MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇓⇓ C7

IME I PREZIME: MATE RADAŠ **BROJ INDEKSA:** 17-2-0183-2092

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Među kompleksnim brojevima odrediti rješenja jednadžbe $z^3 = -(-i)^{548}$. *Prikazati rješenja u kompleksnoj ravnini!* 10+5

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} 5x + 4z + 2t &= 3 \\ x - y + 2z + t &= 1 \\ 4x + y + 2z &= 1 \\ x + y + z + t &= 0 \end{aligned}$$

3. Odrediti kada je $\frac{x+1}{\sqrt{x^2-x}} - 1 > 0$. Obavezno uvrštavanjem provjeriti rješenje jednadžbi koje se javljaju tokom rješavanja nejednadžbe. 13+2 ~~10~~ 11

4. Za funkciju: $f(x) = \sqrt{x^2+4x+3}$ treba:

   (a) pronaći drugu derivaciju 10

   (b) na temelju ispitivanja toka funkcije skicirati graf 20(graf)

5. Odrediti i provjeriti rješenje $\lim_{x\to+\infty} \left(\frac{n}{n-4}\right)^n =$ 8+2

6. Grafički odrediti rješenja nejednadžbe: $\ln(x-1) < \arccos x$. *Provjeriti uvrštavanjem!* 10+5

Ukupno: 56

5. $\lim_{x\to+\infty} \left(\frac{n}{n-4}\right)^n = \lim_{x\to+\infty} \left(1 + \frac{n}{n-4} - 1\right)^n = \lim_{x\to+\infty} \left(1 + \frac{n-(n-4)}{n-4}\right)^n =$

$= \lim_{x\to+\infty} \left(1 + \frac{4}{n-4}\right)^n = \lim_{x\to+\infty} \left(1 + \frac{1}{\frac{n-4}{4}}\right)^n = \lim_{x\to+\infty} \left(1 + \frac{1}{\frac{n-4}{4}}\right)^{\frac{n-4}{4}\cdot\frac{4n}{n-4}}$

$= e^{\frac{4n}{n-4}} = e^{\frac{4}{1}} = e^4 =$

$e^4 = 54.60$ ✓

PROVJERA:

$\left(\frac{10}{10-4}\right)^{10} = 165.38$

$\left(\frac{100}{100-4}\right)^{100} = 59.26$

$\left(\frac{1000}{1000-4}\right)^{1000} = 55.037 \approx e^4$

✓

2. $\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$ 1.r. $\approx$ 4.r

$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right]$ $\begin{array}{l} 2.r-1.r \\ 3.r-4\times 1.r \\ 4.r-5\times 1.r \end{array}$ $\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$ $2.r-3.r$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$ $\begin{array}{l} 1.r-2.r \\ \\ 3.r+3\times 2.r \\ 4.r+5\times 2.r \end{array}$ $\sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 7 & 8 & 1 \\ 0 & 0 & 14 & 17 & 3 \end{array}\right]$ $\begin{array}{l} \\ \\ \\ 4.r-2\times 3.r \end{array}$ $\left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 7 & 8 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$

$\sim$ $x-2z-3t=0 \Rightarrow x-2\cdot(-1)-3\cdot(-1)=0 \Rightarrow x-1=0 \Rightarrow x=1$

$y+3z+4t=0 \Rightarrow y+3\cdot(-1)+4\cdot 1=0 \Rightarrow y=3-4 \Rightarrow y=-1$

$7z+8t=1 \Rightarrow 7z+8=1 \Rightarrow 7z=-7/:7 \Rightarrow z=-1$

$t=1$

PROVJERA:

$5x+4z+2t=3 \Rightarrow 5\cdot 1+4\cdot(-1)+2\cdot 1=3 \Rightarrow 5-4+2=3 \Rightarrow 3=3$ ✓

$x-y+2z+t=1 \Rightarrow 1-(-1)+2\cdot(-1)+1=1 \Rightarrow 1+1-2+1=1 \Rightarrow 1=1$ ✓

$4x+y+2z=1 \Rightarrow 4\cdot 1-y+2\cdot(-1)=1 \Rightarrow 4-1-2=1 \Rightarrow 1=1$ ✓

