MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	POPUNJAVA NASTAVNIK
IME I PREZIME: ANĐELO ŽMIRE **BROJ INDEKSA:** 17-2-0173-2012	Broj ↓ bodova

1. Odrediti integracijom (analitički): $\int_0^3 x^2 \ln x \, dx =$.	~~15~~ 12
2. Izračunati $\int\limits_0^{\pi/2} e^x \cos x \, dx$	15
3. Napiši jednadžbu ravnine koja prolazi točkom $T(1,0,2)$ i okomita je na *os* x.	5+10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y \cancel{f} - e$.	~~20~~ 5
5. Riješi diferencijalnu jednadžbu $(1+e^x)\,yy' = e^x$ uz početni uvjet $y(0) = 1$.	20
6. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x \sin x$. Provjeri dobiveno rješenje	15
	Ukupno: 52

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln \lvert\cos x\rvert$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int e^x \, dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int a^x \, dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x \, dx = \sin x + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

1.

$$\int_0^3 x^2 \ln x \, dx = \left[\begin{array}{ll} \ln x = u \ /' & x^2 dx = dv \ /\int \\ \frac{1}{x} dx = du & \int x^2 dx = \int dv \\ & \frac{x^3}{3} = v \end{array}\right]$$

$$\int u \, dv = u \cdot v - \int v \, du$$

$$\int x^2 \ln x \, dx = \ln x \cdot \frac{x^3}{3} - \int \frac{x^{\cancel{3}\,2}}{3} \frac{1}{\cancel{x}} dx = \ln x \frac{x^3}{3} - \frac{1}{3} \int x^2 dx = \ln x \frac{x^3}{3} - \frac{1}{3} \frac{x^3}{3}$$

$$= \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C$$

$$\int_0^3 x^2 \ln x \, dx = \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right)\Big|_0^3 = \left(9\left(\ln 3 - \frac{1}{3}\right)\right) - \left(0\left(\ln 0 - \frac{1}{3}\right)\right) = 9\ln 3 - 3 \approx 6.89$$

TREBALO JE ISTAKNUTI! NEPRAVI INTEGRAL

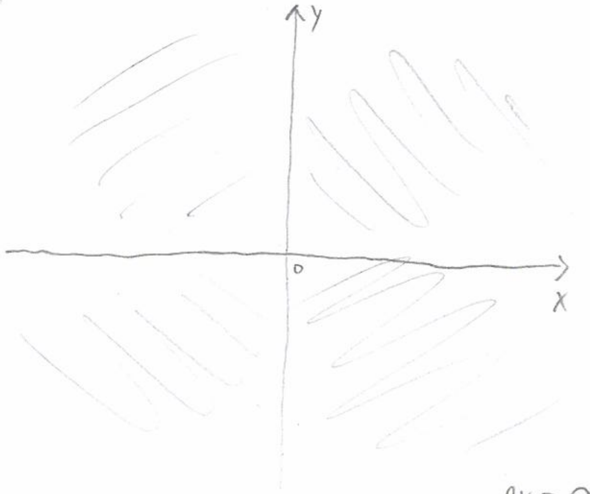
$$\boxed{\lim_{x \to 0} x \ln x = 0}$$

-2,71

4. $f(x,y) = x^2 + y + - e^x$

$f(x,y) = x^2 + y - e^x$

$D_f: \mathbb{R}^2$ ✓



$\frac{\partial f}{\partial x} = 2x$

$\frac{\partial f}{\partial y} = 1$

$\frac{\partial^2 x}{\partial x^2} = 2$

$\frac{\partial^2 y}{\partial y^2} = 0$

$\frac{\partial f}{\partial x \partial y} = 0$

$2x = 0 \qquad y = 0$

$x = 0$

$T_0(0,0)$

$\Delta = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0$

SEDLASTA TOČKA ✗

NIJE UZET U OBZIR ISPRAVAK ZADATKA NA PLOČI.

