y} = 1$

Točka $T_1(0,0)$ je sedlasta točka ✓

15

5. $Z = \arctan\left(\frac{y}{x}\right)$ $\qquad M(1,1,z_0)$

$$z_0 = \arctan\left(\frac{y}{x}\right)$$
$$= \arctan\left(\frac{1}{1}\right)$$
$$\underline{= 0{,}79}$$

$M(1,1,0.79)$

$$f(x) = \frac{\delta f}{\delta x} = \frac{1}{1+x^2}$$
$$f_x(M) = \frac{1}{1+1^2} = \frac{1}{2}$$
$$f_y = \frac{\delta f}{\delta y} = \frac{1}{1+y^2} = \frac{1}{1+1^2} = \frac{1}{2}$$

$$z - z_0 = f_{(x)}(M)(x - x_0) + f_{(y)}(M)(y - y_0)$$
$$z - 0.79 = \frac{1}{2}(x-1) + \frac{1}{2}(y-1)$$
$$z - 0{,}79 = \frac{1}{2}x + \frac{1}{2}y - 1$$
$$\boxed{\frac{1}{2}x + \frac{1}{2}y - z - 1 = 0}$$

$$\frac{x - x_0}{f_{(x)}(M)} = \frac{y - y_0}{f_{(y)}(M)} = \frac{z - z_0}{-1} = \boxed{\frac{x-1}{\frac{1}{2}} = \frac{y-1}{\frac{1}{2}} = \frac{z - 0{,}79}{-1}}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ROMANO FUZUL **BROJ INDEKSA:** 0269060225	

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,2)$, $B(2,0)$ i $C(4,4)$. 15
2. Zadano je $f(x) = \dfrac{2x^2+x+2}{x^2-1}$. Odrediti $\int_3^2 f(x)\,dx$. (3, 2) 15
3. Grafički prikazati funkciju $f(x,y) = \ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. ~~15~~ 8
4. Istražiti domenu i ekstreme funkcije $f(x,y) = xy - x^3 - y^2$. ~~20~~ 15
5. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1,1,z_0)$. 15
6. Riješiti diferencijalnu jednadžbu: ~~$9y'' - 6y' + y = xe^{-x}$ uz početne uvjete $y(0)=0$ i $y'(0)=1$.~~ Provjeri rješenje. ~~20~~

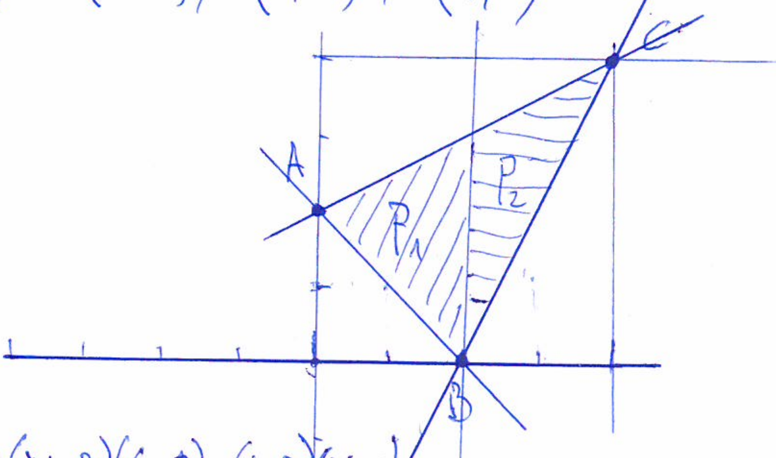
$$y'' + 4y = 4, \quad y(0)=0,\ y'(0)=2$$

Ukupno: 53

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\, dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2\pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1) $A(0,2),\ B(2,0),\ C(4,4)$



$$(y-y_1)(x_2-x_1) = (y_2-y_1)(x-x_1)$$

$$AB: (y-2)\underbrace{(2-0)}_{2} = \underbrace{(0-2)}_{-2}(x-0)$$

$$(y-2)2 = -2x$$

$$2y-4 = -2x$$

$$2y = -2x+4$$

$$\boxed{y = -x+2}$$

$$AC: (y-2)(4-0) = (4-2)(x-0)$$

$$(y-2)4 = 2(x-0)$$

$$4y-8 = 2x$$

$$4y = 2x+8$$

$$y = \frac{2}{4}x + \frac{8}{4}$$

$$\boxed{y = \frac{1}{2}x+2}$$

$$BC: (y-0)(4-2) = (4-0)(x-2)$$

$$(y)(2) = (4)(x-2$$

$$2y = 4x-8$$

$$\boxed{y = 2x-4}$$

1

$$\boxed{P = P_1 + P_2}$$

$$P_1 = \int_0^2 (AC - AB)\,dx$$

$$= \int_0^2 \left(\tfrac{1}{2}x + 2\right) - (-x + 2)\,dx$$

$$= \left[\int_0^2 \tfrac{1}{2}x + \cancel{2} + x - \cancel{2}\right]_0^2$$

$$= \int_0^2 \frac{3}{2}x\,dx = \left[\frac{3}{2} \cdot \frac{x^2}{2}\right]_0^2 = \left[\frac{3x^2}{4}\right]_0^2 = \left[\left(\frac{3 \cdot 2^2}{4}\right) - \left(\frac{3 \cdot 0^2}{4}\right)\right]$$

$$= 3 - 0 = 3$$

$$P_2 = \int_2^4 (AC - BC)\,dx$$

$$= \int_2^4 \left(\tfrac{1}{2}x + 2\right) - (2x - 4)\,dx$$

$$= \int_2^4 \cancel{\tfrac{1}{2}x + 2 - 2x + 4}\,dx$$

$$= \int_2^4 -\frac{1}{2}x + 6\,dx = \int_2^4 -\frac{1}{2}x\,dx + 4\int_2^4 dx$$

$$= -\frac{3}{2}\int_2^4 x\,dx + 6\int_2^4 dx$$

$$= \left[\left(-\frac{3}{2} \cdot \frac{x^2}{2}\right) + 6x\right]_2^4 = \left[-\frac{3x^2}{4} + 6x\right]_2^4$$

$$= \cancel{\left[\left(-\frac{3 \cdot 4^2}{4} + 6 \cdot 4\right) - \left(-\frac{3 \cdot 2^2}{4} + 6 \cdot 2\right)\right]}$$

$$= \cancel{42 - 9}$$

$$= 12 - 9 = 3$$

$$P = P_1 + P_2$$

$$= 3 + 3$$

$$\boxed{P = 6}$$

ROMANO FUZUL

2.) $\displaystyle\int_2 \frac{2x^2+x+2}{x^2-1}\,dx$ $\qquad D_f = \mathbb{R}\setminus\{\pm 1\}$ $\qquad x^2-1 \neq 0$