$-x+y+z+t=0 \Rightarrow 1-1-1+1=0 \Rightarrow 0=0$ ✓

✓

MATÉ RADAŠ C7

$548 : 4 = 137$
28

$z^3 = -(-i)^{548}$

$z^3 = -(-1)^0$ $\quad r = \sqrt{(-1)^2 + y^2}$

$z^3 = -(1)$ $\quad r = \sqrt{1+0} = \sqrt{1} = 1$

$z^3 = -1 / \sqrt[3]{}$ $\quad tg\varphi = \frac{0}{-1} = 0$

$z = \sqrt[3]{-1}$

$$z = \sqrt[n]{r}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right)$$

$k = 0$

$$z_1 = \sqrt[3]{1}\left(\cos\frac{0 + 2\cdot 0\cdot\pi}{3} + i\sin\frac{0 + 2\cdot 0\cdot\pi}{3}\right)$$

$$z_1 = 1(\cos 0 + i\sin 0)$$

$$z_1 = (1 + 0) = 1$$

$k = 1$

$$z_2 = \sqrt[3]{1}\left(\cos\frac{0 + 2\cdot 1\cdot\pi}{3} + i\sin\frac{0 + 2\cdot 1\cdot\pi}{3}\right)$$

$$z_2 = 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

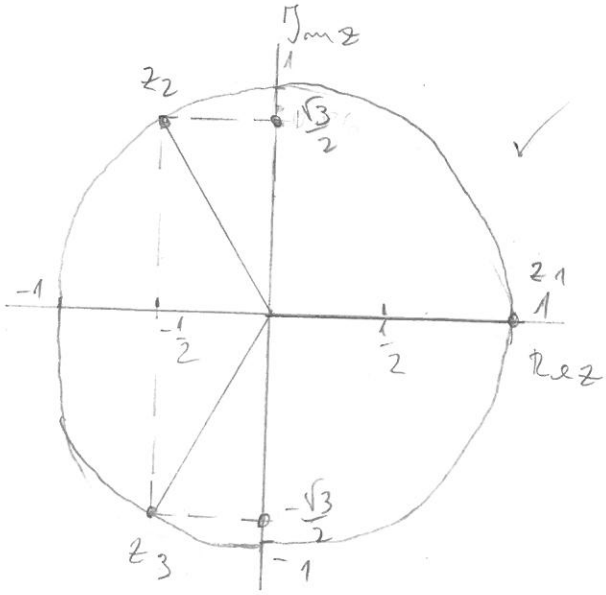
$$z_2 = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$k = 2$

$$z_3 = \sqrt[3]{1}\left(\cos\frac{0 + 2\cdot 2\cdot\pi}{3} + i\sin\frac{0 + 2\cdot 2\pi}{3}\right)$$

$$z_3 = 1\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$$

$$z_3 = \left(-\frac{1}{2} + -\frac{\sqrt{3}}{2}i\right)$$



3. $\frac{x+1}{\sqrt{x^2-x}} - 1 > 0$

$\sqrt{x^2-x}$ $\quad x^2-x \neq 0$

$\sqrt{x^2-x} > 0 \,/^2$ $\quad x(x-1) \neq 0$

$x^2-x > 0$ $\quad x_1 \neq 0$

$x(x-1) > 0$ $\quad x_2 \neq 1$

$x_1 > 0 \quad x < 0$ (ili +)

$x_2 > 1$

$\frac{x+1-\sqrt{x^2-x}}{\sqrt{x^2-x}} > 0$

	$-\infty$		0		1		$+\infty$
$\frac{x+1-\sqrt{x^2-x}}{\sqrt{x^2-x}}$		$-$	0	$-$	0	$+$	

$x \in \langle 1, +\infty \rangle$ ✗

$\frac{-1+1-\sqrt{(-1)^2-(-1)}}{\sqrt{(-1)^2-(-1)}} = \frac{0-\sqrt{2}}{\sqrt{1+1}} = -\frac{\sqrt{2}}{\sqrt{2}} = -1$