5. $(1+e^x)yy' = e^x \; / : (1+e^x)$

$yy' = \frac{e^x}{1+e^x}$

$y \frac{dy}{dx} = \frac{e^x}{1+e^x} \; / \cdot dx$

$y\,dy = \frac{e^x}{1+e^x}\,dx \; / \int$

$\int y\,dy = \int \frac{e^x}{1+e^x}\,dx^{*}$

$\frac{y^2}{2} = x - \ln|e^x+1| + C \; / \cdot 2$ ✓

$y^2 = 2x - 2\ln|e^x+1| \; / \sqrt{}$

$y = \sqrt{2x - 2\ln|e^x+1| + C}$

$y(0) = 1 \qquad x = 0 \quad y = 1$

$1 = \sqrt{0 - 2\ln|e^0+1| + C}$

$1 = \sqrt{-2\ln|2| + C} \; /^2$

$1 = -2\ln|2| + C$

$C = 1 + 2\ln|2|$

$* \int \frac{e^x}{1+e^x}\,dx = \int \frac{e^x+1-1}{e^x+1}\,dx =$

$= \int dx - \int \frac{1}{e^x+1}\,dx = x - \ln|e^x+1| + C$

⑥ $4y'' - y = 2x \sin x$

$4k^2 - 1 = 0$

$4k^2 = 1$

$k^2 = \frac{1}{4} \,/\sqrt{\ }$

$k_{1,2} = \pm \frac{1}{2}$

$k_1 = k_2$

$y_H = (C_1 + C_2 x) e^{kx}$

$$\boxed{y_H = (C_1 + C_2 x) e^{\frac{1}{2}x}}$$

$y = 2x \sin x$

$y_p = (Ax+B)\cos x + \sin x$

$y_p' = ((Ax+B)' \cdot \cos x + (Ax+B) \cdot (\cos x)') + \cos x$

$= (A \cos x + (Ax+B) \cdot (-\sin x)) + \cos x$

$= A \cos x - Ax \sin x - B \sin x + \cos x$

$y_p'' = -A \sin x - (A \sin x + Ax \cos x) - B \cos x - \sin x$

$= -A \sin x - A \sin x - Ax \cos x - B \cos x - \sin x$

$= -2A \sin x - Ax \cos x - B \cos x - \sin x$

$4(-2A \sin x - Ax \cos x - B \cos x - \sin x) - Ax \cos x + B \cos x + \sin x = 2x \sin x$

$-8A \sin x - 4Ax \cos x - 4B \cos x - 4 \sin x - Ax \cos x + B \cos x + \sin x = 2x \sin x$

$A = 0$

$B = 0$

$$\boxed{y_p = \sin x}$$

$y(0) = 0$

$y = y_H + y_p$

$$\boxed{y = (C_1 + C_2 x) e^{\frac{1}{2}x} + \sin x}$$

$0 = (C_1 + C_2 \cdot 0) e^0 + \sin 0$

$0 = (C_1 + 0) \cdot 1 + 0$

$0 = C_1$

(2.)

$$\int_0^{\frac{\pi}{2}} e^x \cos x\, dx = \left[\begin{array}{ll} \cos x = u \;/' & e^x\, dx = dv \;/\int \\ -\sin x\, dx = du & \int e^x\, dx = \int dv \\ & e^x = v \end{array}\right]$$

$$\int e^x \cos x\, dx = \cos x \cdot e^x + \int e^x \cdot \sin x\, dx^{*}$$

$$^{*}\int e^x \sin x\, dx = \left[\begin{array}{ll} \sin x = u \;/' & e^x\, dx = dv \;/\int \\ \cos x\, dx = du & e^x = v \end{array}\right]$$

$$\int e^x \cos x\, dx = \cos x \cdot e^x + \sin x \cdot e^x - \int e^x \cos x\, dx$$

$$2\int e^x \cos x\, dx = \cos x\, e^x + \sin x\, e^x \quad /:2$$

$$\int_0^{\frac{\pi}{2}} e^x \cos x\, dx = \frac{e^x(\cos x + \sin x)}{2}\Big|_0^{\frac{\pi}{2}} = \left(\frac{e^{\frac{\pi}{2}}(\overset{0}{\cos\frac{\pi}{2}} + \overset{1}{\sin\frac{\pi}{2}})}{2}\right) - \left(\frac{e^0(\cos 0 + \sin 0)}{2}\right)$$