$x^2 \neq 1$

$x = \pm 1$

Domena nam ne spada u segment $[2,3]$ podintegralne funkcije tako da integral nije nepravi. ✓

$$\int \frac{2x^2+x+2}{x^2-1}\,dx$$

$$(2x^2+x+2):(x^2-1) = 2 + \frac{x+4}{x^2-1}$$

$$\underline{\cancel{-2x^2} \qquad +2}$$

$$+x+4$$

$$= \int 2\,dx + \int \frac{x+4}{x^2-1}\,dx$$

$$J = 2\int dx + \int \frac{x+4}{\underset{(x-1)(x+1)}{x^2-1}}\,dx$$

$$= 2x + \underbrace{\int \frac{x+4}{(x-1)(x+1)}\,dx}_{J_1}$$

$$J_1 = \int \frac{x+4}{(x-1)(x+1)}\,dx$$

$$\frac{x+4}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} \Big/ (x-1)(x+1)$$

$$x+4 = A(x+1) + B(x-1)$$

$$x+4 = Ax + A + Bx - B$$

$$A + B = 1$$

$$A - B = 4$$

$$\cancel{\boxed{A=5}}$$

$$\cancel{5 + B = 1}$$

$$\cancel{B = 1 - 5}$$

$$\cancel{B = -4}$$

$$2A = 5$$

$$\boxed{A = \frac{5}{2}}$$

$$\frac{5}{2} + B = 1$$

$$B = 1 - \frac{5}{2}$$

$$\boxed{B = -\frac{3}{2}}$$

$$J_1 = \int \frac{\frac{5}{2}\,dx}{x-1} - \int \frac{\frac{3}{2}\,dx}{x+1}$$

$$= \frac{5}{2}\int \frac{dx}{x-1} - \frac{3}{2}\int \frac{dx}{x+1}$$

$$\left|\begin{matrix} x-1 = t \\ dx = dt \end{matrix}\right| \qquad \left|\begin{matrix} x+1 = u \\ dx = du \end{matrix}\right|$$

$$= \frac{5}{2}\int \frac{dt}{t} - \frac{3}{2}\int \frac{du}{u}$$

$$= \frac{5}{2}\ln|x-1| - \frac{3}{2}\ln|x+1|$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odrediti integracijom (analitički): $\int_0^3 x^2 \ln x \, dx =$ 15
2. Izračunati $\int\limits_0^{\pi/2} e^x \cos x \, dx$ 15
3. Napiši jednadžbu ravnine koja prolazi točkom $T(1,0,2)$ i okomita je na *os x*. 5+10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y + -e$. 20
5. Riješi diferencijalnu jednadžbu $(1+e^x)yy' = e^x$ uz početni uvjet $y(0) = 1$. 20
6. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x \sin x$. Provjeri dobiveno rješenje 15

Ukupno:

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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$\int \cos x \, dx = \sin x + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

ROMANO
FOZUL

$$y=\left[2x+\frac{5}{2}\ln|x-1|-\frac{3}{2}\ln|x+1|\right]_{2}^{3}$$

$$=\left[2\cdot 3+\frac{5}{2}\ln|3-1|-\frac{3}{2}\ln|3+1|\right]-\left[2\cdot 2+\frac{5}{2}\ln|2-1|-\frac{3}{2}\ln|2+1|\right]$$

$$=\left[6+1.73-2.079\right]-\left[4+0-1.64\right]$$

$$=6+1.73-2.079-4+1.64=3.291$$

$$P\approx 3$$

3.) $f(x,y) = \ln(x-y+2)$

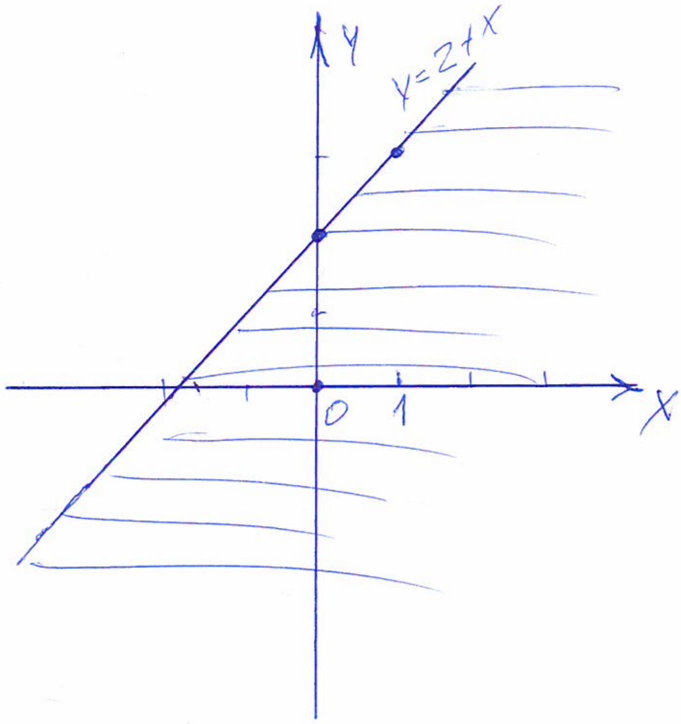
1.) Domena

$x - y + 2 > 0$

$x - y > -2$

$-y > -2 - x \quad / \cdot (-1)$

$y < 2 + x$ ✓



x	0	1	2
y	2	3	4

VRIJEDNOSTI

8

2.) Razinske krivulje

$\ln(x-y+2) = c \quad / e^{c}$

$\cancel{e}^{\ln(x-y+2)} = e^{c}$

$x - y + 2 = e^{c}$

$-y = e^{c} - x - 2 \quad / \cdot (-1)$

$y = x + 2 - e^{c}$

$$\left.\begin{aligned} c=0 &\Rightarrow y = x+2-1 \\ c=1 &\Rightarrow y = x+2-e^{1} \\ c=2 &\Rightarrow y = x+2-e^{2} \end{aligned}\right\}$$ NACRTATI

KOMANO
FUZUL

6.) B) $y'' + 4y = 4$

1.) $\lambda^2 + 4\lambda = 0$ X

$\lambda(\lambda + 4) = 0$

$\lambda_1 = 0 \quad \lambda_2 = -4$

$Y_0(x) = C_1 e^{0x} + C_2 e^{-4x}; \; C_1 C_2 \in \mathbb{R}$

$Y_0(x) = C_1 + C_2 e^{-4x}; \; C_1 C_2 \in \mathbb{R}$

2.) $\cancel{Y = A \cdot 4}$ $\cancel{Y' =}$ $\quad Y = 4A$ X $\quad Y = 4 \cdot \frac{1}{4}$

$\cancel{Y' = 4}$ $\quad Y = 1$

$\cancel{Y'} = 0$

$0 + 4 \cdot (4A) = 4$

$16A = 4$

$A = \frac{4}{16}$

$A = \frac{1}{4}$

$Y(x) = Y_0 + Y$

opće rješenje

$$\boxed{Y(x) = C_1 + C_2 e^{-4x} + 1}$$

6

$y = C_1 + C_2 e^{-4x} + 1$ $\qquad y' = 2 \quad x = 0$

$y' = \cancel{0 -} 4C_2 e^{-4x}$ $\qquad y = 0 \quad x = 0$

$0 = C_1 + C_2 e^{-4 \cdot 0} + 1$

$0 = C_1 + C_2 + 1$

$C_1 + C_2 = -1$

$2 = -4C_2 \cdot e^{-4 \cdot 0}$

$2 = -4C_2$

$-4C_2 = 2$

$C_2 = -\frac{2}{4}$

$\boxed{C_2 = -\frac{1}{2}}$

$C_1 - \frac{1}{2} = -1$

$C_1 = \frac{1}{2} - 1$

$\boxed{C_1 = -\frac{1}{2}}$

$C_2 e^{-4x}$

$C_2 e^{-4x} \cdot (-4x)'$

$-4C_2 e^{-4x}$

Partikularno rješenje

$$\boxed{y(x) = -\frac{1}{2} - \frac{1}{2} e^{-4x} + 1}$$

4.) $f(x,y) = xy - x^3 - y^2$

DOMENA ?

