MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** Antun Žanetić **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$. 20
2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = x^2 + y^2$ u točki $T(1, -2, z_0)$. ~~15~~
4. Analitički ispitati skalarnu funkciju $f(x, y) = (x-1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta. 10+~~10~~
5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. ~~20~~
6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. 15

Ukupno: 50

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2\pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Antun Žanetić

$\boxed{1.}$

$$\int_0^1 \frac{dx}{1+\sqrt{x}} = \left| \begin{array}{l} x = t^2 \;/' \\ dx = 2t\,dt \\ \begin{array}{c|c|c} x & 0 & 1 \\ \hline t & 0 & 1 \end{array} \end{array} \right| = \int_0^1 \frac{2t\,dt}{1+\sqrt{t^2}} = 2\int_0^1 \frac{t}{1+t}\,dt =$$

$$= 2\int_0^1 \frac{t+1-1}{1+t}\,dt = 2\int_0^1 \frac{\cancel{t+1}}{\cancel{t+1}}\,dt - 2\int_0^1 \frac{1}{t+1}\,dt =$$

$$= 2\int_0^1 dt - 2\int_0^1 \frac{dt}{t+1} = \left(2t - 2\ln|t+1|\right)\Big|_0^1 =$$

$$= \left(2\cdot 1 - 2\ln|1+1|\right) - \left(2\cdot 0 - 2\ln|0+1|\right) = \left(2 - 2\ln|2|\right) - \left(0 - 2\ln|1|\right) =$$

$$= 2 - 2\ln|2| \approx 0.6137056389$$

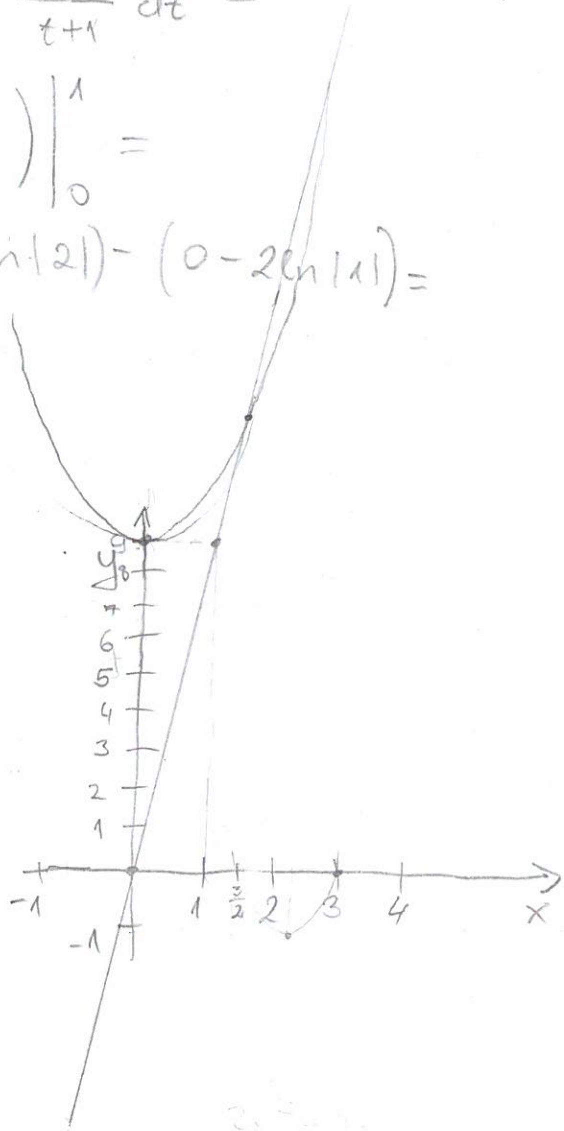
$\boxed{2.}$

$y = 2x^2 + 9$

$y = 9x$

$y = 9x$

x	0	1
y	0	9



$2x^2 + 9 = 9x$

$2x^2 - 9x + 9 = 0$

$$x_{1,2} = \frac{9 \pm \sqrt{81-72}}{4}$$

$$x_{1,2} = \frac{9 \pm \sqrt{9}}{4} = \frac{9 \pm 3}{4}$$

$x_1 = 3$

$x_2 = \frac{3}{2}$

$-\frac{b}{2a}\ ,\ \frac{4ac-b^2}{4a}$

$T(0, 9)$

$$P = \int_{\frac{3}{2}}^{3} \left[(9x) - (2x^2+9)\right] dx = \int_{\frac{3}{2}}^{3} (9x - 2x^2 - 9)\,dx = \int_{\frac{3}{2}}^{3} (-2x^2 + 9x - 9)\,dx =$$

$$= -2\int_{\frac{3}{2}}^{3} x^2\,dx + 9\int_{\frac{3}{2}}^{3} x\,dx - 9\int_{\frac{3}{2}}^{3} dx = \left(-2\cdot\frac{x^3}{3} + \frac{9x^2}{2} - 9x\right)\Big|_{\frac{3}{2}}^{3} =$$

$$= -\frac{9}{2} - \left(-\frac{45}{8}\right) = \frac{9}{8} = 1.125$$

3. $z = x^2 + y^2$

$T(1, -2, z_0)$

$x_0 = 2 \cdot 1 = 2$

$y_0 = 2 \cdot (-2) = -4$

$f_x(T) = \dfrac{\partial f}{\partial x} = 2x$

$f_y(T) = \dfrac{\partial f}{\partial y} = 2y$

$z_0 = x_0^2 + y_0^2$

$z_0 = 2^2 + (-4)^2$

$z_0 = 4 + 16 = 20$

$f_x(T) = 2 \cdot 1$

$f_y(T) = 2 \cdot (-2)$

$z - z_0 = f_x(T) \cdot (x - x_0) + f_y(T) \cdot (y - y_0)$

$\boxed{z - 20 = 2x \cdot (x - 2) + 2y \cdot (y + 4)}$ → Jednadžba tangencijalne ravnine

$z - 20 = 2x^2 - 4x + 2y^2 + 8y$

$z = 2x^2 - 4x + 2y^2 + 8y + 20$

$$\frac{x - x_0}{f_x(T)} = \frac{y - y_0}{f_y(T)} = \frac{z - z_0}{-1}$$

$$\frac{x - 2}{2x} = \frac{y + 4}{2y} = \frac{z - 20}{-1}$$

$$\boxed{\frac{x - 2}{2x} = \frac{y + 4}{2y} = -z + 20}$$ Normala

4. $f(x, y) = (x-1)^2 - y^2$

$(x-1)^2 - y^2 = 0$

$x^2 - 2x + 1 - y^2 = 0$

$-y^2 = -x^2 + 2x - 1$

$y^2 = x^2 - 2x + 1$

$y = \sqrt{x^2 - 2x + 1}$

$x^2 - 2x + 1 \geq 0$

$x_{1,2} = \dfrac{2 \pm \sqrt{4 - 4}}{2}$

$x_{1,2} = \dfrac{2 \pm 0}{2}$

$x_{1,2} = 1, \quad y = 0$

$(x-1)^2 - y^2 = C$

$x^2 - 2x + 1 - y^2 = C$

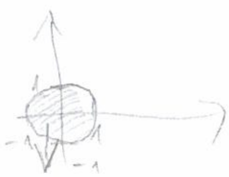
$-y^2 = C - x^2 + 2x - 1$

$y^2 = x^2 - 2x + 1 - C$

$\boxed{y = \sqrt{x^2 - 2x + 1 - C}}$

SKUP SVIH RAZINSKIH KRIVULJA

ELIPSA



Antun Žanetić

5.