$$= \left(\frac{4{,}81(0+1)}{2}\right) - \left(\frac{1(1+0)}{2}\right) \approx 2{,}4 - 0{,}5 \approx 1{,}9$$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	POPUNJAVA NASTAVNIK
IME I PREZIME: Branimir Pijaca **BROJ INDEKSA:** 17-2-0086-2011	Broj ↓ bodova

1. Odrediti integracijom (analitički): $\int_0^3 x^2 \ln x \, \mathrm{d}x =$.	~~15~~
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5. Riješi diferencijalnu jednadžbu $(1+\mathrm{e}^x)\,yy' = \mathrm{e}^x$ uz početni uvjet $y(0) = 1$.	~~20~~
6. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x\sin x$. Provjeri dobiveno rješenje	~~15~~
	Ukupno: 30

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
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$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \cosh x \,\mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

(4) $f(xy) = x^2 + y - e^x$?

$D f = \mathbb{R}$

$\partial_x f = 2x - e^x \cdot x \qquad \partial_y x = 1$

$\partial_{xx} f = 2 - e^x \cdot x \cdot 1 \qquad \partial_{yy} = 0$

$\partial_{xy} f = 0 \qquad \partial_{yx} f = 0$

$\partial_y x = 0$

$1 \neq 0$?!?

$T(0,0)$

$A = 2$

$$\Delta \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 2 \cdot 0 = 0$$

nema ekstrema

$$\int_0^{\pi/2} e^x \cos x\,dx \quad \left[\begin{array}{ll} \cos x = u & e^x = dv \\ -\sin x\,dx = du & e^x = v \end{array}\right.$$

$$\cos x \cdot e^x + \int e^x \sin dx \quad \left[\begin{array}{ll} \sin x = u & e^x = dv \\ \cos x\,dx = du & e^x = v \end{array}\right.$$

$$\cos x \cdot e^x + \left[\sin x \cdot e^x - \int e^x \cdot \cos x\,dx \right]$$

$$\int e^x \cos x\,dx = \cos x \cdot e^x + \sin x \cdot e^x - \int e^x \cdot \cos x\,dx$$

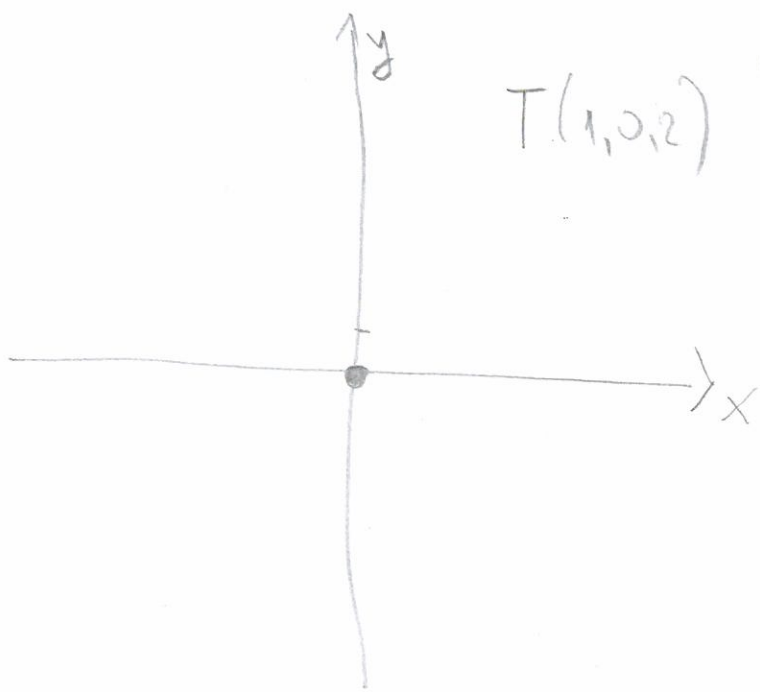
$$2\int e^x \cos x\,dx = \cos x \cdot e^x + \sin x \cdot e^x \quad /2$$