ROMANO FUZUL

$$\frac{\delta f}{\delta x} = y - 3x^2 \Rightarrow y - 3x^2 = 0$$

$$y = 3x^2$$

$$\frac{\delta f}{\delta y} = x - 2y$$

$$x - 2(3x^2) = 0$$

$$x - 6x^2 = 0$$

$$x(1 - 6x) = 0$$

$$\boxed{x_1 = 0} \qquad 1 - 6x = 0$$

$$-6x = -1$$

$$\boxed{x_2 = \frac{1}{6}}$$

$$y_1 = 3 \cdot 0^2$$

$$\boxed{y_1 = 0}$$

$$y_2 = 3 \cdot \left(\frac{1}{6}\right)^2$$

$$y_2 = \frac{1}{12}$$

$T_1(0,0)$ – Sedlasta točka ✓

$T_2\left(\frac{1}{6}, \frac{1}{12}\right)$ – Maksimum ✓

$$\Delta_1 = \begin{vmatrix} \frac{\delta^2 f}{\delta x} & \frac{\delta f}{\delta x \delta y} \\ \frac{\delta f}{\delta x \delta y} & \frac{\delta^2 f}{\delta^2 y} \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 0 - 1 = -1$$

$$\frac{\delta^2 f}{\delta^2 x} = -6x \Rightarrow \text{za } x_1 = -6 \cdot 0 = 0$$

$$\text{za } x_2 = -6 \cdot \frac{1}{6} = -1$$

$$\Delta_2 = \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} = 2 - 1 = 1$$

15

$$\frac{\delta^2 f}{\delta^2 y} = -2$$

$$\frac{\delta f}{\delta y \delta x} = 1$$

$$\frac{\delta f}{\delta x \delta y} = 1$$

ROMANO FUZUL

6.) a) $9y''-6y'+y=xe^{-x}$

1.) $9\lambda^2-6\lambda+1=0$

$$\lambda_{1,2}=\frac{6\pm\sqrt{36-4\cdot 9}}{2\cdot 9}$$

$$\lambda_{1,2}=\frac{6}{18}=\frac{1}{3}$$

$$y_0=e^{\frac{1}{3}x}(C_1+C_2x);\ C_1C_2\in\mathbb{R}$$

$$y=y_0+Y$$

$$Y=(Ax+B)e^{-x}$$

$$Y'=Ae^{-x}+(Ax+B)(-e^{-x})$$

$$Y''=-Ae^{-x}+A(-e^{-x})+(Ax+B)(e^{-x})$$

$$9(-Ae^{-x}-Ae^{-x}+(Ax+B)e^{-x})-6(Ae^{-x}+(Ax+B)\cdot -e^{-x})+(Ax+B)e^{-x}=xe^{-x}$$

$$9(-Ae^{-x}-Ae^{-x}+(Ax+B)e^{-x})-6(Ae^{-x}+(-Ax-B)e^{-x})+(Ax+B)e^{-x}=xe^{-x}\ /e^{-x}$$

$$9(-A-A+Ax+B)-6(A-Ax-B)+Ax+B=x$$

$$-9A-9A+9Ax+9B-6A+6Ax\pm 6B+Ax+B=x$$

$$-9A-9A+9B-6A\pm 6B+B=0$$

$$9A+6A+A=1$$

$$-24A+\cancel{4}\,16B=0$$

$$16A=1$$

$$\boxed{A=\frac{1}{16}}$$

$$-\frac{24}{16}+\cancel{4}\,16B=0$$

$$\cancel{4}\,16B=\pm\frac{3}{2}$$

$$B=\pm\frac{3}{\cancel{8}\,32}$$

$$Y=\left(\frac{1}{16}x+\frac{3}{8}\right)e^{-x}$$

OPĆE RJEŠENJE

$$y=e^{\frac{1}{3}x}(C_1+C_2x)+\left(\frac{1}{16}x-\frac{3}{8}\right)e^{-x}$$

TREBALO JE PROVJERITI PARTIK. RJEŠENJE, PA PRONAĆI I ISPRAVITI GREŠKU.

$y = e^{\frac{1}{3}x}(C_1 + C_2 x) + \left(\frac{1}{16}x - \frac{3}{8}\right)e^{-x}$ $y(0) = 0$ $y = 0$ $x = 0$

$0 = e^{\frac{1}{3}\cdot 0}(C_1 + C_2 \cdot 0) + \left(\frac{1}{16}\cdot 0 - \frac{3}{8}\right)e^{-0}$

$0 = 1(C_1) - \frac{3}{8}$

$-C_1 = -\frac{3}{8} \quad /(-1)$

$\boxed{C_1 = \frac{3}{8}}$

$y' = 1 \quad x = 0$

$y' = \frac{1}{3}e^{x}(C_1 + C_2 x) + e^{\frac{1}{3}x}(C_2) + \left(\frac{1}{16}\right)e^{-x} + \left(\frac{1}{16}x - \frac{3}{8}\right)\cdot(-e^{-x})$

$1 = \frac{1}{3}\cdot e^{0}(C_1 + C_2 \cdot 0) + e^{\frac{1}{3}\cdot 0}(C_2) + \frac{1}{16}\cdot e^{-0} + \left(\frac{1}{16}\cdot 0 - \frac{3}{8}\right)\cdot(-e^{-0})$

$1 = \frac{1}{3}\cdot\frac{3}{8} + C_2 + \frac{1}{16} + \frac{3}{8}$

$-C_2 = \frac{1}{3}\cdot\frac{3}{8} + \frac{1}{16} + \frac{3}{8} - 1$

$-C_2 = -\frac{7}{16} \quad /(-1)$

$\boxed{C_2 = \frac{7}{16}}$

Partikularno rješenje

$$\boxed{y = e^{\frac{1}{3}x}\left(\frac{3}{8} + \frac{7}{16}x\right) + \left(\frac{1}{16}x - \frac{3}{8}\right)e^{-x}}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Jelena Maleš **BROJ INDEKSA:** 17-2-0103-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,2)$, $B(2,0)$ i $C(4,4)$. — 15
   $x_1 y_1$ $x_2 y_2$ $x_3 y_3$
2. Zadano je $f(x) = \dfrac{2x^2+x+2}{x^2-1}$. Odrediti $\int_3^2 f(x)\,\mathrm{d}x$. — 15
3. Grafički prikazati funkciju $f(x,y) = \ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. — 15
4. Istražiti domenu i ekstreme funkcije $f(x,y) = xy - x^3 - y^2$. — 20
5. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1,1,z_0)$. — 15
6. Riješiti diferencijalnu jednadžbu: $9y'' - 6y' + y = xe^{-x}$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$. Provjeri rješenje. — 20

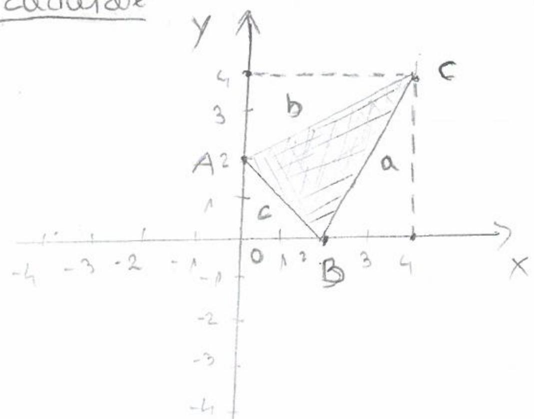
$$y'' + 4y = 4, \quad y(0) = 0,\ y'(0) = 2$$

Ukupno: 50

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x + \sqrt{x^2\pm a^2}\right\| + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

I zadatak



1. $Y - Y_1 = \dfrac{Y_2 - Y_1}{X_2 - Y_1}(X - Y_1)$

$Y - 0 = \dfrac{4-0}{4-2}(X-2)$

$Y = 2(X-2)$

$Y = 2X - 4$

2. $Y - 2 = \dfrac{0-2}{2-0}(X-0)$

$Y - 2 = -1(X)$

$Y - 2 = -X$

$Y = -X + 2$

3. $Y - 2 = \dfrac{4-2}{4-0}(X-0)$

$Y - 2 = \frac{1}{2}\frac{2}{4}X$

$Y - 2 = \frac{1}{2}X$

$Y = \frac{1}{2}X + 2$

1

$P = P_b - P_a - P_c$

$P_b = \int_0^4 (\frac{1}{2}x + 2)\,dx$

$P_b = \frac{16}{4} + 8 - 0 = 12_{\shortparallel}$

$P_a = \int_2^4 (2x-4)\,dx =$

$= 16 - 16 - (4-8) = 4_{\shortparallel}$

$P_c = \int_0^2 (-x+2)\,dx =$

$= -2 + 4 - 0 = 2_{\shortparallel}$

$P = 12 - 4 - 2$

$\boxed{P = 6_{\shortparallel}}$ ✓

II. $f(x) = \frac{2x^2+x+2}{x^2-1}$ $\qquad \int_3^2 f(x)\,dx$

$\int \left(2 + \frac{x+4}{(x-1)(x+1)}\right) dx =$

$= 2\int dx + \int \frac{A}{(x-1)(x+1)}\,dx + \int \frac{B}{(x-1)(x+1)}\,dx$

$= 2x + \frac{5}{2}\int \frac{dx}{x-1} - \frac{3}{2}\int \frac{dx}{x+1}$

$= \left. 2x + \frac{5}{2}\ln(x-1) - \frac{3}{2}\ln(x+1) \right|_3^2$

$= \left(2\cdot(2) + \frac{5}{2}\ln(2-1) - \frac{3}{2}\ln(2+1)\right) - \left(2\cdot 3 + \left(\frac{5}{2}\ln(3-1) - \frac{3}{2}\ln(3+1)\right)\right)$