TREBALO JE JOŠ RIJEŠITI JEDNADŽBU:

$\frac{x+1}{\sqrt{x^2-x}} - 1 = 0$

PROVJERA:

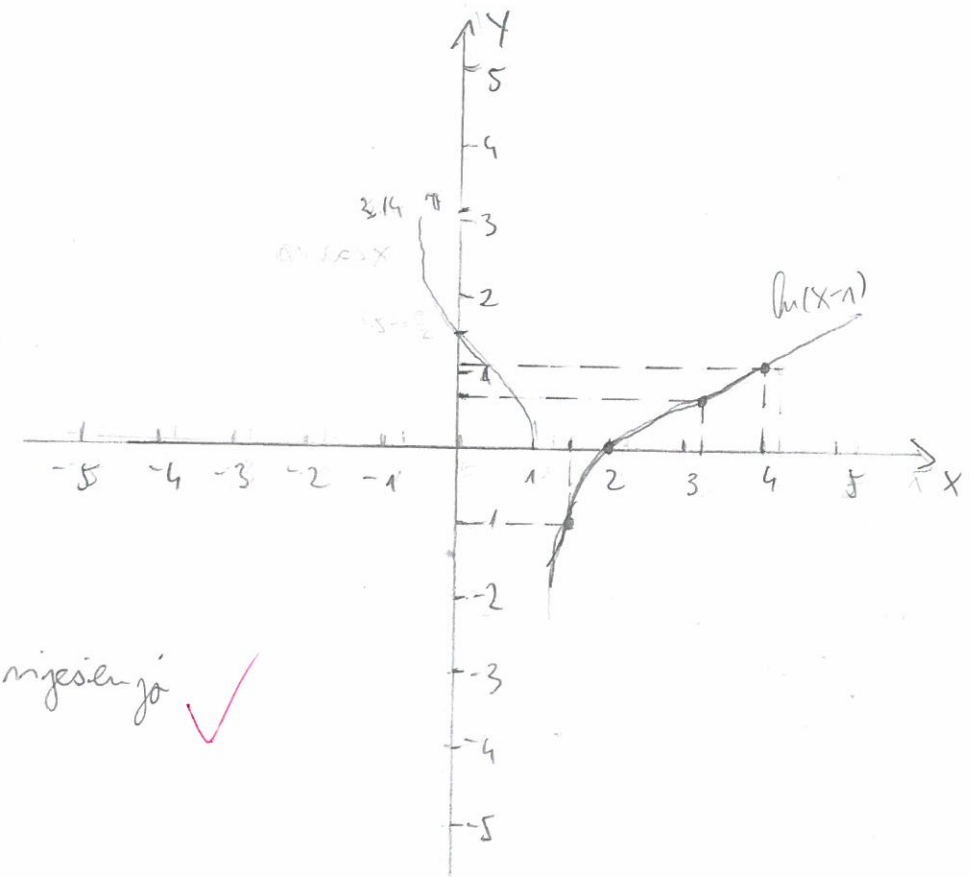
$\frac{2+1-\sqrt{2^2-2}}{\sqrt{2^2-2}} = \frac{3-\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}-(\sqrt{2})^2}{(\sqrt{2})^2} = \frac{3\sqrt{2}-2}{2} \neq$

VIDI KNEŽEVIĆ.

6. $\ln(x-1) < \arccos x$

$y = \ln(x-1)$

x	y
2	0
3	0.7
4	1.1
$\frac{3}{2}$	-1



nejednadžba nema rješenja ✓

MATE ŽADAS C7

4) $f(x) = \sqrt{x^2+4x+3}$

$f'(x) = \left(\sqrt{x^2+4x+3}\right)' = \left((x^2+4x+3)^{\frac{1}{2}}\right)' \cdot (x^2+4x+3)' = \frac{1}{2}(x^2+4x+3)^{-\frac{1}{2}} \cdot (2x+4)$

$f''(x) = \left(\frac{1}{2}(x^2+4x+3)^{-\frac{1}{2}}\right)' \cdot (2x+4) = -\frac{1}{4}(x^2-4x+3)^{-\frac{3}{2}} \cdot 2 = -\frac{1}{2}(x^2-4x+3)^{-\frac{3}{2}}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ C7