$$\int e^x \cos x\,dx = \frac{\cos x \cdot e^x}{2} + \frac{\sin x \cdot e^x}{2} \Big|_0^{\pi/2} \checkmark$$

$$= 0 + \frac{1 e^{\frac{\pi}{2}}}{2} - \left[\frac{e^0}{2} + 0\right]$$

$$= \frac{e^{\frac{\pi}{2}}}{2} - \frac{1}{2} \ // \checkmark$$

(3)



T(1,0,2) ?

$$\int_0^3 x^2 \ln x \, dx \qquad \left[\begin{array}{ll} \ln x = u & dv = x^2 \\ \frac{1}{x} dx = du & v = \frac{x^3}{3} \end{array}\right]$$

$$\ln x \cdot \frac{x^3}{3}\Big|_0^3 - \int_0^3 \frac{x^3}{3} \cdot \frac{1}{x} \, dx$$

$$\ln x \cdot \frac{x^3}{3}\Big|_0^3 - \frac{1}{3}\int_0^3 x^2 \, dx$$

$$\ln x \cdot \frac{x^3}{3}\Big|_0^3 - \frac{1}{3}\frac{x^3}{3}\Big|_0^3$$

$$\ln 3 \cdot 1 - \ln 0 \cdot \frac{0}{3} - \left[\frac{1}{3} \cdot \frac{3^3}{3} - \frac{1}{3} \cdot \frac{0^3}{3}\right]$$

$$\ln 3 - [3 - 0]$$

$$\ln 3 - 3 =$$ Nepostoji

NEPRAVI INTEGRAL!

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2) $$\int_0^{\pi/2} e^x \cos x \, dx \qquad \left[\begin{array}{ll} \cos x = u & e^x = dv \\ -\sin x \, dx = du & e^x = v \end{array}\right] \qquad \begin{array}{l} x = t \\ dx = dt \\ e^t dt \\ e^x \end{array}$$

$$\cos x \cdot e^x\Big|_0^{\pi/2} - \int^{\pi/2} e^x - \sin x \, dx \qquad \left[\begin{array}{ll} e^x = u & \sin x = v \\ e^x \cdot x \, \cancel{dx} = du & -\cos x = dv \end{array}\right]$$

$$\cos x \cdot e^x\Big|_0^{\pi/2} + \int_0^{\pi/2} e^x \sin x \, dx$$

$$\cos x \cdot e^x\Big|_0^{\pi/2} + \left[e^x \cdot -\cos x - \int \sin x \, e^x \cdot x \, dx\right.$$

(5) $(1+e^x)yy' = e^x$

$$yy' = \frac{e^x}{(1+e^x)}$$

$$y\frac{dy}{dx} = \frac{e^x}{1+e^x} \quad / \cdot dx$$

$$y\,dy = \frac{e^x}{1+e^x}dx$$

$$\frac{y^2}{2} = \frac{e^x}{1+e^x}dx \quad / 2$$

$$y^2 = \frac{\frac{e^x}{1+e^x}}{\frac{2}{1}}$$

$$y = \sqrt{\frac{e^x}{2+2e^x}}$$

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6) $4y'' - y = 2x\sin x$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: LUKA KNEŽEVIĆ **BROJ INDEKSA:** 17-2-1120-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti integracijom (analitički): $\int_0^3 x^2 \ln x \, \mathrm{d}x = .$	~~15~~ 12
2. Izračunati $\int\limits_0^{\pi/2} e^x \cos x \, \mathrm{d}x$	~~15~~
3. Napiši jednadžbu ravnine koja prolazi točkom $T(1,0,2)$ i okomita je na *os* x.	5+10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y\cancel{4} - \mathrm{e}$. x	~~20~~
5. Riješi diferencijalnu jednadžbu $(1+\mathrm{e}^x)\,yy' = \mathrm{e}^x$ uz početni uvjet $y(0)=1$.	20
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Ukupno: 27