$= \left(4 + \frac{5}{2}\ln(1) - \frac{3}{2}\ln 3\right) - \left(6 + \frac{5}{2}\ln 2 - \frac{3}{2}\ln 4\right) =$

$= (4 + 0 - 1{,}647918433) - (6 + 1{,}732867951 - 2{,}079\ldots$

$= \ldots$

2

2. zad *

Jelena Matić

$(2x^2+x+2):(x^2-1)=2$

$\frac{2x^2-2}{\cancel{x}+4}$

$$\frac{x+4}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1} \quad /\cdot(x+1)(x-1)$$

$x+4=A(x+1)+B(x-1)$

$x+4=Ax+A+Bx-B$

$A+B=1$

$A-B=4$

$2A=5$

$A=\frac{2}{5}$

$\frac{5}{2}+B=1$

$B=1-\frac{5}{2}$

$B=-\frac{3}{2}$

3

4. $f(x,y) = xy - x^3 - y^2$

$f(x) = y - 3x^2$

$f(y) = x - 2y$

$f(x) = 0 \qquad -3x^2 + y = 0$

$f(y) = 0 \qquad -2y + x = 0 \Rightarrow x = 2y$

$-3 \cdot 4y^2 + y = 0$

$-12y^2 + y = 0$

$y(-12y + 1) = 0$

$y_1 = 0$

$\wedge$

$-12y + 1 = 0$

$y_2 = \frac{1}{12}$

$\boxed{T_1(0,0)}$

$\boxed{T_2\left(\frac{1}{6}, \frac{1}{12}\right)}$

$x_1 = 0$

$x_2 = \cancel{2} \cdot \frac{1}{\cancel{12}} = \frac{1}{6}$

$x_2 = \frac{1}{6}$

$f_{xx} = -6x$

$f_{xy} = -1$

$f_{yy} = -2$

$T(0,0)$

$A = 0$

$B = 1$

$C = -2$

$\Delta = AC - B^2$

$\Delta = 0 \cdot (-2) - 1^2$

$\Delta = -1 < 0$ Nije ekstrem ✓

$T\left(\frac{1}{6}, \frac{1}{12}\right)$

$A = -1$

$B = 1$

$C = -2$

$\Delta = 2 - 1 = 1$

$A = -1$ MAX ✓

$D(f) = \mathbb{R} \cdot \mathbb{R} = \mathbb{R}^2$ ✓

4

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: RANKO BRKIĆ **BROJ INDEKSA:** 17-1-0091-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,2)$, $B(2,0)$ i $C(4,4)$. 15
2. Zadano je $f(x) = \dfrac{2x^2 + x + 2}{x^2 - 1}$. Odrediti $\int_3^2 f(x)\,\mathrm{d}x$. 15 / 15
3. Grafički prikazati funkciju $f(x,y) = \ln(x - y + 2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. 15
4. Istražiti domenu i ekstreme funkcije $f(x,y) = xy - x^3 - y^2$. ~~20~~
5. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1,1,z_0)$. 15
6. Riješiti diferencijalnu jednadžbu: $9y'' - 6y' + y = xe^{-x}$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$. Provjeri rješenje. 20

$$y'' + 4y = 4, \quad y(0) = 0,\ y'(0) = 2$$

Ukupno: 45

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x\,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
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(2) $f(x) = \dfrac{2x^2 + x + 2}{x^2 - 1}$

$$\frac{2x^2 + x + 2}{(x^2 - 1)} = 2$$

$$-2x^2 + 2$$

$$x + 4$$

$$\int_2^3 2\,dx + \int_2^3 \frac{(x+4)}{(x^2-1)} \rightarrow x^2 - 1 = (x-1)\cdot(x+1)$$

$$\frac{x+4}{(x-1)\cdot(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \quad /\cdot (x-1)\cdot(x+1)$$

NOVI LIST →

1. $AB \to (y-2)\cdot(2-0)=(0-2)\cdot(x-0)$

$2y-4=-2x$

$2y=-2x+4$

$y=-x+2$ $\quad BC \to (y-0)\cdot(4-2)=(4-0)\cdot(x-2)$

$2y=4x-8$

$y=2x-4$

$AC \to (y-2)\cdot(4-0)=(4-2)\cdot(x-0)$

$4y-8=2x$

$y=\frac{1}{2}x+2$

$P_1=\int_0^2 \frac{1}{2}x+2+x-2$

$P_1=\int_0^2 \frac{3}{2}x$

$P_1=\frac{3}{2}\cdot\frac{x^2}{2}\Big|_0^2$

$P_1=\frac{3}{\cancel{2}}\cdot\frac{4}{2}$

$P_1=3$

$P_2=\int_2^4 \frac{1}{2}x+2-2x+4$

$P_2=-\frac{3}{2}\cdot\frac{x^2}{2}\Big|_2^4+6x\Big|_2^4$

$P_2=-\frac{3}{2}\cdot\frac{4^2}{2}+\left(\frac{3}{2}\right)\cdot\left(\frac{2^2}{2}\right)+(6\cdot4)-(6\cdot2)$

$P_2=-\frac{3}{2}\cdot8+\frac{3}{2}\cdot2+24-12$

$P_2=-12+3+12$

$P_2=3$ $\quad P=P_1+P_2$

$P=6$ ✓

② $x+4 = A\cdot(x+1) + B\cdot(x-1)$

$x+4 = A\cdot x + A + B\cdot x - B$

$A-B=4$

$A+B=1$

$A=B+4$

$B+4+B=1$

$2B=-3$

$B=-\frac{3}{2}$

$A=1+\frac{3}{2}$

$A=\frac{5}{2}$

$$\int_2^3 \frac{\frac{5}{2}}{x-1}\,dx + \int_2^3 \frac{-\frac{3}{2}}{x+1}\,dx$$

$$f(x) = 2x\Big|_2^3 + \frac{5}{2}\cdot \ln|x-1|\Big|_2^3 - \frac{3}{2}\cdot \ln|x+1|\Big|_2^3$$

$$f(x) = 2\cdot3 - 2\cdot2 + \frac{5}{2}\cdot(0.693-0) - \frac{3}{2}\cdot(1.386-1.099)$$

$$f(x) = 2 + 1.7325 - 0.4305$$

$$f(x) = 3.302$$ ✓

15

⑤ $z_0 = \arctan\left(\frac{y}{x}\right)$

$z_0 = \arctan\left(\frac{1}{1}\right)$

$z_0 = 0.785$

$$\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{-y}{x^2} = $$

$$= \frac{1}{1+1}\cdot\frac{(-1)}{1} = \frac{-1}{2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x} = $$

$$= \frac{1}{1+1}\cdot 1 = \frac{1}{2}$$

$$z - z_0 = -\frac{1}{2}\cdot(x-1) + \frac{1}{2}\cdot(y-1)$$ ✓

$$z - 0.785 = -\frac{1}{2}x + \frac{1}{2} + \frac{1}{2}y - \frac{1}{2}$$

$-\frac{1}{2}x + \frac{1}{2}y - z + 0{,}785 = 0$ Tangencijalna ravnina

$$\frac{x-1}{-\frac{1}{2}} = \frac{y-1}{\frac{1}{2}} = \frac{z-0{,}785}{-1}$$ Jednadžba normale