$$y'' - y = -x + 1$$

$$y'' - y = 0$$

$a = 1$
$b = 0$
$c = -1$

$y = e^{kx}$

$k^2 - 1 = 0$

$y_1 = e^{x}$

$k^2 = 1$

$y_2 = e^{-x}$

$k_1 = 1$

$k_2 = -1$

$$\boxed{y_H = c_1 \cdot e^{x} + c_2 \cdot e^{-x}} \checkmark$$

$f(x) = -x + 1$

$y_p = -Ax + B \quad /'$

$y_p' = -A \quad /'$

$y_p'' = 0$

$0 - (-Ax + B) = -x + 1$

$0 + Ax - B = -x + 1$

$\boxed{A = -1}$

$-B = 1$

$\boxed{B = -1}$

$y' = 0$

$y = y_H + y_P$

$$\boxed{y = C_1 \cdot e^{x} + C_2 \cdot e^{-x} + x - 1} \checkmark$$

$C_1 \cdot (e^{x})' \cdot x' + C_2 \cdot (e^{-x})' \cdot (-x)' + x'$

Uvjeti:
$x = 0$
$y = 0$
$y' = 0$

$x=0 \Rightarrow c_1 - c_2 + 1 = 0$

$C_1 \cdot e^{x} - C_2 \cdot e^{-x} + 1 = 0$ $\boxed{c_1 - c_2 = -1}$

$C_1 \cdot e^{0} + C_2 \cdot e^{-0} + 0 - 1 = 0$

$C_1 \cdot e^{x} - (1 - C_1) \cdot e^{-x} + 1 = 0$

$C_1 + C_2 - 1 = 0$

$\boxed{C_1 + C_2 = 1} \checkmark$

$C_1 \cdot e^{x} - e^{x} \cdot C_1 \cdot e^{-x} + 1 = 0$

$C_1 \cdot e^{x} - C_1 \cdot e^{x} \cdot e^{-x} + 1 = 0$

$C_2 = 1 - C_1$

$C_1 \cdot e^{x} - C_1 \cdot e^{-2x} + 1 = 0 \quad / \ln$

$C_1 \cdot e^{x} - C_1 \cdot e^{-2x}$

$\ln(C_1 \cdot e^{x}) - \ln(C_1 \cdot e^{-2x}) + 0 = 1$

$C_1 \cdot x - C_1 \cdot (-2x) = 1$ ✗

$\boxed{6}$

$xy' + y - e^x = 0$

$y(1) = 1$

$\boxed{xy' + y = e^x}$

$xy' + y = 0$

$x \cdot \frac{dy}{dx} = -y \quad / : x \; / \cdot dx$

$dy = -\frac{y}{x} dx \quad / : y$

$\frac{dy}{y} = -\frac{dx}{x} \quad / \int$

$\int \frac{dy}{y} = -\int \frac{dx}{x}$

$\ln|y| = -\ln|x| + C$

$\ln|y| = -\ln|x \cdot c| \quad / e^{\square}$

$e^{\ln|y|} = e^{\ln \frac{c}{x}}$

$y = \frac{c}{x}$

$C = C'(x)$

$y = \frac{C(x)}{x} \; /'$

$y' = \frac{C'(x) \cdot x - C(x) \cdot x'}{x^2}$

$y' = \frac{C'(x) \cdot x - C(x)}{x^2}$

$\cancel{x} \cdot \left(\frac{C'(x) \cdot x - C(x)}{x^{\cancel{2}}} \right) + \frac{C(x)}{x} - e^x = 0$

$\frac{C'(x) \cdot x - C(x)}{x} + \frac{C(x)}{x} = e^x$

$\frac{C'(x) \cdot \cancel{x}}{\cancel{x}} - \cancel{\frac{C(x)}{x}} + \cancel{\frac{C(x)}{x}} = e^x$

$C'(x) = e^x \quad / \int$

$C(x) = \int e^x dx$

$C(x) = e^x + C$

$\boxed{y = \frac{e^x + C}{x}}$ ✓

UVJET

$y(1) = 1$

$\frac{e^1 + C}{1} = 1$

$e + C = 1$

$\boxed{C = 1 - e}$ ✓

PROVJERA: $y = \frac{e^x + 1 - e}{x}$

$y' = \frac{xe^x - e^x - 1 + e}{x^2}$

odj:

$\cancel{x} \cdot \frac{\cancel{xe^x} - \cancel{e^x} - \cancel{1} + \cancel{e}}{x^{\cancel{2}}} + \frac{\cancel{e^x} + \cancel{1} - \cancel{e}}{x} - \frac{\cancel{xe^x}}{x} = 0$ ✓

$y(1) = \frac{e^1 + 1 - e}{1} = 1$ ✓

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Ukupno: 55

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\, dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int e^x\, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2\pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

① $\int_0^1 \frac{dx}{1+\sqrt{x}} = \left[\begin{matrix} x = t^2 \\ dx = 2t\,dt \end{matrix}\right] = \int \frac{dt\, 2t}{1+t} = 2\int \frac{1+t-1}{1+t}\, dt =$

$= 2\int \frac{1+t}{1+t}\, dt + 2\int \frac{1}{1+t}\, dt$

$= 2t - 2 \cdot \ln|1+t| \Big|_0^1 = 2(\sqrt{x}) - 2\cdot\ln|1+\sqrt{x}|$

$= 2(\sqrt{1}) - 2\cdot\ln|1+\sqrt{1}| - \left(2(\sqrt{0}) - 2\cdot\ln|1+\sqrt{0}|\right)$

$= 2 - 1.386 - 0$

$= 0.6137$ ✓

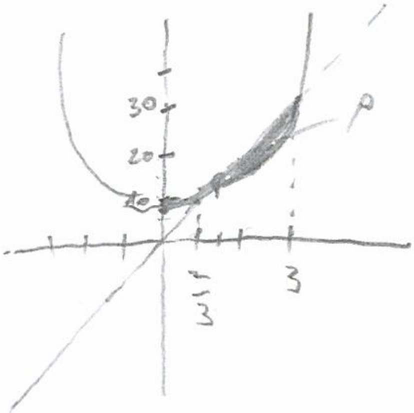
② $Y=2x^2+9$ $\quad Y=9x$

$x_{1,2}=\frac{9\pm\sqrt{81-72}}{4}$?

$x_{1,2}=\frac{9\pm3}{4}$ $\quad x_1=3$ $\quad x_2=\frac{2}{3}$ $\frac{3}{2}$

$2x^2+9=9x$

$2x^2-9x+9=0$



$P=\int_{\frac{2}{3}}^{3} 9x-2x^2-9$?

$P=9\frac{x^2}{2}-2\frac{x^3}{3}-9x\Big|_{\frac{2}{3}}^{3}$?

$P=9\frac{3^2}{2}-2\frac{3^3}{3}-9\cdot3-\left(9\cdot\frac{\left(\frac{3}{2}\right)^2}{2}-2\frac{\left(\frac{3}{2}\right)^3}{3}-9\cdot\frac{2}{3}\right)$?

$P=-\frac{9}{2}+\frac{45}{8}$

$P=\frac{9}{8}$?

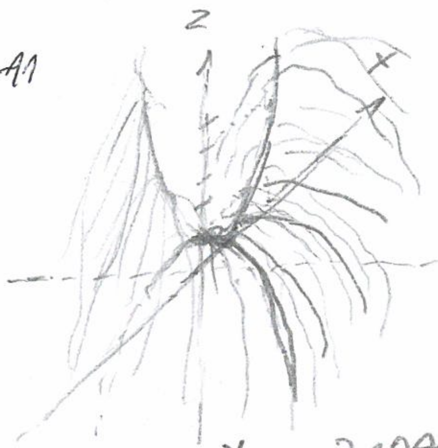
JURE MIČIĆ

17-1-0083-11

④ $f(x,y) = (x-1)^2 - y^2$

$\frac{\partial f}{\partial x} = 2x-2 \qquad \frac{\partial f}{\partial x} = -2y$

$\frac{\partial^2 f}{\partial x^2} = 2 \qquad \frac{\partial^2 f}{\partial y} = -2$



$2x-2=0 \qquad -2y=0$

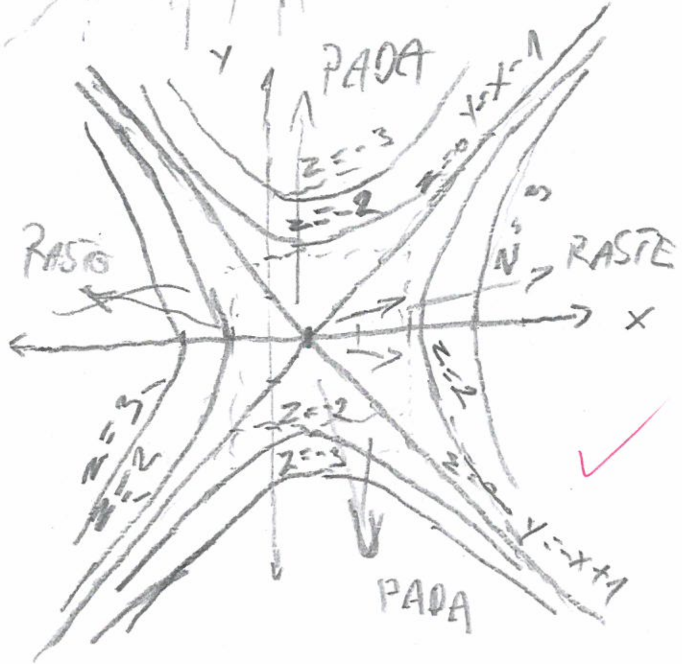
$-2=-2x \,/:2 \qquad y=0$

$x=1$

$T(1,0)$

$$\begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = -4 < 0$$

T(1,0) JE SEDLASTA TOČKA ✓



$(x-1)^2 - y^2 = 0$

$-y^2 = -(x-1)^2$

$y_1 = x-1$

$y_2 = -x+1$

$(x-1)^2 - y^2 = 2 \,/:2$

$\frac{(x-1)^2}{2} - \frac{y^2}{2} = 1$ – HIPERBOLA

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⑥ $xy' + y - e^x = 0$

$xy' + y = e^x \,/:x$

$y' + \frac{y}{x} = \frac{e^x}{x}$

$y' + \frac{y}{x} = 0$

$\frac{dy}{dx} = -\frac{y}{x} \,/ \cdot \frac{dx}{y}$

$\frac{dy}{y} = -\frac{dx}{x}$

$\ln|y| = -\ln|x| + \ln|c|$

$y = \frac{c}{x}$

$y = \frac{c(x)}{x}$

$y' = \frac{c'(x) \cdot x - c(x)}{x^2}$

$\cancel{x} \cdot \frac{c'(x) \cdot \cancel{x} - \cancel{c(x)}}{x^2} + \frac{\cancel{c(x)}}{x^2} = \frac{e^x}{x}$