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \, \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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$\int \mathrm{e}^x \, \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \sin x \, \mathrm{d}x = -\cos x + C$	$\int \sinh x \, \mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2 \pm a^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \cosh x \, \mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2} \, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

① $\int_0^3 x^2 \ln x \, dx$

$x^2 = dv$ $\quad \ln x = u$

$\int x^2 dx = \int dv$ $\quad \frac{1}{x} = du$

$\frac{x^3}{3} = v$

$= [u \cdot v]_a^b - \int v\,du$

$= \left[\ln x \cdot \frac{x^3}{3}\right]_0^3 - \int_0^3 \left(\frac{x^3}{3} \cdot \frac{1}{x}\, dx\right)$

$= [\ln|3| \cdot 9] - \underbrace{\left[\ln 0 \cdot \frac{0^3}{3}\right]}_{0} - \int_0^3 \frac{x^2}{3}$

$9\ln|3| - \frac{1}{3}\int_0^3 \frac{x^3}{3}$

$9\ln|3| - \frac{x^3}{9}\Big|_0^3$

$9\ln|3| - 3 - 0$

$9\ln|3| - 3$

$9\ln|3| - \frac{1}{3} \cdot |9-0|$

$\boxed{9\ln|3| - 3}$

VIDI ŽMIRE

12

① ~~x^2+y+e~~

x^2+y-e^x ✗

D $x^2+y \in R$

~~[illegible]~~ 0 $e \in R^+$

D $f(x,y) \in R$

$T(0,0)$

$\frac{\partial f_x}{\partial x} = 2x$

$\frac{\partial f_y}{\partial y} = 1$

$\frac{\partial^2 f_x}{\partial x^2} = 2$

$\frac{\partial^2 f_y}{\partial y^2} = 0$

$\Delta = \begin{vmatrix} 2 & 2 \\ 2 & 0 \end{vmatrix} = -2$

TOČKA T(0,0) IMA MAKSIMUM -2.

✗

② $\int_0^{\frac{\pi}{2}} e^x \cos x \, dx$

$\cos x = u$ $\quad e^x dx = dv$

$\sin x = du$ $\quad \int e^x dx = \int dv$

$e^x = v$

$\left|\cos x \cdot e^x\right|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \sin x \, dx$

$\sin x = u$ $\quad e^x = dv$

$\cos x = du$ $\quad e^x = v$

$\underbrace{\cos\frac{\pi}{2}}_{1} \cdot e^{\frac{\pi}{2}} - \cos 0 \cdot e^0 - \int_0^{\frac{\pi}{2}} e^x \sin x \, dx$

$e^{\frac{\pi}{2}} - (1 \cdot 1) + \left|\sin x \cdot e^x\right|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \cos x \cdot dx$

~~$\cos\frac{\pi}{2} \cdot e^{\frac{\pi}{2}}$~~

$2\int \cos x \cdot e^x - \left|\sin x \cdot e^x\right|_0^{\frac{\pi}{2}}$

$2\int_0^{\frac{\pi}{2}} \cos x \cdot e^x - \left|1 \cdot e^{\frac{\pi}{2}} - 0\right|$

$2\int_0^{\frac{\pi}{2}} \cos x \cdot e^x - e^{\frac{\pi}{2}}$

$\int_0^{\frac{\pi}{2}} \cos x \cdot e^x = \frac{e^{\frac{\pi}{2}}}{2}$ ✗

VIDI ZMIRE.

③ $T(1,0,2)$ $\quad \vec{n}(1,0,0)$

$\pi \equiv 1(x-1) + 0(y-0) + 0(z-2)$

$\pi \equiv x - 1 = 0$ ✓ ILI $\boxed{x = 1}$

⑤ $(1+e^x)y\, y' = e^x$ $\quad$ UVJET $y(0) = 1$

?

26. 9. 2013.