4. $f : \mathbb{D} \to \mathbb{R}^2$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	POPUNJAVA NASTAVNIK
IME I PREZIME: BORIS KREŠIĆ **BROJ INDEKSA:** 17-1-0022-2010	Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,2)$, $B(2,0)$ i $C(4,4)$. — 15
2. Zadano je $f(x) = \dfrac{2x^2+x+2}{x^2-1}$. Odrediti $\int_3^2 f(x)\,\mathrm{d}x$. — 15
3. Grafički prikazati funkciju $f(x,y) = \ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. — ~~15~~ 13
4. Istražiti domenu i ekstreme funkcije $f(x,y) = xy - x^3 - y^2$. — 20
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6. Riješiti diferencijalnu jednadžbu: $9y'' - 6y' + y = xe^{-x}$ uz početne uvjete $y(0)=0$ i $y'(0)=1$. Provjeri rješenje. — 20

$$y'' + 4y = 4, \quad y(0) = 0,\ y'(0) = 2$$

Ukupno: 43

?PREPISIVANJE?
VIDI IZA

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

(1) Integriranjem odredi površinu trokuta. ~~ZADATAK~~

$A(0,2) \quad B(2,0) \quad C(4,4)$

BORIS KRESIĆ

$\boxed{\text{Formula za pravac}}$

$$(y-y_1)(x_2-x_1)=(y_2-y_1)(x-x_1)$$

$$AC = (y-2)\underset{4}{(4-0)} = \underset{2}{(4-2)}(x-0)$$

$$=(y-2)\cdot 4 = 2\cdot(x-0)$$

$$=4y-8=2x \;/:4$$

$$=4y=2x+8 \;/:4$$

$$\boxed{y=\frac{1}{2}x+2}$$

$$AB = (y-2)\underset{2}{(2-0)} = \underset{-2}{(0-2)}(x-0)$$

$$=(y-2)\cdot 2 = -2(x-0)$$

$$=2y-4=-2x \;/:2$$

$$=2y=-2x+4 \;/:2$$

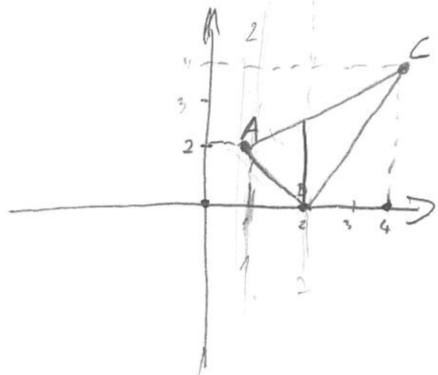
$$\boxed{y=-x+2}$$

$$BC = (y-0)\underset{2}{(4-2)} = \underset{4}{(4-0)}(x-2)$$

$$=(y-0)\cdot 2 = 4\cdot(x-2)$$

$$=2y=4x-8 \;/:2$$

$$\cancel{y=2x-8} \quad \boxed{y=2x-4}$$



$P_1 = AC-AB$

$P_2 = AC-BC$

$\boxed{\text{Ukupno} = P_1+P_2}$

$P_1 = AC-AB$

$$P_1=\int_0^2 \frac{1}{2}x+2-(-x+2)\,dx=\int_0^2 \frac{1}{2}+2+x-2\,dx$$

$$=\frac{1}{2}\int_0^2 x\,dx+2\int_0^2 dx+\int_0^2 x\,dx-2\int_0^2 dx=$$

$$=\frac{1}{2}\frac{x^2}{2}\Big|_0^2+2x\Big|_0^2+\frac{1}{2}x^2\Big|_0^2$$

$$=\frac{1}{4}(2^2-0^2)+2(2-0)+\frac{1}{2}(2^2-0^2)-$$

$$-2\cdot(2-0)=\frac{1}{4}\cdot 4+4+2-4=3$$

$$=P_1=\boxed{3} \rightarrow$$

BORIS KREŠIĆ

$$P_2 = \int_{2}^{4} \frac{1}{2}x^2 - 2x + 4\,dx = \frac{1}{2}\int_{2}^{4} x\,dx + 2\int_{2}^{4} dx - 2\int_{2}^{4} x\,dx + 4\int_{2}^{4} dx$$

$$= \frac{1}{2}\cdot\frac{x^2}{2}\Big|_{2}^{4} + 2x\Big|_{2}^{4} - \cancel{2}\frac{x^2}{\cancel{2}}\Big|_{2}^{4} + 4x\Big|_{2}^{4}$$

$$= \frac{1}{4}\left(4^2 - 2^2\right) + 2(4-2) - \left(4^2 - 2^2\right) + 4(4-2)$$

$$= \frac{1}{4}\cdot 12 + 2\cdot 2 - 12 + 8 = \boxed{3}$$

$P_{ukupno} = P_1 + P_2$

$P_{ukupno} = 3 + 3 = \boxed{6}$

ZADATAK

(2) $f(x)=\dfrac{2x^2+x+2}{x^2-1}$. Odrediti $\int_3^2 f(x)\,dx$.

BORIS KRESIC

$$\int_2^3 \frac{2x^2+x+2}{x^2-1} \qquad (2x^2+x+2):(x^2-1)=2$$

$$\begin{array}{l} -2x^2-2 \\ \quad + \\ \hline \boxed{x+4} \end{array}$$

$$=\int_2^3 2dx+\int_2^3 \frac{x+4}{x^2-1}dx=2\int_2^3 dx+\int_2^3 \frac{x\,dx}{x^2-1}+4\int_2^3 \frac{dx}{x^2-1}$$

$$=\int_2^3 \frac{x\,dx}{x^2-1}=\left[\begin{array}{l} x^2-1=t \\ 2x\,dx=dt \\ dx=\frac{1}{2}dt \end{array} \qquad \begin{array}{c|c|c} x & 2 & 3 \\ \hline t & 3 & 8 \end{array}\right]$$

$$=\int_3^8 \frac{\frac{1}{2}dt}{t}=\frac{1}{2}\int_3^8 \frac{dt}{t}=\frac{1}{2}\ln|t|\Big|_3^8=\frac{1}{2}\ln|8|-\frac{1}{2}\ln|3|$$

$$=2x\Big|_2^3+\frac{1}{2}\ln|8|-\frac{1}{2}\ln|3|+4\cdot\frac{1}{2}\ln\frac{x-1}{x+1}\Big|_2^3$$

$$=2\cdot3-2\cdot2+\frac{1}{2}\ln|2^3|-\frac{1}{2}\ln|3|+2\ln\left|\frac{3-1}{3+1}\right|-2\ln\left|\frac{2-1}{2+1}\right|$$

$$=2+\frac{3}{2}\ln|2|-\frac{1}{2}\ln|3|+2\ln\left|\frac{2}{4}\right|-2\ln\left|\frac{1}{3}\right|$$

$$=2+\frac{3}{2}\ln|2|-\frac{1}{2}\ln|3|+2\ln\left|\frac{1}{2}\right|-2\ln|3^{-1}|$$

$$=2+\frac{3}{2}\ln|2|-\frac{1}{2}\ln|3|+2\ln|2^{-1}|+2\ln|3|$$

$$=2+\frac{3}{2}\ln|2|+\frac{3}{2}\ln|3|-2\ln|2|=2-\frac{1}{2}\ln|2|+\frac{3}{2}\ln|3|$$