$c'(x) = \frac{e^x}{x} \,/\int dx \qquad x = \ln(t)\,' \quad dx = \frac{1}{t}dt$

$C(x) = \int \frac{e^{\ln(t)}}{\ln(t)} \cdot \frac{1}{t}\,dt$

$C(x) = \int \frac{t}{\ln t} \cdot \frac{1}{t}$

$C(x) = \int \frac{1}{\ln(t)} \qquad \int \begin{matrix} \ln(t) = u \,' \\ \frac{1}{x}\,dx = du \end{matrix}$

$\int e^x \cdot \frac{1}{x}\,dx = \left[\begin{matrix} \frac{1}{x} = u' \\ \frac{1}{x}dx = du \end{matrix} \right.$

$e^x \cdot \ln|x| - \int \ln|x| \cdot e^x\,dx$

$e^x = t$

$e^x \ln|x| - \int$

$\frac{e^x}{x} = -\frac{1 e^x}{x} + \int e^x \cdot \frac{1}{x^2}\,dx$

$\frac{e^x}{x} = \frac{e^x}{x^2} - \frac{e^x}{x}\,dx$

$\frac{e^x}{x} \int \frac{e^x - xe^x}{x^2}$

RUBNI UVJET ?

⑤ $y'' - y = -x + 1$

$r^2 - 1 = 0$ $\qquad Y_H = C_1 e^{x} + C_2 e^{-x}$ ✓

$r_1 = 1 \quad r_2 = -1$

$-x + 1 = e^{\alpha x}\left(P_n \cos(\beta x) + Q_n \sin(\beta x)\right)$

$\alpha = 0$

$\beta = 0$

$P_n = -1$

$Q_n = 0$

$2B + 2x - xA + Bx^2 = -x + 1$

$B = 0$

$x(2 - A) = -x + 1$

$2 - A = -1$

$-A = -3$

$A = 3$

$Y_p = x^{0} e^{\alpha x}\left(P_n \cos(\beta x) + Q_n \sin(\beta x)\right)$

$Y_p = \cancel{x} 1 \cdot (A + Bx)$

$Y_p' = A + Bx + x \cdot B$

$= 2xB + A$

$Y_p'' = 2(B + x)$

$= 2B + 2x$

DALJE...

RUBNI UVJETI?

$z = x^2 + y^2 \qquad T(1, -2, z_0)$

$\Pi_1 = \frac{\partial f(1,-2)}{\partial x} \cdot x + \frac{\partial f(1,-2)}{\partial y} y$ ✗

$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial x} = 2y$

$z = 2 \cdot 1 \cdot x + 2 \cdot -2 \cdot y$

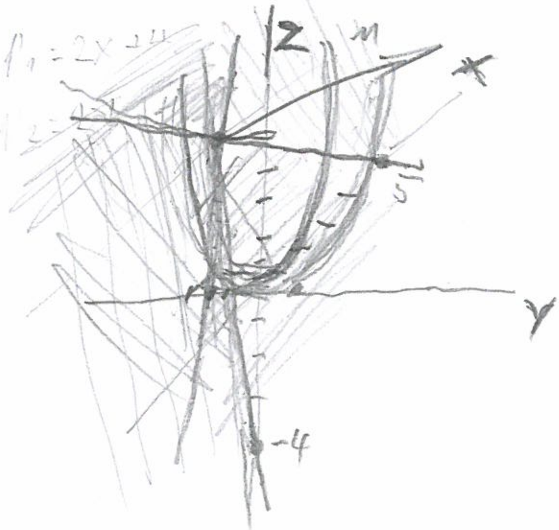
$z = 2x - 4y \qquad 2x - 4y - z = 0 \,/ : 4$

$\frac{x}{2} + \frac{y}{-1} + \frac{z}{-4} = 0$

$z = 1^2 + (-2)^2$

$z = 5$

$T(1, -2, 5)$



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IME I PREZIME: Ivan Kovačević **BROJ INDEKSA:** 17-2-0125-2012

POPUNJAVA NASTAVNIK
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Ukupno: 50

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \cot x \,\mathrm{d}x = \ln\|\sin x\| + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x \,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sinh x \,\mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \cosh x \,\mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

3) $z = x^3 + y^2$ Ivan Karačević

$T = (1, -2, z_0)$ $z_0 = (1)^3 + (-2)^2 = 5$

$\frac{df}{dx} = 2x$ $\frac{df}{dy} = 2y$

$z - z_0 = \frac{df}{dx}(x_0, y_0)(x - x_0) + \frac{df}{dy}(x_0, y_0) \cdot (y - y_0)$

$z - 5 = 2 \cdot (x - 1) \cdot (4) \cdot (y + 2)$ ✗ $5-2-8=-5$ ✓

$+$ $-$

tangencijalna ravnina $2x - 4y - z - 5 = 0$ ✓

$\frac{x - x_0}{\frac{df}{dx}(x_0, y_0)} = \frac{y - y_0}{\frac{df}{dy}(x_0 y_0)} = \frac{z - z_0}{-1}$

$\frac{x - 1}{2} = \frac{y + 2}{-4} = \frac{z - 5}{-1}$ normala na ~~ravninu~~ pravac ✓

2.) Izračunaj površinu lika omeđenog

Porobolu $\Rightarrow y = 2x^2 + 9$

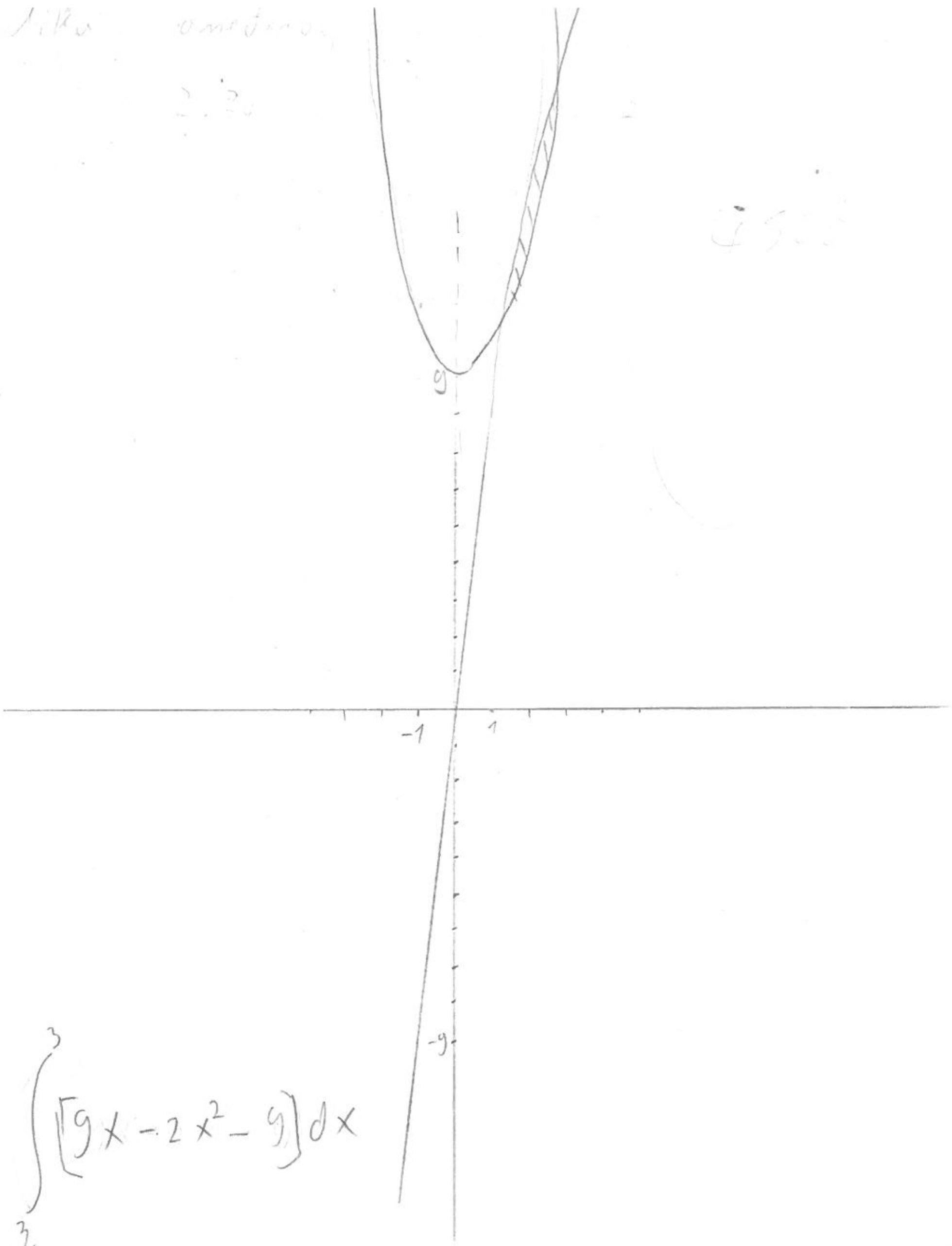
Provoć $\Rightarrow y = 9x$

Provoć

x	-1	0	1
y	-9	0	9

Porobolu

x	-2	-1	0	1	2
y	17	11	9	11	17



$2x^2 + 9 = 9x$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \dfrac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 9}}{4}$