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	POPUNJAVA NASTAVNIK
IME I PREZIME: Matija Miočić **BROJ INDEKSA:** 17-1-0110-2012	Broj ↓ bodova

1. Odrediti integracijom (analitički): $\int_0^3 x^2 \ln x \, \mathrm{d}x = .$ — 15
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3. Napiši jednadžbu ravnine koja prolazi točkom $T(1, 0, 2)$ i okomita je na *os* x. — 5+10
4. Ispitati domenu i ekstreme funkcije $f(x, y) = x^2 + y \cancel{+} e.$ — 20
5. Riješi diferencijalnu jednadžbu $(1 + e^x)\, yy' = e^x$ uz početni uvjet $y(0) = 1$. — 20
6. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x \sin x$. Provjeri dobiveno rješenje — 15

Ukupno: 15

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x \,\mathrm{d}x = e^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \sinh x \,\mathrm{d}x = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \cosh x \,\mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

③ $T(1,0,2)$

$p \equiv \frac{x+1}{1} = \frac{y+1}{1} = \frac{z+1}{1} \to \vec{c}(1,1,1)$

$\pi \perp p$

$1(x-1) + 1(y-0) + 1(z-2)$

$x - 1 + y - 0 + z - 2 = 0$

$x + y + z - 3 = \pi$

RJEŠENJE: $x = 1$

② $\int_0^{\pi/2} e^x \cos x\,dx = e^x \cos x \Big|_0^{\pi/2} + \sin x\, e^x \Big|_0^{\pi/2} - \int_0^{\pi/2} e^x \cos x\,dx$

$= \int_0^{\pi/2} e^x \cos x\,dx - e^x \cos x \Big|_0^{\pi/2} + \sin x\, e^x \Big|_0^{\pi/2}$

$= \int_0^{\pi/2} e^x \cos x\,dx = \dfrac{e^x \cos x \Big|_0^{\pi/2} + \sin x\, e^x \Big|_0^{\pi/2}}{2}$

$= \dfrac{0 - 1 + e^{\frac{\pi}{2}} - 0}{2} = \dfrac{-1 + e^{\frac{\pi}{2}}}{2} \approx 1{,}905239$ ✓

④ $f(x,y) = x^2 + y - e^x$

?

① $\int_0^3 x^2 \ln x\,dx = x^2 \ln x \Big|_0^3 + \frac{x^3}{3} \ln x \Big|_0^3 - \int_0^3 x^2 \ln x\,dx$ ✗

$= \int_0^3 x^2 \ln x - x^2 \ln x \Big|_0^3 + \frac{x^3}{3} \ln x \Big|_0^3$

$= \dfrac{x^2 \ln x \Big|_0^3 + \frac{x^3}{3} \ln x \Big|_0^3}{2} = \dfrac{3^2 \ln(3) + \frac{3^3}{3} \ln(3)}{2} = 9{,}8875106$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: MARIN MATEK **BROJ INDEKSA:** 17-1-0111-12

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti integracijom (analitički): $\int_0^3 x^2 \ln x \, dx =$. — ~~15~~
2. Izračunati $\int\limits_0^{\pi/2} e^x \cos x \, dx$ — ~~15~~
3. Napiši jednadžbu ravnine koja prolazi točkom $T(1, 0, 2)$ i okomita je na *os x*. — 5+10
4. Ispitati domenu i ekstreme funkcije $f(x, y) = x^2 + y \cancel{+} -e$. — ~~20~~
5. Riješi diferencijalnu jednadžbu $(1 + e^x)\, yy' = e^x$ uz početni uvjet $y(0) = 1$. — 20
6. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x \sin x$. Provjeri dobiveno rješenje — ~~15~~

Ukupno: 20

Tablični integrali

$\int dx = x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, dx = \ln\lvert\sin x\rvert + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int e^x dx = e^x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cos x \, dx = \sin x + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

2. $\int\limits_0^{\frac{\pi}{2}} e^x \cos x \, dx =$ $\left[\begin{array}{ll} e^x = u & dv = \cos x \, dx \ /\int \\ e^x dx = du & v = \sin x \end{array}\right]$

$= u \cdot v - \int v \cdot du$

$= e^x \sin x - \int \sin x \, e^x dx$

$\cos x = u$ $\quad dv = e^x dx$

$-\sin x \, dx = du$ $\quad v = e^x$

$\cos x \, e^x + \int e^x \sin x \, dx \, dx$

$e^x = u$ $\quad dv = \sin x \, dx \ /\int$

$e^x dx = du$ $\quad v = -\cos x$

?