IME I PREZIME: Antonijo Knežević **BROJ INDEKSA:** 57672

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Među kompleksnim brojevima odrediti rješenja jednadžbe $z^3 = -(-i)^{548}$. *Prikazati rješenja u kompleksnoj ravnini!* 10+5

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Ukupno: 49

2)

$$\left|\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right| \Rightarrow \left|\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right| \begin{array}{l} \\ R_2-R_1 \\ R_3-4R_1 \\ R_4-5R_1 \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right| \cdot\left(\frac{1}{2}\right) \left|\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right| \begin{array}{l} R_1-R_2 \\ \\ R_3+3R_2 \\ R_4+5R_2 \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & \frac{3}{2} & 1 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{7}{2} & -4 & -\frac{1}{2} \\ 0 & 0 & -\frac{7}{2} & -3 & \frac{1}{2} \end{array}\right| \cdot\left(\frac{2}{7}\right) \left|\begin{array}{cccc|c} 1 & 0 & \frac{3}{2} & 1 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & \frac{7}{2} & -3 & \frac{1}{2} \end{array}\right| \begin{array}{l} R_1-\frac{3}{2}R_2 \\ R_2+\frac{1}{2}R_3 \\ \\ R_4+\frac{7}{2}R_3 \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{5}{7} & -\frac{2}{7} \\ 0 & 1 & 0 & \frac{4}{7} & -\frac{3}{7} \\ 0 & 0 & 1 & \frac{8}{7} & -\frac{1}{7} \\ 0 & 0 & 0 & 1 & 1 \end{array}\right| \begin{array}{l} R_1 + \frac{5}{7}R_4 \\ R_2 - \frac{4}{7}R_4 \\ R_3 - \frac{8}{7}R_4 \end{array} \left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right|$$

PROVJERA:

$$\left|\begin{array}{cccc} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right| \left|\begin{array}{c} 1 \\ -1 \\ -1 \\ 1 \end{array}\right| = \left|\begin{array}{l} 5 - 4 + 2 \\ 1 + 1 - 2 + 1 \\ 4 - 1 - 2 \\ 1 - 1 - 1 + 1 \end{array}\right| = \left|\begin{array}{c} 7 - 4 \\ 3 - 2 \\ 4 - 3 \\ 2 - 2 \end{array}\right|$$

$$= \left|\begin{array}{c} 3 \\ 1 \\ 1 \\ 0 \end{array}\right| \quad \checkmark$$

(1) $z^3 = -(-i)^{548}$

$z^3 = -(-i)^{182 \cdot 3 + 2}$

$z^3 = -\left[(-i)^3\right]^{182} \cdot (-i)^2$

$z^3 = -1 \cdot i^2$

$z^3 = -i^2$

$z^3 = -(-1)$

$z^3 = 1$

$$z = \sqrt[3]{1}$$

① $\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right)$

$k = 0, 1, \dots, n-1$

$x + yi = r(\cos\varphi + i\sin\varphi)$

$1 = ?$

$1 + 0i = x + yi$ $\quad x = 1$, $y = 0$

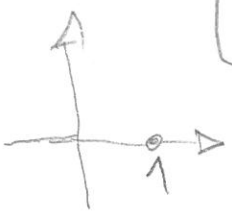
$r = |z| = \sqrt{x^2+y^2}$

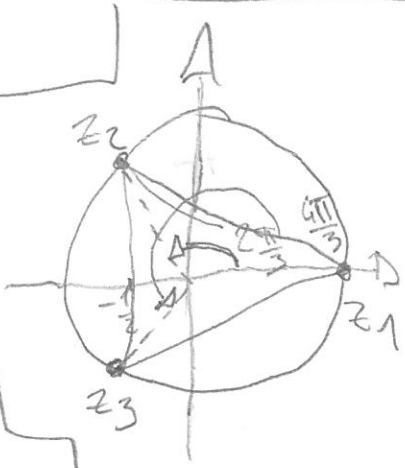
$r = \sqrt{1^2+0^2} = 1$

$\operatorname{tg}\varphi = \frac{y}{x}$

$\operatorname{tg}\varphi = 0$

$\varphi = 0$





RJEŠENJE SE NALAZI NA KRUGU JEDINICE

$r = 1$

$1 = 1\cdot(\cos 0 + i\sin 0)$

$\sqrt[3]{1} = \sqrt[3]{1}\cdot\left(\cos\frac{0+2k\pi}{3} + i\sin\frac{0+2k\pi}{3}\right)$