$x_1 = 3 \qquad x_2 = \frac{3}{2}$

$$\int_{\frac{3}{2}}^{3} \left[(9x) - (2x^2 + 9)\right] dx = \int_{\frac{3}{2}}^{3} \left[9x - 2x^2 - 9\right] dx$$

$$= \left[\frac{-2x^3}{3} + \frac{9x^2}{2} - 9x\right]\Bigg|_{\frac{3}{2}}^{3} = \left[\left(\frac{-2 \cdot 3^3}{3} + \frac{9 \cdot 3^2}{2} - 9 \cdot 3\right) - \left(\frac{-2 \cdot 1.5^3}{3} + \frac{9 \cdot 1.5^2}{2} - 9 \cdot 1.5\right)\right.$$

$$= \frac{9}{8} \checkmark$$

1) $\int_0 \frac{dx}{1+\sqrt{x}} = \left| dx = 2t\,dt \right| = \int_0^1 \frac{2t\,dt}{1+t}$ Ivan Kovačević

$= 2\int_0^1 \frac{t}{1+t}\,dt = \left| \begin{matrix} 1+t = u \\ dt = du \end{matrix} \right|$

$= 2\int_1^2 \frac{u-1}{u}\,du = 2\int_1^2 \left(\frac{u}{u} - \frac{1}{u}\right) du$

$= 2\int_1^2 \left(1 - \frac{1}{u}\right) du = 2\cdot(u - \ln|u|)\Big|_1^2$

$= 2\cdot\left[(2-\ln 2) - (1-\ln 1)\right] = 0.6137$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: TENA KRMPOTIĆ **BROJ INDEKSA:** 5970

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$. — 20 / 16
2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. — 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = x^2 + y^2$ u točki $T(1, -2, z_0)$. — 15
4. Analitički ispitati skalarnu funkciju $f(x, y) = (x-1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta. — 10+10
5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. — 20
6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. — 15

Ukupno: 46

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\, dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

(1) $\int_0^1 \frac{dx}{1+\sqrt{x}} = \left| \begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right| \begin{array}{c|c} x & t \\ 1 & 1 \\ 0 & 0 \end{array}$

$\int \frac{2t\,dt}{1+\sqrt{t^2}} = 2\int \frac{t}{1+t}\,dt = 2\int \frac{t+1-1}{1+t}\,dt = 2\int \frac{t+1}{1+t}\,dt + 2\int \frac{-1}{1+t}\,dt$

$= 2\int dt + 2\int \frac{-1}{1+t}\,dt$

$= 2t \underbrace{- 2\int \frac{dt}{1+t}}_{*}$

$= 2\sqrt{x}\Big|_0^1 - 2\ln\lvert 1+\sqrt{x}\rvert\Big|_0^1$

$* \; -2\int \frac{dt}{1+t} \quad \left| \begin{array}{l} 1+t = u \\ dt = du \end{array} \right|$

$-2\int \frac{du}{u}$

$-2\ln\lvert u\rvert$

$-2\ln\lvert 1+t\rvert$

$-2\ln\lvert 1+x\rvert$

→ OSTATAK NA DRUGOJ STRANI.

$= 2x \Big|_0^1 - 2\ln|1+x| \Big|_0^1$

$= 2\cdot(1-0) - 2\ln|1+1| - 2\ln|1-0|$

$= 2 - 2\ln|2| - 2\ln|1|$

$= 2 - 2\ln|2| - 0$ ✓ ~~$\ln 2$~~

$= 2 - 2\ln|2| + C$ ✗

ODREĐENI INTEGRAL JE BROJ !!!

$\underline{16}$

ZBOG DODAVANJA NEODREĐENE KONST. ODUZETA 4 BODA.

(4) Domena svi $\mathbb{R}$ krivi!

(3)

$z = x^2 + y^2$ $\qquad T(1, -2, z_0)$

$z_0 = x^2 + y^2$
$= 1^2 + (-2)^2$
$= 1 + 4$
$= 5$

$T(1, -2, 5)$

$\frac{df}{dx} = 2x \qquad f_{(T)} = 2 \cdot 1$
$\underline{f(T) = 2}$ ✓

$\frac{df}{dy} = 2y \qquad f(T) = 2 \cdot (-2)$
$\underline{= -4}$ ✓

$z - z_0 = f(x)T(x - x_1) + f(y)T(y - y_1)$
$z - 5 = 2(x - 1) - 4(y + 2)$
$z - 5 = 2x - 2 - 4y - 8$
$z - 5 = 2x - 4y - 10$ ✓

$2x - 4y - 10 - z + 5$
$\underline{2x - 4y - z - 5}$

$\underline{\pi \ldots\ 2x - 4y - z - 5 = 0}$

$\frac{x - x_0}{f(x)T} = \frac{y - y_0}{f(x)T} = \frac{z - z_0}{-1}$

$\underline{\frac{x - 1}{2} = \frac{y + 2}{-4} = \frac{z - 5}{-1}}$ ✓

TENA KRUPSTIU

(5)

$y = 2x^2 + 9 \quad ; \quad y = 9x$

$2x^2 + 9 = 9x$

$2x^2 + 9 - 9x = 0$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \dfrac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 9}}{4}$

$= \dfrac{9 \pm \sqrt{81 - 72}}{4}$

$= \dfrac{9 \pm \sqrt{9}}{4}$

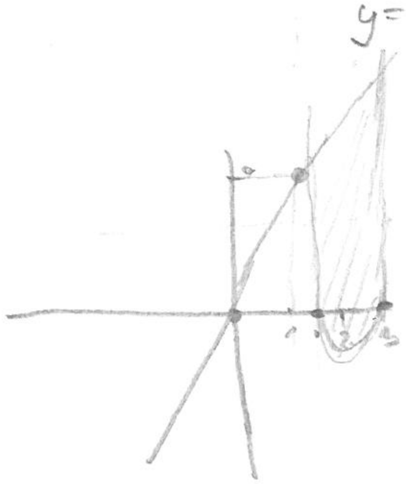
$= \dfrac{9 \pm 3}{4}$

$x_1 = \dfrac{9 - 3}{4} \qquad x_2 = \dfrac{9 + 3}{4}$

$x_1 = \dfrac{6}{4} \qquad x_2 = \dfrac{12}{4}$

$\boxed{x_2 = 3}$

$\boxed{x_1 = \dfrac{3}{2}}$



$y=9x$	x
0	0
9	1

$y = 2x^2 + 9$

$y_{1,2} = \dfrac{0 \pm \sqrt{0 - 4 \cdot 2 \cdot 9}}{4}$

$y_{1,2} = \dfrac{0 \sqrt{-72}}{4}$

$P = \int_{\frac{3}{2}}^{3} 9x - 2x^2 + 9 \, dx$

$P = \int_{\frac{3}{2}}^{3} 9x \, dx - \int_{\frac{3}{2}}^{3} 2x^2 \, dx + \int_{\frac{3}{2}}^{3} 9 \, dx$

$P = 9 \int_{\frac{3}{2}}^{3} x \, dx - 2 \int_{\frac{3}{2}}^{3} x^2 \, dx + 9 \int_{\frac{3}{2}}^{3} dx$

$= 9 \dfrac{x^2}{2} \Big|_{\frac{3}{2}}^{3} - 2 \dfrac{x^3}{3} \Big|_{\frac{3}{2}}^{3} + 9x \Big|_{\frac{3}{2}}^{3}$

$= \dfrac{9}{2} x^2 \Big|_{\frac{3}{2}}^{3} - \dfrac{2}{3} x^3 \Big|_{\frac{3}{2}}^{3} + 9x \Big|_{\frac{3}{2}}^{3}$

$= \dfrac{9}{2}\left(3^2 - \left(\dfrac{3}{2}\right)^2\right) - \dfrac{2}{3}\left(3^3 - \left(\dfrac{3}{2}\right)^3\right) + 9\left(3 - \dfrac{3}{2}\right)$

$= \dfrac{9}{2}\left(9 - \dfrac{9}{4}\right) - \dfrac{2}{3}\left(27 - \dfrac{27}{8}\right) + 9\left(\dfrac{3}{2}\right)$

$= \dfrac{9}{2}\left(\dfrac{27}{4}\right) - \dfrac{2}{3}\left(\dfrac{189}{8}\right) + \dfrac{27}{2}$