1. $\int_0^3 x^2 \ln x \, dx = \lim_{x \to 0}$?

5. $(1+e^x)yy' = e^x$

$$yy' = \frac{e^x}{1+e^x}$$

$$y\frac{dy}{dx} = \frac{e^x}{1+e^x} \quad / \cdot dx$$

$$\int y\,dy = \int \frac{e^x}{1+e^x}\,dx$$

$$\frac{y^2}{2} = \int \frac{dt}{t}$$

$$\frac{y^2}{2} = \int \frac{dt}{t}$$

$$\frac{y^2}{2} = \ln|1+e^x| + C \quad \checkmark$$

$$\left[\begin{array}{l} 1+e^x = t \\ e^x dx = dt \end{array}\right]$$

4. $f(x,y) = x^2 + y - e^x$

$$\frac{\partial f}{\partial x} = 2x - e^x$$

$$\frac{\partial f}{\partial y} = 1 - e^x \Rightarrow e^x = 1$$

$$\boxed{x = 0}$$

$$\frac{\partial^2 f}{\partial x^2} = 2 - e^x$$

$$\frac{\partial^2 f}{\partial y^2} = -e^x$$

$$\frac{\partial f}{\partial x \partial y} = 0$$

$$\Delta = \begin{vmatrix} 2-e^x & 0 \\ 0 & -e^x \end{vmatrix}$$

$$\frac{\partial f}{\partial x} = 0$$

$$2x - e^x = 0$$

$$x_1 = \frac{e^x}{2} \quad \times$$

$$\Delta = -1 < 0$$

NEMA EKSTREM

6. $4y'' - y = 2x \sin x$ MARIN MATEK

$4r^2 - r = 0$ $y = y_p + y_0$

$r(4r-1) = 0$

$r_1 = 0 \quad 4r_2 - 1 = 0$

$r_2 = \frac{1}{4}$

$y_0 = C_1 \cdot e^{0x} + c_2 e^{\frac{1}{4}x}$

$y_0 = c_1 + c_2 e^{\frac{1}{4}x}$

$y_p = ?$

$e^x = u \qquad dv = \cos x \, dx$

$e^x dx = du \qquad v = \sin x$

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Tibor Rah

BROJ INDEKSA: 17-1-0060-2017

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Odrediti integracijom (analitički): $\int_0^3 x^2 \ln x \, \mathrm{d}x =$. ~~15~~
2. Izračunati $\displaystyle\int_0^{\pi/2} e^x \cos x \, \mathrm{d}x$ ~~15~~
3. Napiši jednadžbu ravnine koja prolazi točkom $T(1,0,2)$ i okomita je na *os* x. 5+10
4. Ispitati domenu i ekstreme funkcije $f(x,y) = x^2 + y + -\mathrm{e}$. ~~20~~
5. Riješi diferencijalnu jednadžbu $(1+\mathrm{e}^x)\,yy' = \mathrm{e}^x$ uz početni uvjet $y(0)=1$. ~~20~~
6. Riješiti diferencijalnu jednadžbu: $4y'' - y = 2x \sin x$. Provjeri dobiveno rješenje 15

Ukupno: 0

Tablični integrali

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int x^\alpha \, \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, \mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \cot x \, \mathrm{d}x = \ln\lvert\sin x\rvert + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \mathrm{e}^x \, \mathrm{d}x = \mathrm{e}^x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
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$\int \cos x \, \mathrm{d}x = \sin x + C$	$\int \cosh x \, \mathrm{d}x = \sinh x + C$	$\int \sqrt{a^2-x^2}\, \mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

1.