(3) ZADATAK

BORIS CREŠIĆ

$\ln(x-y+2)$

$D_f = \{(x,y) \in \mathbb{R}^2 \mid$ $x-y+2>0$

$x-y>-2$

$y<x+2\}$ ✓

$a=\ln(x-y+2)$

$e^a = x-y+2$

$y = x-2-e^a < x+2$

$f(x,y)=a \Rightarrow (x,y) = (x, x+2-e^a) \Rightarrow a \in \mathbb{R}$

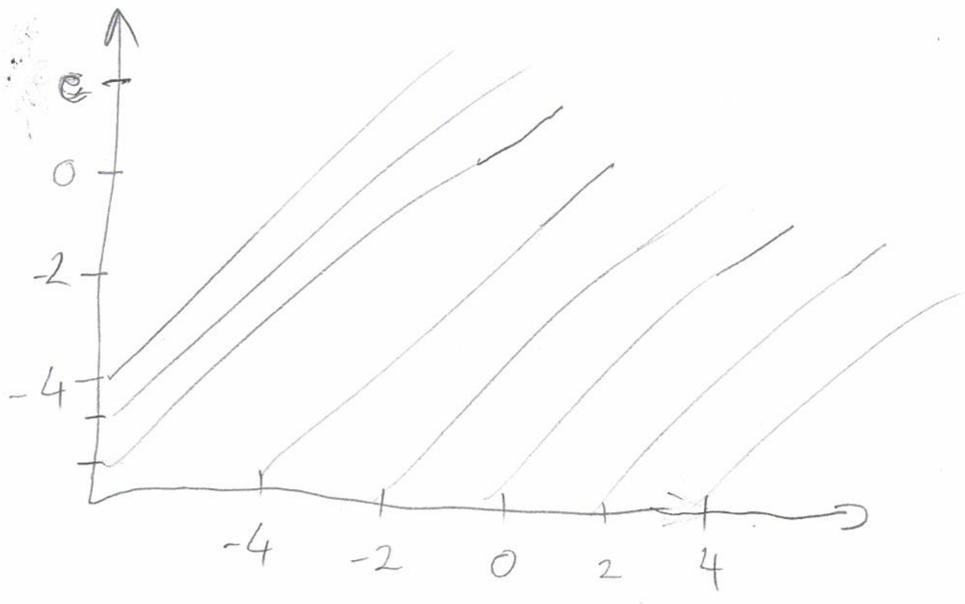
$f(x, x+2-e^a) = \ln(x-(x+2-e^a)+2)$

$= \ln(x-x-2+e^a+2)$

$= \ln e^a = a$

KODOMENA $\boxed{\text{svi realni brojevi}}$

13



✓

POTPUNO JEDNAKO KAO JOZIĆ.

⇒

① str.

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Zoko Matijašević **BROJ INDEKSA:** 17-1-0973-2011

POPUNJAVA NASTAVNIK

	Broj ↓ bodova
1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,2)$, $B(2,0)$ i $C(4,4)$.	~~15~~
2. Zadano je $f(x) = \dfrac{2x^2+x+2}{x^2-1}$. Odrediti $\int_2^3 f(x)\,dx$.	~~15~~
3. Grafički prikazati funkciju $f(x,y) = \ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.	15
4. Istražiti domenu i ekstreme funkcije $f(x,y) = xy - x^3 - y^2$.	~~20~~ 5
5. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1,1,z_0)$.	15
6. Riješiti diferencijalnu jednadžbu: $9y'' - 6y' + y = xe^{-x}$ uz početne uvjete $y(0)=0$ i $y'(0)=1$. Provjeri rješenje. $y'' + 4y = 4, \quad y(0)=0,\ y'(0)=2$	20
Ukupno:	5

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\,dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cos x\,dx = \sin x + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

2. $\int_2^3 \frac{2x^2+x+2}{x^2-1}\,dx$

$\int_2^3 \left(2 \cdot \frac{-x-2}{x^2-1}\right) dx$

$2\int_2^3 \left(\frac{-x-2}{\underbrace{(x-1)}_{A}\cdot\underbrace{(x+1)}_{B}}\right) dx$

$2\int_2^3 \left(\frac{A}{(x+1)}, \frac{B}{(x-1)}\right) dx$

$2\int_2^3 \left(\frac{2}{(x+1)} \cdot \frac{-2}{(x-1)}\right)$

$(\cancel{2}x^2 + x + 2) : (x^2-1) = 2$
$-\cancel{2}x^2 - 2x - 4$
$-x - 2$

$(x-1) + (x+1)$

$x_1 = 1$
$x_2 = -1$

$A = 1 + 1 = 2$
$B = -1 - 1 = -2$

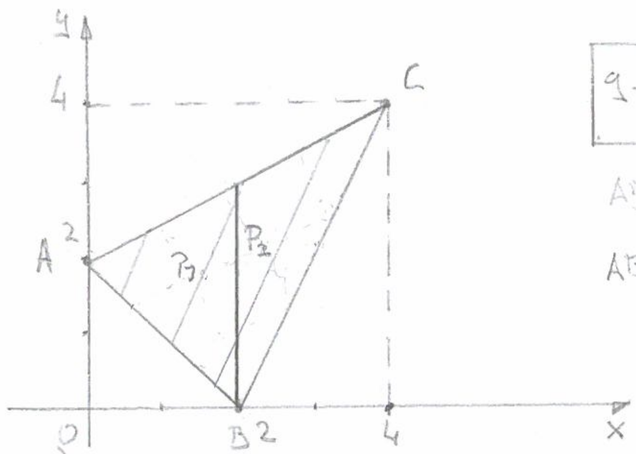
x	0
$y=-x+2$	2

x	2
$y=x-2$	0

x	2
$y=-x+2$	0

x	4
$y=x-2$	2

1. $A(0,2)$ $\quad B(\overset{x_1\ y_1}{2,0})$ $\quad ; C(\overset{x_2\ y_2}{4,4})$



$P = P_1 + P_2$

$P_1 = \int_0^2 (-x+2)\,dx$

$P_1 = -\int_0^2 x\,dx + 2\int_0^2 dx$

$= \left(-\frac{x^2}{2} + 2x\right)\Big|_0^2$

$= \left(-\frac{2^2}{2} + 2\cdot 2\right) - \left(-\frac{0^2}{2} + 2\cdot 0\right)$

$= 2$ kvadratnih jedinica

$P = 2 + 2 = 4$ kvad. jed.

$P_2 = \int_2^4 (x-2)\,dx$

$= \int_2^4 x - 2\int_2^4 dx$

$= \left(\frac{x^2}{2} - 2x\right)\Big|_2^4$

$= \left(\frac{4^2}{2} - 2\cdot 4\right) - \left(\frac{2^2}{2} - 2\cdot 2\right)$

$= 8 - 8 - 2 + 4$

$= 0 + 2 = 2$

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot x - x_1}$$

$AB = y - 2 = \frac{0-2}{2-0} \cdot x - 0$

$AB = y - 2 = \frac{-2}{2} \cdot x - 0$

$y - 2 = -x$

$y = -x + 2$

$BC = y - 0 = \frac{4-2}{4-2} \cdot x - 2$

$= y = \frac{2}{2} x - 2$

$= y = x - 2$

3. $\ln(x - y + 2)$

$x - y + 2 > 0$

$x - y > -2$

$D = \{(x,y) \in \mathbb{R}^2 / x - y > -2\}$

4. $f(x,y) = xy - x^3 - y^2$ $\boxed{D = \mathbb{R}^2}$

Bolo Matjašević

$$\frac{\partial f}{\partial x} = y - 3x^2$$

$y - 3x^2 = 0$

$y = 3x^2$ $y = 3 \cdot (-4)^2$

$= 48$

$T(-4, 48)$

$$\frac{\partial f}{\partial y} = x - 2y$$

$x - 2y = 0$

$x = 2y$

$x = 2 \cdot (-2)$

$x = -4$

?