$= \dfrac{243}{8} - \dfrac{63}{4} + \dfrac{27}{2}$

$= \dfrac{117}{8} - \dfrac{27}{2} = \boxed{\dfrac{9}{8}}$ ✓

(5) $y'' - y = -x + 1$

$r^2 - r = 0$ X

$\boxed{r^2 - 1 = 0}$

$r(r-1) = 0$

$\boxed{r_1 = 0}$

$r - 1 = 0$

$\boxed{r_2 = 1}$

$y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$

$= C_1 e^{0x} + C_2 e^{1x}$

$= C_1 + C_2 e^{x} + 1$ X

$f(x) = -x + 1$

$Y = Ax^2 + B$

$Y' = 2Ax$

$Y'' = A$

$Y = 0x^2 + 1$

$Y = 1$

$A - Ax^2 + B = -x + 1$

$-A = 0$ $\boxed{A = 0}$

$A + B = 1$

$B = 1 - A$

$B = 1 - 0$

$\boxed{B = 1}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	POPUNJAVA NASTAVNIK
IME I PREZIME: MATE ĆOSIĆ **BROJ INDEKSA:** 55924	Broj ↓ bodova

1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$. — 20
2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. — 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = x^2 + y^2$ u točki $T(1, -2, z_0)$. — 15
4. Analitički ispitati skalarnu funkciju $f(x, y) = (x-1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta. — 10+10
5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. — 20
6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. — 15

Ukupno: 15

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

2. $y = 2x^2 + 9$ $\qquad y = 9x$

$4x = 0$

$x = 0$

$y = 9$

$T(0, 9)$

$2x^2 + 9 = 9x$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{9 \pm \sqrt{9^2 - 4\cdot 2\cdot 9}}{2\cdot 2} = \frac{9 \pm \sqrt{9}}{4}$

$S_1 \begin{cases} x_1 = 3 \\ y_2 = 27 \end{cases} \qquad \begin{rcases} x_2 = \frac{3}{2} \\ y_2 = \frac{27}{2} \end{rcases} S_2$

$\int_{\frac{3}{2}}^{3} 9x\,dx - \int_{\frac{3}{2}}^{3} 2x^2 + 9\,dx$

$2x^2 + 9 = 0$

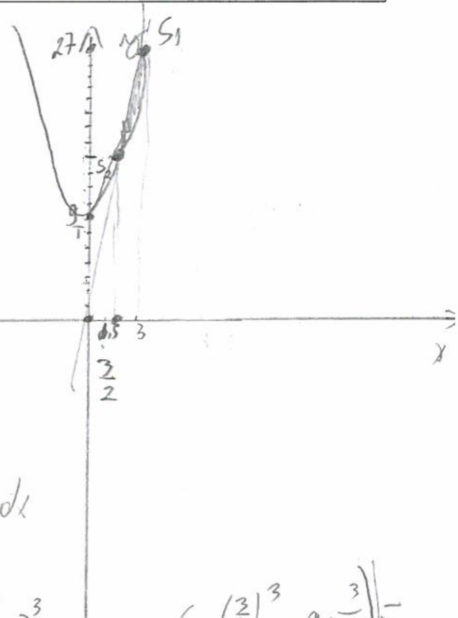
$2x^2 = -9$

$x^2 = -\frac{9}{2}$

NEMA N.T.

$P = 9\frac{x^2}{2}\Big|_{\frac{3}{2}}^{3} - \left(2\frac{x^3}{3} + 9x\right)\Big|_{\frac{3}{2}}^{3} \Rightarrow P = 9\cdot\frac{3^2}{2} - 9\frac{\left(\frac{3}{2}\right)^2}{2} - \left(2\frac{3^3}{3} + 9\cdot 3 - \left(2\frac{\left(\frac{3}{2}\right)^3}{3} + 9\cdot\frac{3}{2}\right)\right) =$

$= \frac{243}{8} - \frac{117}{4} = \frac{9}{8}$



$$\int_0^1 \frac{dx}{1+\sqrt{x}}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	POPUNJAVA NASTAVNIK
IME I PREZIME: KREŠIMIR KALCINA **BROJ INDEKSA:** 57181-2009	Broj ↓ bodova

1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$. ~~20~~

2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. ~~15~~ 12

3. Odredi tangencijalnu ravninu i normalu na plohu $z = x^2 + y^2$ u točki $T(1, -2, z_0)$. ~~15~~

4. Analitički ispitati skalarnu funkciju $f(x, y) = (x-1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta. 10+10

5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. 20

6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. ~~15~~

Ukupno: 12

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha\, dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\, dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x\, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x\, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

(1) $\int_0^1 \frac{dx}{1+\sqrt{x}} \left[\begin{matrix} x = t^2 \,/' \\ dx = 2dt \end{matrix}\right] = \int_0^1 \frac{2dt}{1+t^2} = 2\int_0^1 \frac{dt}{1+t^2} = \frac{1}{1} \arctan \frac{t}{1} \Big|_0^1$

$= \left(\arctan\frac{1}{1}\right) - \left(\arctan\frac{0}{1}\right) = 0.785 - 0 = 0.785$ X

(3)

$z = x^2 + y^2 \quad T(1, -2, z_0)$

$z = 1^2 + (-2)^2$

$z = 1 + 4$

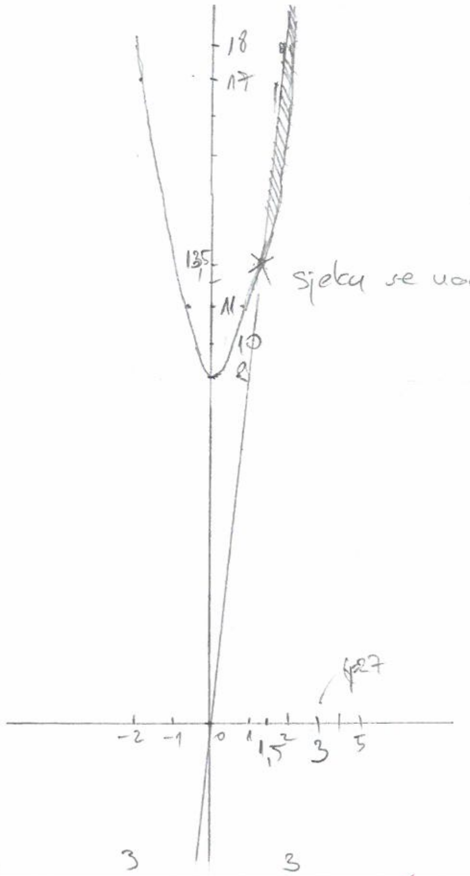
$z = 5$

0

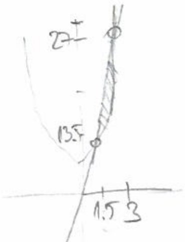
(2) $y = 2x^2 + 9$ i $y = 9x$

-2	-1	0	1	2	5
17	11	9	11	17	59

-2	-1	0	1	2	5
-18	-9	0	9	18	45



sjeku se u ovoj točci



$2x^2 + 9 = 9x$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{9 \pm \sqrt{9^2 - 4 \cdot 2 \cdot 9}}{2 \cdot 2} = \frac{9 \pm \sqrt{81 - 72}}{4}$

$x_1 = \frac{9 - \sqrt{9}}{4} = \frac{9-3}{4} = \frac{6}{4} = \frac{3}{2}$

$x_2 = \frac{9+3}{4} = \frac{12}{4} = 3$

$\int_{1.5}^{3} (9x - 2x^2 - 9)\,dx$ ✓ $= \int_{1.5}^{3} 9x\,dx - \int_{1.5}^{3} 2x^2\,dx - \int_{1.5}^{3} 9\,dx = 9\int_{1.5}^{3} x\,dx - 2\int_{1.5}^{3} x^2\,dx - 9\int_{1.5}^{3} dx$ ✓

$= 9\frac{x^2}{2}\Big|_{1.5}^{3} - 2 \cdot \frac{x^3}{3}\Big|_{1.5}^{3} - 9x\Big|_{1.5}^{3}$ ✓

$= \left(\left(9 \cdot \frac{3^2}{2}\right) - \left(9 \cdot \frac{1.5^2}{2}\right)\right) - \left(\left(2 \cdot \frac{3^3}{3}\right) - \left(2 \cdot \frac{1.5^3}{3}\right)\right) - \left((9 \cdot 3) - (9 \cdot 1.5)\right)$

$= (40.5 - 10.125) - (18 - 2.25) - (27 - 13.5)$

12

$= 29.875 - 15.75 - 13.5$

$\boxed{P = 0.625}$ ✗

(6) $xy' + y - e^x = 0 \quad y(1) = 1$

$x\frac{dy}{dx} + y = e^x \;/ \cdot dx$

$x\,dy + y = e^x dx \;/ : x$

$\int dy + y = \int \frac{e^x}{x} dx$ ✗

I $\int dy + \int y\,dy = y + \frac{y^2}{2} = y + \frac{1}{2}y^2$

II $\int \frac{e^x}{x} dx = \int$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	POPUNJAVA NASTAVNIK
IME I PREZIME: RINO KURTIN **BROJ INDEKSA:** 17-2-0112-2011	Broj ↓ bodova

1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$. 20
2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. 15
3. Odredi tangencijalnu ravninu i normalu na plohu $z = x^2 + y^2$ u točki $T(1, -2, z_0)$. 15
4. Analitički ispitati skalarnu funkciju $f(x, y) = (x-1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta. 10+10
5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. 20
6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. 15

Ukupno: 15

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

6. $xy' + y - e^x = 0$

$$xy' = e^x - y$$

$$y' = \frac{e^x - y}{x}$$

$$\frac{dy}{dx} = \frac{e^x - y}{x}$$

$$dy = \frac{e^x - y}{x}\, dx$$

?