$$\int_0^3 x^2 \ln x \, dx = \left(2x \cdot \frac{1}{x}\right)\Bigg|_0^3$$

$$= \left(6 \cdot \frac{1}{3}\right) - \left(0 \cdot \frac{1}{0}\right) = \frac{6}{3} - 0 = 2$$

5. $(1+e^x)\,y\,y' = e^x$ $\qquad y' = \frac{dy}{dx}$

$(1+e^x)\,y\,\frac{dy}{dx} = e^x \quad \Big| \cdot dx$

$(1+e^x)\,y\,dy = e^x\,dx \quad \Big| : (1+e^x)$

$y\,dy = \frac{e^x}{1+e^x}\,dx$ ✗

2. $\int_0^{\frac{\pi}{2}} e^x \cos x\,dx = \left(e^x \cdot (-\sin x)\right)\Big|_0^{\frac{\pi}{2}}$

$= \left(e^{\frac{\pi}{2}} \cdot \left(-\sin \frac{\pi}{2}\right)\right) - \left(e^0 \cdot (-\sin 0)\right)$

$= -e^{\frac{\pi}{2}} \sin \frac{\pi}{2} - 0 = -e^{\frac{\pi}{2}} \sin \frac{\pi}{2}$ ✗

4. $f(x,y) = x^2 + y + (-e)$ $D_f : \mathbb{R} \backslash$ ×

NIJE UZET U OBZIR ISPRAVAK ZADATKA NA PLOČI

$\frac{\partial f}{\partial x} = 2x$ $\frac{\partial f}{\partial y} = 1$ $\frac{\partial f}{\partial x} = 0$ $\frac{\partial f}{\partial y} = 0$

$\frac{\partial^2 f}{\partial x^2} = 2$ $\frac{\partial^2 f}{\partial y^2} = 0$ $2x = 0 / :2$ $1 = 0$

$\frac{\partial f}{\partial x \partial y} = 0$ $\frac{\partial f}{\partial y \partial x} = 0$ $\boxed{x = 0}$ $\boxed{y = 0}$

$$\Delta = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial f}{\partial y \partial x} \\ \frac{\partial f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0 - 0 = 0$$

$\boxed{\Delta = 0}$

$-2{,}72$ $1{,}72$ $f(x,y) = (0,0,0)$

6. $4y'' - y = 2x \sin x$ 0

MATEMATIKA 2: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ANTE TROSKOT **BROJ INDEKSA:** 17-1-0007-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Odrediti integracijom (analitički): $\int_0^3 x^2 \ln x \, dx = .$	~~15~~
2. Izračunati $\int\limits_0^{\pi/2} e^x \cos x \, dx$	~~15~~
3. Napiši jednadžbu ravnine koja prolazi točkom $T(1,0,2)$ i okomita je na *os* x.	~~5+10~~
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5. Riješi diferencijalnu jednadžbu $(1+e^x)\, yy' = e^x$ uz početni uvjet $y(0) = 1$.	20
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Ukupno: 0

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$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1},\ \alpha \neq -1$	$\int \tan x \, dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
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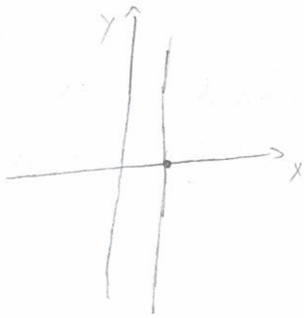
$\int \cos x \, dx = \sin x + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

3) $T \ \overset{x\ y}{(1,0,2)}$



$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$

1) $\int_0^3 x^2 \ln x \, dx$

$\ln x = t$

$\frac{1}{x} dx = dt \ / \cdot x$

$dx = dtx$

$x^2 + dtx$

2) $\int_{0}^{\frac{\pi}{2}} e^x \cos x \, dx$

$e^x = u$

$e^x dx = du$

$\cos x \, dx = dv$

$\int \cos x \, dx = v$

$v = \sin x$

$u\,dv = uv - \int v\,du$

$e^x \sin x - \int \sin x \cdot e^x \, dx$