$$\frac{\partial^2 f}{\partial^2 x} = -3 \cdot 2x = -6x$$

$$\frac{\partial^2 f}{\partial^2 y} = -2$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**
IME I PREZIME: SABOLIĆ BORIS   **BROJ INDEKSA:** 17-2-0010-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1.	Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,2)$, $B(2,0)$ i $C(4,4)$.	15
2.	Zadano je $f(x) = \dfrac{2x^2+x+2}{x^2-1}$. Odrediti $\int_2^3 f(x)\,\mathrm{d}x$.	15
3.	Grafički prikazati funkciju $f(x,y) = \ln(x-y+2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije.	15
4.	Istražiti domenu i ekstreme funkcije $f(x,y) = xy - x^3 - y^2$.	20
5.	Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1,1,z_0)$.	15
6.	Riješiti diferencijalnu jednadžbu: $9y'' - 6y' + y = xe^{-x}$ uz početne uvjete $y(0)=0$ i $y'(0)=1$. Provjeri rješenje.	20

$$y'' + 4y = 4, \quad y(0) = 0,\ y'(0) = 2$$

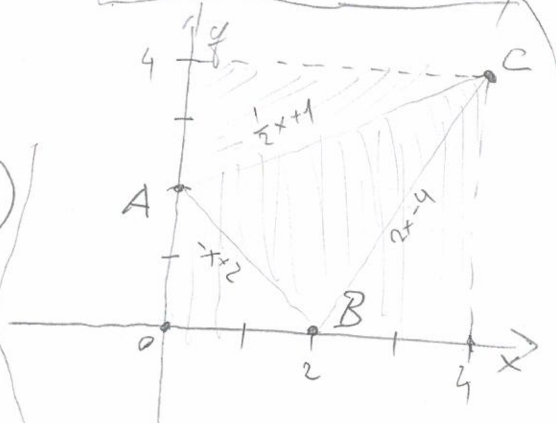
Ukupno: 0

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1. $A(0,2)$, $B(2,0)$, $C(4,4)$

$$\boxed{y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)}$$



AB:
$y - 2 = \frac{0-2}{2-0}(x-0)$
$y - 2 = -1x$
$y = -x + 2$

BC:
$y - 0 = \frac{4-0}{4-2}(x-2)$
$y = \frac{4}{2}(x-2)$
$y = 2(x-2)$
$y = 2x - 4$

AC:
$y - 2 = \frac{4-2}{4-0}(x-2)$
$y - 2 = \frac{2}{4}(x-2)$
$y - 2 = \frac{1}{2}(x-2)$
$y - 2 = \frac{1}{2}x - 1$
$y = \frac{1}{2}x - 1 + 2$

$y = \frac{1}{2}x + 1$

$P = \int_0^4 \left(\frac{1}{2}x+1\right)dx - \int_2^4 (2x-4)\,dx - \int_0^2 (-x+2)$

$P_1 = \left(\frac{1}{2}\cdot 4+1\right) - \left(\frac{1}{2}\cdot 0+1\right) = 2$

$P_2 = (2\cdot 4-4) - (2\cdot 2-4) = 4$

$P_3 = (-2+2) - (-0+2) = -2$

$P = 2 - 4 + 2 = 0\ ?$

④ $f(x,y) = xy - x^3 - y^2$

$$\left.\begin{aligned}\partial_x f &= y - 3x^2\\ \partial_y f &= x - 2y\end{aligned}\right\}$$

$y - 3x^2 = 0 \rightarrow y = 3x^2$

$x - 2y = 0$

$x - 6x^2 = 0$

$\partial_{xx} f = -6x \Rightarrow A$

$-6x^2 + x = 0$

$\partial_{xy} = 1 \Rightarrow B$

$x_{12} = \frac{-1 \pm \sqrt{1+24}}{-12} = \frac{-1+5}{-12} = \frac{4}{-12} = -\frac{1}{3} = x_1$

$\partial_{yy} = -2 \Rightarrow C$

$x_2 =$

$x_2 = \frac{-1-5}{-12} = \frac{-6}{-12} = \frac{1}{2}$

$y_1 = 3\left(-\frac{1}{3}\right)^2 = \frac{1}{3}$

$y_2 = 3 \cdot \left(\frac{1}{2}\right)^2 = \frac{3}{4}$

$$\boxed{\begin{aligned}&T_1\left(-\frac{1}{3}, \frac{1}{3}\right)\\ &T_2\left(\frac{1}{2}, \frac{3}{4}\right)\end{aligned}}$$

$T_1\left(-\frac{1}{3}, \frac{1}{3}\right)$

$A = -6 \cdot x = -6 \cdot \left(-\frac{1}{3}\right)$

$A = 2 > 0 \rightarrow \underline{min}$

$\Delta = A \cdot C - B^2$

$\Delta = 2 \cdot (-2) - 1$

$\Delta = -4 - 1$

$\Delta = -5 < 0$ – SEDLASTA TOČKA

– NEMA EXTREM

$T_2\left(\frac{1}{2}, \frac{3}{4}\right)$

$A = -6 \cdot \frac{1}{2} = -3 < 0$ $\underline{max}$

$\Delta = -3 \cdot (-2) - 1^2$

$\Delta = 5 > 0$ ima extrem

$f_{max} = \frac{1}{2} \cdot \frac{3}{4} - \left(\frac{1}{2}\right)^3 - \left(\frac{3}{4}\right)^2$

$f_{max} = -\frac{5}{16} \approx -0.31$

(2) $f(x) = \dfrac{2x^2+x+2}{x^2-1}$ $\qquad \displaystyle\int_2^3 f(x)\,dx$

$(x^2-1) = (x+1)(x-1)$

$$\int_2^3 \frac{2x^2+x+2}{x^2-1}\,dx = \int \frac{2x^2+x+2}{(x+1)}\,dx + \int \frac{2x^2+x+2}{x-1}\,dx$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: TOMISLAV TUTA **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,2)$, $B(2,0)$ i $C(4,4)$. ~~15~~
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$$y''+4y=4,\quad y(0)=0,\ y'(0)=2$$

Ukupno: 0

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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1.) $(x_2-x_1)\cdot(y-y_1)=(y_2-y_1)\cdot(x-x_1)$

$J_{AB}=(2-0)\cdot(y-2)=(0-2)\cdot(x-0)$

$=2y-4=-2x$

$2y=-2x+4/:2$

$y=-x+2$

$J_{AC}=(4-0)\cdot(y-2)=(4-2)(x-0)$

$=4y-8=2x$

$4y=2x+8/:4$

$y=\frac{1}{2}x+2$

$J_{BC}=(4-2)\cdot(y-0)=(4-0)\cdot(x-2)$

$=2y=4x-8/:2$

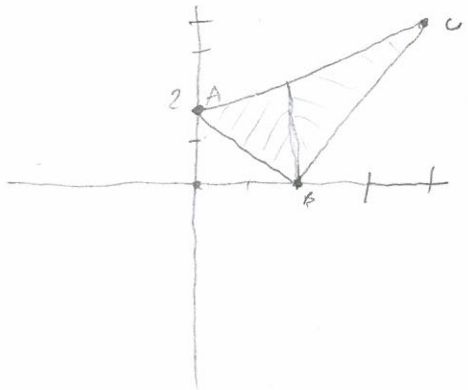
$=y=2x-4$

3. $f(x,y) = \ln(x-y+2)$

$$x-y+2 > 0$$

$$x-y > -2$$

1.1



$$\int_0^2 \left(\frac{1}{2}x+2-(-x+2)\right)$$

$$\int_0^2 \left(\frac{1}{2}x+2+x+2\right)$$

$$\int_0^2 \left(\frac{5}{2}x+4\right) = \Big|_0^2 \frac{5}{2}\left|\frac{x^2}{2}\right| + 4x = \frac{5}{2}(2)+8 = 13$$

$$\int_2^4 \left(\frac{1}{2}x+2-(2x-4)\right) = \int_2^4 \left(\frac{1}{2}x+2-2x+4\right) = \int_2^4 \left(-\frac{3}{2}x+6\right) =$$

$$= \Big|_2^4 -\frac{3}{2}\left|\frac{x^2}{2}\right| + 6x = -\frac{3}{2}\left(\frac{16}{2}-\frac{4}{2}\right)+6(4-2) =$$