1. $\int_0^1 \frac{dx}{1+\sqrt{x}} = \Big[\quad 1+\sqrt{x} = t \;/'$

$\frac{1}{2}x^{-\frac{1}{2}}\,dx = dt$

$\frac{1}{2\sqrt{x}}\,dx = dt$

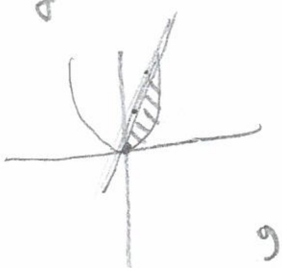
$dx = \frac{dt}{\frac{1}{2\sqrt{x}}}$

$dx = 2\sqrt{x}\,dt$

$\int_1^2 \frac{2\sqrt{x}\,dt}{t}$

$2\int_1^2 \frac{\sqrt{x}\,dt}{t}$?

2. $y = 2x^2+9$ $\qquad y = 9x$



$9x = 2x^2+9$

x	0	1	2
y	0	9	12

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{9 \pm \sqrt{81 - 4\cdot 2\cdot 9}}{2\cdot 2}$

$x_{1,2} = \frac{9\pm 3}{4}$

$x_1 = 3 \quad x_2 = \frac{3}{2}$

$\int_{\frac{3}{2}}^{3} 9x - 2x^2 - 9\,dx =$

$9\int_{\frac{3}{2}}^{3} x\,dx - 2\int_{\frac{3}{2}}^{3} x^2\,dx - 9\int dx =$

$9\left(\frac{x^2}{2}\right)\Bigg|_{\frac{3}{2}}^{3} - 2\left(\frac{x^3}{3}\right)\Bigg|_{\frac{3}{2}}^{3} - 9\,(x)\Bigg|_{\frac{3}{2}}^{3} =$

$= 9\left(\frac{3^2}{2} - \frac{\left(\frac{3}{2}\right)^2}{2}\right) - 2\left(\frac{3^3}{3} - \frac{\left(\frac{3}{2}\right)^3}{3}\right) - 9\left(3-\frac{3}{2}\right)$

$= 9\cdot\frac{27}{8} - 2\cdot\frac{63}{8} - 9\cdot\frac{3}{2} = \frac{9}{8}$

5. $y'' - y = -x + 1$

$y_H =$

$y_{(P)} = C_1 e^{x} + C_2 e^{-x}$ ✓

$\lambda'' - \lambda = 0$

$\lambda_{1,2} = \frac{0 \pm \sqrt{0 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1}$

$\lambda_{1,2} = \frac{0 \pm 2}{2}$

$\lambda_1 = 1$

$\lambda_1 = -1$

$y = -Ax + B$

$y' = -A$

$y'' = 0$

? $\cancel{\emptyset}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: IVAN VIDAKOVIĆ

BROJ INDEKSA: 57188-2009

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$.	~~20~~
2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$.	~~15~~
3. Odredi tangencijalnu ravninu i normalu na plohu $z = x^2 + y^2$ u točki $T(1, -2, z_0)$.	15
4. Analitički ispitati skalarnu funkciju $f(x, y) = (x-1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta.	10+10
5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.	20
6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$.	15

Ukupno: 0

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}$, $\alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

1.) $\int_0^1 \frac{dx}{1+\sqrt{x}} = \int_0^1 \frac{1}{1+x^{\frac{1}{2}}} dx = \int_0^1 \frac{1}{1+\sqrt{x}} \cdot \frac{1-\sqrt{x}}{1-\sqrt{x}} dx = \int_0^1 \frac{1-\sqrt{x}}{1-\cancel{\sqrt{x}}+\cancel{\sqrt{x}}-\sqrt{x}^2} dx$

$= \int_0^1 \frac{1-\sqrt{x}}{1-x} dx = \textcircled{1} \int_0^1 \frac{1}{1-x} dx - \textcircled{2} \int_0^1 \frac{\sqrt{x}}{1-x} dx = \textcircled{1} \begin{bmatrix} u = 1-x \; dx \\ du = -dx \\ -du = dx \end{bmatrix} \quad \textcircled{2} \left[\frac{\sqrt{x}}{1-x} \cdot \frac{x}{x} = \frac{x\sqrt{x}}{x-x^2} \right]$

$-\int_0^1 \ln|u|$

0

2.) $y = 2x^2 + 9$ $\Rightarrow a > 0 \rightarrow$ 

$y = 9x$

1.) Sjecišta

$9x = 2x^2 + 9$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 9}}{2 \cdot 2}$

$x_{1,2} = \frac{9 \pm \sqrt{81 - 72}}{4}$

$x_{1,2} = \frac{9 \pm 3}{4}$

$x_1 = 3$

$x_2 = \frac{3}{2}$

$$P = \int_{3/2}^{3} 9x\,dx - \int_{3/2}^{3} (2x^2 + 9)\,dx$$

$$P = 9 \Big|_{3/2}^{3} \frac{1}{2} x^2 - \left[\left(2 \Big|_{3/2}^{3} \frac{1}{3} x^3\right) + 9\right]$$

$$P = \left[\left(9 \frac{1}{2} \cdot 3^2\right) - \left(9 \frac{1}{2} \cdot \left(\frac{3}{2}\right)^2\right)\right] - \left[2 \frac{1}{3} \cdot 3^3 - 2 \frac{1}{3} \cdot \left(\frac{3}{2}\right)^3\right] + 9$$

$$P = \left[\left(\frac{9}{\cancel{2}_1} \cdot \cancel{6}^3\right) - \left(\frac{9}{2} \cdot \frac{\cancel{6}^3}{\cancel{4}_2}\right)\right] - \left[\frac{2}{\cancel{3}_1} \cdot \cancel{9}^3 - \frac{\cancel{2}^1}{3_1} \cdot \frac{\cancel{27}^9}{\cancel{8}_4}\right] + 9$$

$$P = 27 - \frac{27}{4} - 6 - \frac{9}{4} + 9$$

$$P = 27 - \frac{27}{4} - \frac{9}{4} + 3$$

$$P = 27 - \frac{27 - 9}{4} + 3 = 30 - \frac{18}{4}$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: MLADEN BULIĆ **BROJ INDEKSA:** 17-1-0018-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$. 20
2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. 15
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4. Analitički ispitati skalarnu funkciju $f(x, y) = (x-1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta. 10+10
5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. 20
6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. 15

Ukupno: 15

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
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$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

1. $\int_0^1 \frac{dx}{1+\sqrt{x}} = \begin{bmatrix} x = t \\ dx = dt \end{bmatrix} =$

$\begin{bmatrix} x = t^2 \\ dx = 2t\,dt \end{bmatrix} = \int_0^1 \frac{2t\,dt}{1+\sqrt{t^2}} = \int_0^1 \frac{2t\,dt}{1+t} = \int_0^1 \frac{dx}{1+x} = \int_0^1 \operatorname{arctg} x = \operatorname{arctg} 1 - \operatorname{arctg} 0 = 45$

X X

2.)

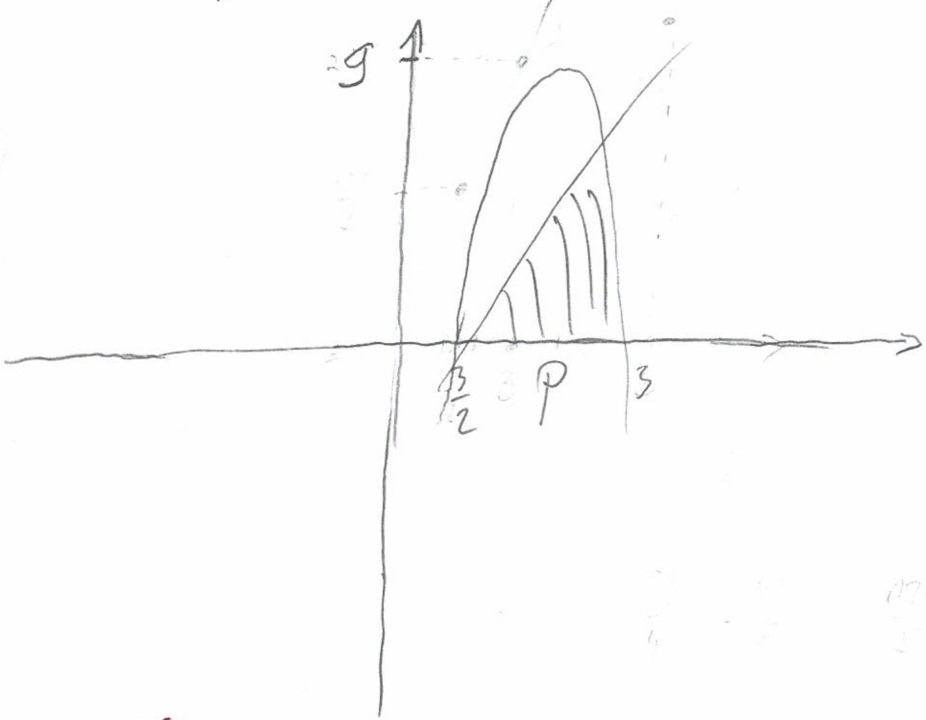
$y = 2x^2 + 9$

$y = 9x$

$2x^2 + 9 = 9x$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \dfrac{9 \pm \sqrt{9^2 - 4 \cdot 2 \cdot 9}}{2 \cdot 2} = \dfrac{9 \pm 3}{4} \Rightarrow x_1 = \dfrac{12}{4} = 3 \;,\; x_2 = \dfrac{6}{4} = \dfrac{3}{2}$