$k = 0, 1, 2$

Za $k = 0$

$z_1 = 1\cdot(\cos 0 + i\sin 0) = 1$

Za $k = 1$

$z_2 = 1\cdot\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

Za $k = 2$

$z_3 = 1\cdot\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$= -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

(3) $\frac{x+1}{\sqrt{x^2-x}} - 1 > 0$

$$\frac{x+1-\sqrt{x^2-x}}{\sqrt{x^2-x}} > 0$$

$$x+1-\sqrt{x^2-x} > 0$$

$$x+1 > \sqrt{x^2-x} \quad /^2$$

$$(x+1)^2 > x^2-x$$

$$x^2+2x+1 > x^2-x$$

$$2x+x > -1$$

$$3x > -1$$

$$\boxed{x > -\frac{1}{3}}$$

TREBALO JE RIJEŠITI JEDNADŽBU I UVRSTITI U TABLICU ISPOD

OVAJ POSTUPAK OPĆENITO NE VODI K RJEŠENJU

UVJET

$x^2-x > 0$

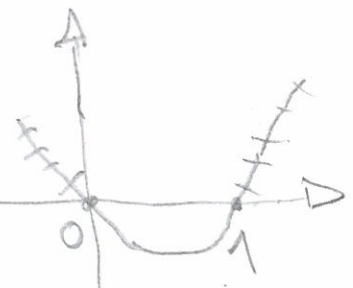
N.T. $x^2-x = 0$

$x(x-1) = 0$

$x_1 = 0 \quad x_2 = 1$

$N_1(0,0) \; N_2(1,0)$

$\boxed{x \in \langle -\infty, 0 \rangle \cup \langle 1, +\infty \rangle}$



9

OVO DVOJE ZAJEDNO DAJE RJEŠENJE, ALI TREBALO JE ZAPISATI KRAJNJI REZULT.

ISPITUJE SE TABLICOM

	$-\infty$		-1		$-\frac{1}{3}$		0		1		$+\infty$
$f(x) = \frac{x+1}{\sqrt{x^2-x}} - 1$		$-$	0	$+$	0	$+$	N/D	$+$		$+$	

RJEŠENJE NEJEDNADŽBE

$x \in \langle -\frac{1}{3}, 0 \rangle \cup \langle 1, +\infty \rangle$

$f(-1) = \ldots = -1 < 0$

$f(2) = \ldots > 0$

$f(-\frac{1}{6}) = \frac{5/6}{\sqrt{7}/6} - 1 = \frac{5}{\sqrt{7}} - 1 \neq 0$

ANTONIJU KNEŽEVIĆ

5] $\lim_{x\to\infty}\left(\frac{n}{n-4}\right)^n = \lim_{x\to\infty}\left(1+\frac{n}{n-4}-1\right)^n =$

$= \lim_{x\to\infty}\left(1+\frac{n-n-4}{n-4}\right)^n = \lim_{x\to\infty}\left(1+\frac{-4}{n-4}\right)^n$

$= \lim_{x\to\infty}\left(1+\frac{1}{\frac{n-4}{-4}}\right)^n = \lim_{x\to\infty}\left(1+\frac{1}{\frac{n-4}{4}}\right)^{\frac{n-4}{-4}\cdot\frac{-4\cdot n}{n-4}}$

$= e^{\lim_{x\to\infty}\frac{-4n}{n-4}} = \lim_{x\to\infty}\frac{-4}{1} = e^{-4}$ //X

$\lim_{n\to 10000}\left(\frac{n}{n-4}\right)^n = \left(\frac{10000}{10000^{-4}}\right)^{10000} = 54.6418...$