$$-\frac{3}{2}\left(\frac{12}{2}\right)+6(2) = -9+12 = 3$$

$P = P_1 + P_2 = 13+3 = 16$ X

4.) $f(x,y) = xy - x^3 - y^2$ $\quad D f(x) = \mathbb{R}$ ×

$$\frac{\partial f}{\partial x} = y - 3x^2 \qquad \frac{\partial^2 f}{\partial x^2} = -6x \qquad \begin{vmatrix} -6x & 1 \\ 1 & -2 \end{vmatrix}$$

$$\frac{\partial f}{\partial y} = x - 2y \qquad \frac{\partial^2 f}{\partial y^2} = -2$$

$$\Delta = -6x \cdot (-2) - 1$$

$$\frac{\partial^2 f}{\partial x \partial y} = 1 \qquad \Delta = 12x - 1$$

$$\Delta = 12 \cdot 0 - 1 = -1$$

NIJE EKSTREM I NEMA
COK. MAX I MIN

×

$$y - 3x^2 = 0 \qquad x - 2y = 0$$

$$3x^2 = y \qquad 2y = x$$

$$x^2 = y \qquad y = x$$

$$x = \sqrt{y}$$

$$0 - 2y = 0$$

$$2y = 0$$

$$x - 3x^2 = 0 \qquad y = 0$$

$$2x^2 = 0$$

$$x = 0$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: TONI PASTUOVIĆ **BROJ INDEKSA:** 0269068933

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,2)$, $B(2,0)$ i $C(4,4)$. ~~15~~
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Ukupno: 0

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$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

① $A(0,2), B(2,0)\ C(4,4)$

$\overline{AB} = (x_B - x_A)i + (y_B - y_A)j$

$\overline{AB} = 2i - 2j$

$\overline{BC} = (4-2)i + (4-0)j$

$\overline{BC} = 2i + 4j$

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} i & j \\ 2 & -2 \\ 2 & 4 \end{vmatrix} = i(8+4) - j(-4-8)$$

$$= 12i + 12j$$

$$|\vec{a}| = \sqrt{a \cdot a} = \sqrt{12^2 + 12^2} = \sqrt{288}$$

$$P = \frac{\sqrt{288}}{2} = \frac{\sqrt{144 \cdot 2}}{2} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$$

② $\int_{2}^{3} \frac{2x^2+x+2}{x^2-1} = \left| \begin{array}{l} x^2-1=t \\ 2x\cdot dx = dt \end{array} \right| =$

0

TONI PASTUOVIĆ MAT 2

(3) $f(x,y) = \ln(x-y+2)$

$Df: x-y+2>0$ Df

$x>y-2$

0

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: DENI MILETIĆ **BROJ INDEKSA:** 57143

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Integriranjem odrediti površinu trokuta koji je zadan točkama $A(0,2)$, $B(2,0)$ i $C(4,4)$. 15

2. Zadano je $f(x) = \dfrac{2x^2 + x + 2}{x^2 - 1}$. Odrediti $\int_3^2 f(x)\,\mathrm{d}x$. 15

3. Grafički prikazati funkciju $f(x,y) = \ln(x - y + 2)$ pomoću razinskih krivulja. Koja je domena i vrijednosti ove funkcije? Strelicama označiti smjer rasta funkcije. 15

4. Istražiti domenu i ekstreme funkcije $f(x,y) = xy - x^3 - y^2$. 20

5. Odredi tangencijalnu ravninu i normalu na plohu $z = \arctan(\frac{y}{x})$ u točki $M(1,1,z_0)$. 15

6. Riješiti diferencijalnu jednadžbu: $9y'' - 6y' + y = x\mathrm{e}^{-x}$ uz početne uvjete $y(0) = 0$ i $y'(0) = 1$. Provjeri rješenje. 20

$$y'' + 4y = 4, \quad y(0) = 0, \; y'(0) = 2$$

Ukupno: 0

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

2. $f(x) = \dfrac{2x^2+x+2}{x^2-1}$

$$x^2+x+2 : x^2-1 = 1 + \frac{x+3}{x^2-1}$$
$$\frac{-x^2 \mp 1}{x+3}$$

$$\int \frac{2x^2+x+2}{x^2-1}dx = 2\int \frac{x^2+x+2}{x^2-1}dx = 2\underbrace{\int 1\,dx}_{\mathrm{I}} + 2\underbrace{\int \frac{x+3}{x^2-1}dx}_{\mathrm{II}}$$

$$\Rightarrow \mathrm{I} \quad 2\int_3^2 dx = [2x]_3^2$$

$$x^2-1=0$$
$$x^2=1$$

$$\Rightarrow \mathrm{II} \quad \int \frac{x+3}{x^2-1}dx = \int \frac{x+3}{(x-1)(x+1)}dx = \frac{A}{x-1} + \frac{B}{x+1} \Big/ \cdot (x-1)(x+1) \quad x_{1,2} = \pm 1 \Rightarrow (x-1)(x+1)$$

$x+3 = A(x+1) + B(x-1)$

$x+3 = Ax + A + Bx - B$

za x; $A+B=1 \Rightarrow A=-B+1 \Rightarrow A=-4+1=-3$

za 1; $A-B=3 \Rightarrow -B+1-B=3$

$B=4$

$$\Rightarrow \int \frac{-3}{x-1}dx + \int \frac{4}{x+1}dx = -3\int \frac{dx}{x-1} + 4\int \frac{dx}{x+1} = 2\cdot\Big[-3\ln|x-1| + 4\ln|x+1|\Big]_3^2$$

$$\Rightarrow \ldots = 2[2x]_3^2 + 2\cdot[-3\ln|x-1|]_3^2 + 2[4\ln|x-1|]_3^2 = 2[2\cdot 3 - 2\cdot 2] + 2[-3\ln|3-1| + 3\cancel{\ln|2-1|}^{\,0}]$$
$$+ 2[4\ln[3-1] - 4\cancel{\ln[2-1]}^{\,0}] = 4 + 2[2{,}079] + 2\cdot[2{,}772] = 13{,}702$$

4. $f(x,y) = xy - x^3 - y^2$ DOMENA?

MIKETIC DENI

$T_1(0,0)$ $T_2(0.70, 0.35)$

$T_3(-0.70, -0.35)$

$\partial f_x = y - \overset{3}{2}x^2 \Rightarrow y - 2x^2 = 0 \Rightarrow y - 2\cdot(2y)^2 = 0$

$\partial f_y = x - 2y \Rightarrow x = 2y$

$y - 2\cdot 4y^2 = 0$

$y - 8y^3 = 0$

$y(1 - 8y^2) = 0$

$x_1 = 0$

$x_2 = 2\cdot\sqrt{\frac{1}{8}}$

$x_3 = 2\cdot\left(-\sqrt{\frac{1}{8}}\right)$

$y_1 = 0$ $\quad y_2 = 8y^2 = 1 \Rightarrow 8y^2 = 1 /:8$

$y^2 = \frac{1}{8} /\sqrt{\ }$

$y_{2,3} = \pm\sqrt{\frac{1}{8}}$

$\partial_{xx} f = -4x$

$\partial_{xy} f = 1$

$\partial_{yy} f = -2$

uvjeti:

1. za $T(0,0)$ $\quad D_2 = -1 < 0 \Rightarrow$ NIJE EKSTREM

$\partial_{xx} f = -4x \Rightarrow A = 0$ neznamo imali ekstrem u ovoj točki

2. za $T(0.70, 0.35)$ $\quad \Delta_2 = AC - B^2 = 4.6$

$\partial_{xx} f = -4\cdot 0{,}70 = -2.8 < 0$

3 za $T(-0.70, -0.35)$ $\quad \Delta_3 = AC - B^2 = -7.6 < 0$

$\partial_{xx} f = -4\cdot(-0.70) = 2.8 > 0$ u ovoj točki imamo minimum