$P = \displaystyle\int_{\frac{3}{2}}^{3} 2x^2 + 9 - 9x = 2\int x^2 + 9\int dx - 9\int x = 2\frac{x^3}{3} + 9x - 9\frac{x^2}{2}\Big|_{\frac{3}{2}}^{3}$ ✗

$P = 2 \cdot \dfrac{3^3}{3} + 9 \cdot 3 - 9 \cdot \dfrac{3^2}{2} - \left(2 \cdot \dfrac{\left(\frac{3}{2}\right)^3}{3} + 9 \cdot \dfrac{3}{2} - 9 \cdot \dfrac{\left(\frac{3}{2}\right)^2}{2}\right) = \dfrac{45}{8}$ ✗

MLADEN BULIC

3.) $z = x^2 + y^2$ u točki $T(1, -2, z_0)$

tangenta

$z - z_0 = f(x)T\,(x - x_0) + f(y)(T)\,(y - y_0)$

normala

$$\frac{x - x_0}{f(x)T} = \frac{y - y_0}{f(y)T} = \frac{z - z_0}{-1}$$

$z_0 = x^2 + y^2$

$z_0 = 1^2 + (-2)^2$

$z_0 = 5$

$x_0 = 1$

$y_0 = -2$

$z_0 = 5$

$f(x)T = 2x = 2 \cdot 1 = 2$

$f(y)T = 2y = 2 \cdot (-2) = -4$

~~tangenta~~ TANGENCIJALNA RAVNINA

$z - 5 = 2\,(x - 1) + (-4)\,(y - (-2))$

normala

$$\frac{x - 1}{2} = \frac{y - (-2)}{-4} = \frac{z - 5}{-1}$$

✓

5.) $y'' - y = -x + 1$ $x = 0,\ y = 0,\ y' = 0$

$\lambda^2 - 1 = 0$

$x_{1,2} = \pm i$ ✗

$y = e^{0 \cdot x}(C_1 \cos xB + C_2 \sin xB)$

$y = C_1 \cos Bx + C_2 \sin Bx$ ✗

$y_p = ax + b$

$y' = a$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: MASTR TOSIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$. ~~20~~
2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. 15
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Ukupno: 15

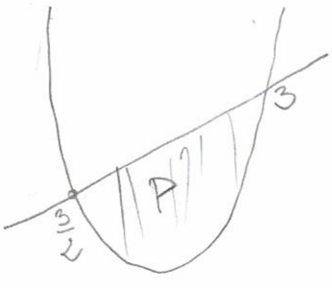
Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\, dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x\, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

2

PARABOLA $y = 2x^2 + 9$

pravac $y = 9x$



$$\int_{\frac{3}{2}}^{3} \left[(9x) - (2x^2+9)\right] dx$$

$$\int_{\frac{3}{2}}^{3} (9x - 2x^2 - 9)\, dx = \left(-2\frac{x^3}{3} + 9\frac{x^2}{2} - 9x\right)\Big|_{\frac{3}{2}}^{3}$$

$$= \left(-2\cdot\frac{3^3}{3} + 9\frac{3^2}{2} - 9\cdot 3\right) - \left(-2\cdot\frac{(\frac{3}{2})^3}{3} + 9\frac{(\frac{3}{2})^2}{2} - 9\cdot\frac{3}{2}\right)$$

$$= \left(-\frac{9}{2}\right) - \left(-\frac{45}{8}\right)$$

SJECIŠTA

$$2x^2 + 9 = 9x$$

$$2x^2 + 9 - 9x = 0$$

$$a = 2 \quad b = -9 \quad c = 9$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{9 \pm \sqrt{81 - 72}}{4}$$

$$x_1 = \frac{9 + \sqrt{9}}{4} = \frac{12}{4} = 3$$

$$x_2 = \frac{9 - \sqrt{9}}{4} = \frac{6}{4} = \frac{3}{2}$$

$$P = \frac{9}{8} = 1{,}125$$

MTSTR JOSIP

$$\int_0^1 \frac{dx}{1+\sqrt{x}} \qquad \left| \begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right|$$

$$\int_0^1 \frac{2t\,dt}{1+t} = \int_0^1 \frac{2t}{1+t} \cdot dt = 2 \cdot \frac{t}{1+t} \quad \times$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ANTONIO SEKULA **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓
bodova

1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$. ~~20~~
2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. ~~15~~
3. Odredi tangencijalnu ravninu i normalu na plohu $z = x^2 + y^2$ u točki $T(1, -2, z_0)$. 15
4. Analitički ispitati skalarnu funkciju $f(x, y) = (x - 1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta. 10+10
5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. 20
6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. 15

Ukupno: ~~0~~

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

1. $\int_0^1 \frac{dx}{1+\sqrt{x}} = \left| \begin{matrix} t^2 = x \\ 2dt = dx \end{matrix} \right| = \int_0^1 \frac{2dt}{1+t^2} = 2\int_0^1 \frac{dt}{1+t^2} =$

$2 \cdot \frac{1}{1} \arctan \frac{t}{1} \Big|_0^1 = 2 \arctan t = 2 \arctan 1 -$

$- 2 \arctan 0 = 90$

$t^2 = 1$

$t = \sqrt{1}$

$t^2 = \sqrt{0}$

$t = 0$

2. $y = 2x^2 + 9$

$y = 9x$

$2x^2 + 9 = 9x$

$2x^2 + 9 - 9x = 0$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

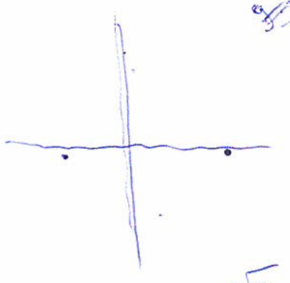
$x_{1,2} = \frac{9 \pm \sqrt{9^2 - 4 \cdot 2 \cdot 9}}{2 \cdot 2}$

$x_{1,2} = \frac{9 \pm \sqrt{81 - 24}}{4}$

$x_{1,2} = \frac{9 \pm \sqrt{57}}{4}$

$x_1 = \frac{9 + \sqrt{57}}{4}$

$x_2 = \frac{9 - \sqrt{57}}{4}$



$P = \int_{\frac{9-\sqrt{57}}{4}}^{\frac{9+\sqrt{57}}{4}} (2x^2 + 9 - 9x)\,dx = 2\int x^2\,dx + 9\int dx - 9\int x\,dx$

$P = 2 \frac{x^3}{3} \Big|_{\frac{9-\sqrt{57}}{4}}^{\frac{9+\sqrt{57}}{4}} + 9x \Big|_{\frac{9-\sqrt{57}}{4}}^{\frac{9+\sqrt{57}}{4}} - 9 \frac{x^2}{2} \Big|_{\frac{9-\sqrt{57}}{4}}^{\frac{9+\sqrt{57}}{4}}$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: MATEJA PEĆARIĆ

BROJ INDEKSA: 17-0032-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$. 20
2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. ~~15~~
3. Odredi tangencijalnu ravninu i normalu na plohu $z = x^2 + y^2$ u točki $T(1, -2, z_0)$. 15
4. Analitički ispitati skalarnu funkciju $f(x, y) = (x-1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta. 10+10
5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. 20
6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. 15

Ukupno: ~~0~~

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

2. $y = 2x^2 + 9$
$y = 9x$

$2x^2 + 9 = 9x$

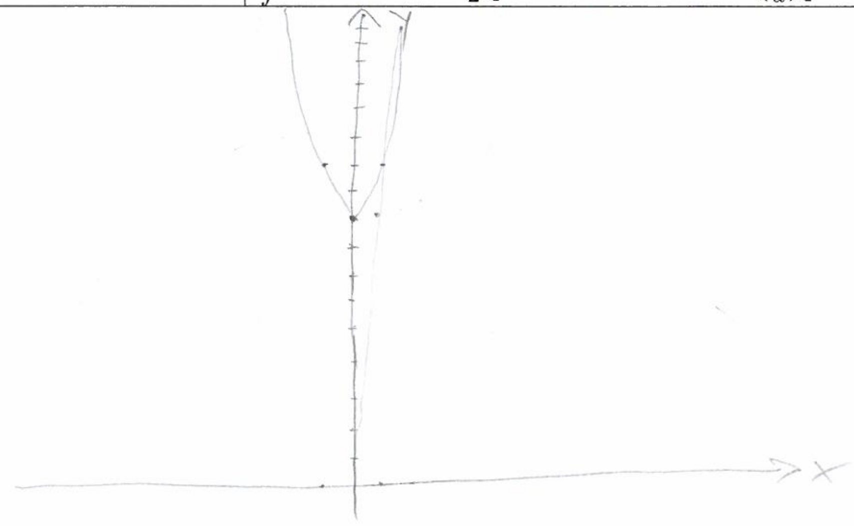
$2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 9}}{4}$