PA KAKO ONDA DOBITI REZULTAT e^{-4} SA e4

$e^4 \approx 54.59815003...$

TO NIJE PROVJERA, TO JE IGNORIRANJE INDIKATORA DA JE REZULTAT POGREŠAN VIDI RADAS

4) $f(x) = \sqrt{x^2+4x+3}$

$f''(x) = ?$

$f(x) = (x^2+4x+3)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x^2+4x+3)^{\frac{1}{2}-1}\cdot(x^2+4x+3)'$

$f(x) = \frac{1}{2}(x^2+4x+3)^{-\frac{1}{2}}\cdot(2x+4)$

$f'(x) = \frac{1}{2\sqrt{x^2+4x+3}}\cdot 2(x+2)$ $\quad f''(x) = ?$

ANTONIJO KNEŽEVIĆ

$$f'(x) = \frac{x+2}{\sqrt{x^2+4x+3}}$$

$$f''(x) = \frac{(x+2)' \cdot \sqrt{x^2+4x+3} - (x+2)\left(\sqrt{(x^2+4x+3)}\right)}{\left(\sqrt{x^2+4x+3}\right)^2}$$

$$f''(x) = \frac{1 \cdot \sqrt{x^2+4x+3} - (x+2) \cdot \frac{1}{2}(x^2+4x+3)^{-\frac{1}{2}} \cdot (x^2+4x+3)}{x^2+4x+3}$$

$$f''(x) = \frac{\sqrt{x^2+4x+3} - \frac{x+2}{2\sqrt{x^2+4x+3}} \cdot (2x+4)}{x^2+4x+3}$$

$$f''(x) = \frac{\sqrt{x^2+4x+3} - \frac{x+2}{2\sqrt{x^2+4x+3}} \cdot 2(x+2)}{x^2+4x+3}$$ ✓

$$f''(x) = \frac{\frac{\left(\sqrt{x^2+4x+3}\right)^2 - (x+2)^2}{\sqrt{x^2+4x+3}}}{x^2+4x+3}$$

$$f''(x) = \frac{x^2+4x+3-(x^2+4x+4)}{(x^2+4x+3)\sqrt{x^2+4x+3}}$$

$$f''(x) = \frac{x^2+4x+3-x^2-4x-4}{(x^2+4x+3)\sqrt{(x^2+4x+3)}}$$

$$f''(x) = \frac{-1}{\sqrt{(x^2+4x+3)^3}}$$

$$f''(x) < 0$$

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IME I PREZIME: Ivan Vukašinac **BROJ INDEKSA:** 1720182-12

POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 30

1. $z^3 = -(-i)^{548}$ $\quad 548:4 = 137$

$i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

$z^3 = -(-i)^4$

$z^3 = -(-i)^4$

$z^3 = -(-1)$

$z^3 = 1 = w$

$z = \sqrt[3]{1} = 1$

$w = 1$ // $\quad w = \sqrt{1^2+0^2} = \sqrt{1^2} = 1$ //

$\operatorname{tg}\varphi = \frac{y}{x} = \frac{0}{1} = 0$

$\varphi = 0$ //

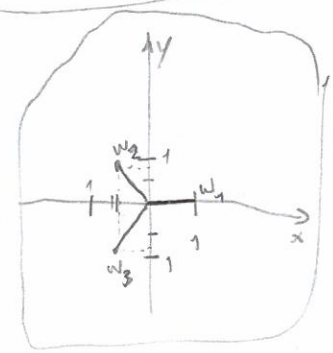
$w = (\cos 0 + i \sin 0)$ //

$k_1 = 0$, $k_2 = 1$, $k_3 = 2$

$w_1 = 1\left(\cos\frac{0+0}{3} + i\sin\frac{0+0}{3}\right) = 1$

$w_2 = 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{2}}{3}i$

$w_3 = 1\left(\cos\frac{2+4\pi}{3} + i\sin\frac{4\pi}{3}\right) = -\frac{1}{2} - \frac{\sqrt{2}}{3}i$