$= \frac{9 \pm \sqrt{81 - 72}}{4}$

$= \frac{9 \pm 3}{4}$

$x_1 = \frac{12}{4} = 3$ $x_2 = \frac{6}{4} = \frac{3}{2}$



$x = 0$ $y = 9$
$x = -1$ $y = 11$
$x = 1$ $y = 11$

$x = 0$ $y = 0$
$x = 1$ $y = 9$
$x = 2$ $y = 18$

$P = \int_3^3 (9x + 2x^2 + 9)\, dx$

$= \int 9x\, dx + \int 2x^2 dx + \int 9 dx$

1 $\int \frac{dx}{1+\sqrt{x}} =$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ŠIME-BORNA MAGAŠ **BROJ INDEKSA:** 17-2-0108-2011

UGLEŠIĆ

	POPUNJAVA NASTAVNIK Broj ↓ bodova
1. Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$.	20
2. Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$.	~~15~~
3. Odredi tangencijalnu ravninu i normalu na plohu $z = x^2 + y^2$ u točki $T(1, -2, z_0)$.	15
4. Analitički ispitati skalarnu funkciju $f(x, y) = (x-1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta.	10+10
5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje.	20
6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$.	15
Ukupno:	~~0~~

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha\, dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x + \sqrt{x^2\pm a^2}\right\| + C$
$\int e^x\, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x\, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2\pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1. $\int_0^1 \frac{dx}{1+\sqrt{x}} =$

2. $9\frac{x^2}{2}+2\frac{x^3}{3}+9x\Big|_{3/2}$

$$=\left(9\frac{3^2}{2}+2\frac{3^3}{3}+9\cdot 3\right)-\left(9\frac{\left(\frac{3}{2}\right)^2}{2}+2\frac{\left(\frac{3}{2}\right)^3}{3}+9\cdot\frac{3}{2}\right)$$

$$=\left(9\frac{9}{2}+2\cdot\frac{27}{3}+27\right)-\left(9\frac{\left(\frac{9}{4}\right)}{\frac{2}{1}}+2\frac{\frac{27}{8}}{3}+\frac{27}{2}\right)$$

$$=\left(9\frac{9}{2}+2\frac{27}{3}+27\right)-\left(9\frac{9}{8}+2\frac{27}{24}+\frac{27}{2}\right)$$

$$=\left(\frac{81}{2}+18+27\right)-\left(\frac{81}{8}+\frac{9}{4}+\frac{27}{2}\right)$$

$$=\left(\frac{81+36+54}{2}\right)-\left(\frac{81+18+108}{8}\right)$$

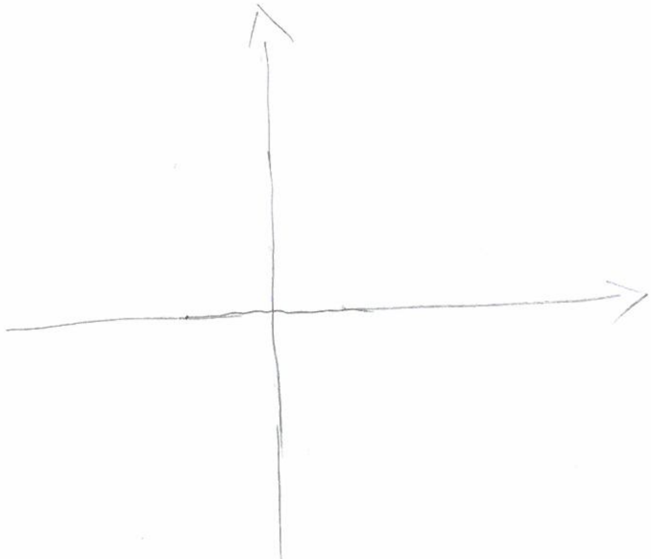
$$=\frac{171}{2}-\frac{207}{8}=\frac{684-207}{8}=\frac{477}{8}=59.62$$

4. $f(x,y)=(x-1)^2-y^2$

$c=(x-1)^2-y^2$

$y^2=(x-1)^2-c$

$y=\sqrt{(x-1)^2-c}$



$c=0$

$x=0 \quad y=1$

$x=1 \quad y=0$

$x=-1 \quad y=2$

$c=1$

$x=0 \quad y=-1$

$x=1 \quad y=1$

$6.\ xy' + y - e^x = 0 \qquad y(1) = 1$

$xy' = e^x + y$

$x \cdot \frac{dy}{dx} = e^x + y \ / dx$

$x\,dy = e^x dx + y\,dx$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: HIKOLINA KOMLJENOVIĆ **BROJ INDEKSA:** 17-2-0114-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

~~1.~~ Izračunati $\int_0^1 \frac{dx}{1+\sqrt{x}}$. ~~20~~

~~2.~~ Izračunati površinu lika omeđenog parabolom $y = 2x^2 + 9$ i pravcem $y = 9x$. 15

3. Odredi tangencijalnu ravninu i normalu na plohu $z = x^2 + y^2$ u točki $T(1, -2, z_0)$. 15

4. Analitički ispitati skalarnu funkciju $f(x, y) = (x-1)^2 - y^2$. Nacrtati razinske krivulje i strelicama označiti smjer rasta. 10+10

5. Riješiti $y'' - y = -x + 1$ i odredimo posebno rješenje koje udovoljava početnom uvjetu $x = 0$, $y = 0$, $y' = 0$. Provjeri rješenje. 20

6. Nađi partikularno rješenje koje zadovoljava: $xy' + y - e^x = 0$ i $y(1) = 1$. ~~15~~

Ukupno: 15

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
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$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x\, dx = \sin x + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

2.

$a > 0$

$y = 2x^2 + 9$

$y = 9x$

x	0	1
y	0	9

$2x^2 + 9 = 9x$

$2x^2 - 9x + 9 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{9 \pm \sqrt{(9)^2 - 4 \cdot 2 \cdot 9}}{4}$

$x_{1,2} = \frac{9 \pm 3}{4}$

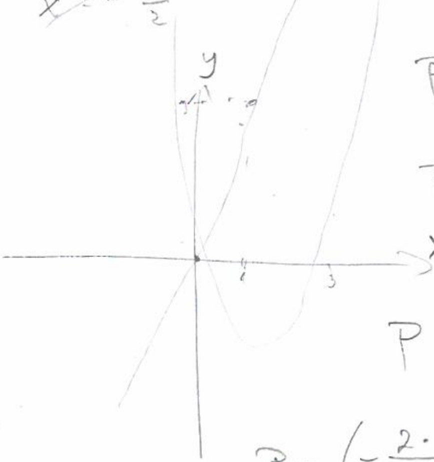
$x_1 = 3$

$x_2 = \frac{3}{2} \approx 1,5$

$2x^2 + 9 = 0$

$2x^2 = -9 \quad / : 2$

$\cancel{x^2 = -\frac{9}{2}}$



$P = \int_{3/2}^{3} 9x - (2x^2 + 9)\, dx$

$P = \int_{3/2}^{3} 9x - 2x^2 - 9\, dx$

$P = \int_{3/2}^{3} -2x^2 + 9x - 9\, dx$

$P = -\frac{2x^3}{3} + \frac{9x^2}{2} - 9x \Big|_{3/2}^{3}$

$P = \left(-\frac{2 \cdot 3^3}{3} + \frac{9 \cdot 3^2}{2} - 9 \cdot 3\right) - \left(-\frac{2 \cdot \left(\frac{3}{2}\right)^3}{3} + \frac{9 \cdot \left(\frac{3}{2}\right)^2}{2} - 9 \cdot \frac{3}{2}\right)$

$P = -\frac{9}{2} - \left(-\frac{45}{8}\right) = \frac{9}{8}$ //

② $\int_0^1 \frac{dx}{1+\sqrt{x}} = \ln|1+\sqrt{x}| + c \Big|_0^1$

$= \left(\ln|1+\sqrt{1}|\right) - \left(\ln|1+\sqrt{0}|\right)$

$= 0,6931471806$

⑥ $xy' + y - e^x = 0$ $\qquad y(1) = 1 \qquad y' = \frac{dy}{dx}$

$x\frac{dy}{dx} + y = e^x$

$y = e^{\int P(x)dx} \cdot \left(\int Q(x) \cdot e^{\int P(x)dx} dx\right)$

$y = e^{\frac{1}{x}dx} \cdot \left(\int e^x dx \cdot e^{x dx} dx\right)$

$y = e^{\ln|x|}\left(e^x \cdot e^x\right) + c$

DALJE . . .