2. $5x + 4z + 2t = 3$
   $x - y + 2z + t = 1$
   $4x + y + 2z = 1$
   $x + y + z + t = 0$

$$\begin{bmatrix} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{bmatrix} \cdot(-1) \cdot(-4) \cdot(-5) \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{bmatrix} \cdot(-1)$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{bmatrix} \cdot(-1/3) \cdot(-5) \sim \begin{bmatrix} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 7 & 8 & 1 \\ 0 & 0 & 14 & 17 & 3 \end{bmatrix} \cdot(-2) \sim \begin{bmatrix} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 7 & 8 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} /:7 \sim \begin{bmatrix} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim$$

④b)

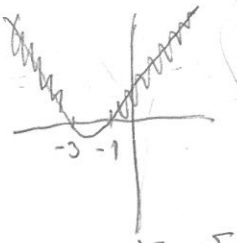
$$f(x) = \sqrt{x^2+4x+3}$$

Domena

$x^2+4x+3 \geq 0$

$x_1 = -1$

$x_2 = -3$



$D_f = \langle -\infty, -3] \cup [-1, +\infty \rangle$

Asimptote

V. A. Nema.

H. A $\quad x \to \infty$

D. H. A $\lim\limits_{x \to \infty} \sqrt{x^2+4x+3} \cdot \frac{\sqrt{x^2+4x+3}}{\sqrt{x^2+4x+3}}$

$\lim\limits_{x \to \infty} \frac{x^2+4x+3}{\sqrt{x^2+4x+3}} \; / : x^2 = \frac{1}{0} = +\infty$

Nema desna horizontalne asimptote!

Parnost

$f(-x) = \sqrt{(-x)^2+4(-x)+3}$

$f(-x) = \sqrt{x^2-4x+3}$

Ni parna ni neparna

D.K.A.

$$\lim_{x \to +\infty} \frac{f(x)}{x} = \frac{\sqrt{x^2+4x+3} \; / : x}{x \; / : x} = \lim_{x \to \infty} \frac{\sqrt{\frac{x^2}{x^2} + \frac{4x}{x^2} + \frac{3}{x^2}}}{\frac{x}{x}} = \frac{1}{1} = 1$$

$$f(x) - 1x = \lim_{x \to \infty} = \sqrt{x^2+4x+3} - x \cdot \frac{\sqrt{x^2+4x+3} + x}{\sqrt{x^2+4x+3} + x}$$

$$= \lim_{x \to \infty} = \frac{x^2+4x+3-x^2 \; / : x}{\sqrt{x^2+4x+3} + x \; / : x} = \frac{4}{1+1} = 2$$

Desna kosa. asimptota $= y = 1+2x$

Ivan Vukašinac

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 1 & \frac{8}{7} & \frac{1}{7} & \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \leftarrow \leftarrow \cdot(-3) \cdot(-2) \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{5}{7} & \frac{2}{7} \\ 0 & 1 & 0 & \frac{4}{7} & -\frac{3}{7} \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \leftarrow \leftarrow \cdot\left(-\frac{8}{7}\right) \cdot\left(-\frac{4}{7}\right) \cdot\left(\frac{5}{7}\right) \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$x = 1$
$y = -1$
$z = -1$
$t = 1$

- $5x + 4z + 2t = 3$
  $5 - 4 + 2 = 3$ //
- $4x + y + 2z = 1$
  $4 - 1 - 2 = 1$ //
- $x - y + 2z + t = 1$
  $1 + 1 - 2 + 1 = 1$ //
- $x + y + z + t = 0$
  $1 - 1 - 1 + 1 = 0$ //

✓

⑤ $\lim\limits_{x \to +\infty} \left(\frac{n}{n-4}\right)^4 = \lim\limits_{x \to +\infty} = \left(1 + \frac{n}{n-4} - \frac{n-4}{n-4}\right)^4 = \lim\limits_{x \to +\infty} \left(1 + \frac{-4}{n-4}\right)^n$

$\lim\limits_{x \to +\infty} \left(1 + \frac{1}{\frac{n-4}{-4}}\right)^{\frac{n-4}{-4} \cdot \frac{-4(n)}{n-4}} =$

$e^{\lim\limits_{x \to +\infty} \frac{-4n}{n-4}} = e^{\lim\limits_{x \to +\infty} \frac{-4n}{n}} = e^{\frac{-4}{1}} = e^{-4}$ //

Provjera:

$\lim\limits_{x \to 10} \left(\frac{10}{10-4}\right)^{10}$

$\lim\limits_{x \to 100} \left(\frac{100}{100-4}\right)^{100}$

$\lim\limits_{x \to 1000} \left(\frac{1000}{1000-4}\right)^{1000}$ ✗

KALKULATOR

idemo u beskonačno, VIDI RADAS →

⑷ $f(x) = \sqrt{x^2 + 4x + 3}$

a)

$f'(x) = (x^2 + 4x + 3)^{\frac{1}{2}} \cdot (x^2 + 4x + 3)$

$f'(x) = \frac{1}{2}(x^2 + 4x + 3)^{-\frac{1}{2}} \cdot (2x + 4)$ ✓

$\left(\frac{a}{b}\right)' = \frac{a' \cdot b - a \cdot b'}{b^2}$

$f'(x) = \frac{1}{\frac{1}{2}\left(\sqrt{x^2 + 4x + 3}\right)} \cdot 2x + 4 \Rightarrow \frac{2x + 4}{\frac{1}{2}\left(\sqrt{x^2 + 4x + 3}\right)}$ //

$f'(x) = \frac{(2x+4)'}{\frac{1}{2}\left(\sqrt{x^2 + 4x + 3}\right)} = \frac{2 \cdot \frac{1}{2}\left(\sqrt{x^2 + 4x + 3}\right) - (2x+4) \cdot \frac{1}{2}\left(\sqrt{x^2 + 4x + 3}\right)'}{\left(\frac{1}{2} \cdot \sqrt{x^2 + 4x + 3}\right)^2} = \sqrt{x^2 + 4x + 3} - 2x + 4$

... DALJE ...

3. $\frac{x+1}{\sqrt{x^2-x}} - 1 > 0$

$\sqrt{x^2-x} \neq 0 \;/^2 \cdot$

$x^2 - x \neq 0$

$x(x-1) \neq 0$

$x \neq 0$

$x - 1 = 0$

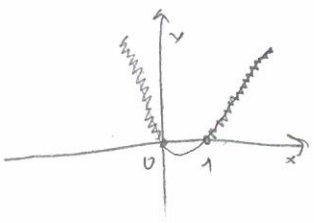
$x \neq 1$

$x^2 - x > 0$

$x(x-1) > 0$

$x = 0$

$x = 1$



$D_f = \langle -\infty, 0 \rangle \cup \langle 1, +\infty \rangle$ //

$\frac{x+1}{\sqrt{x^2-x}} - 1 > 0$

$\frac{x+1}{\sqrt{x^2-x}} > 1 \;/ \cdot \sqrt{x^2-x}$

$x + 1 > \sqrt{x^2 - x}\;/^2$

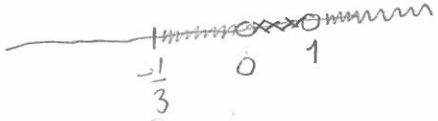
$(x+1)^2 > x^2 - x$

$x^2 + 2x + 1 > x^2 - x$

$x^2 + 2x - x^2 + x > -1$

$3x > -1$

$x > -\frac{1}{3}$ //



NPR ZA $x = 0$

NEJEDNADŽBA NIJE

ZADOVOLJENA

Ivan Vukašina

4.b)

Lijeva asimptota Horiz

$$\lim_{x \to -\infty} = \sqrt{x^2+4x+3} = \lim_{x \to \infty} \sqrt{x^2-4x+3} \cdot \frac{\sqrt{x^2-4x+3}}{\sqrt{x^2-4x+3}}$$

$$\lim_{x \to +\infty} = \frac{x^2-4x+3}{\sqrt{x^2-4x-3}} \to +\infty \;//$$ Nema lijevu hor. asimtotu

Lijeva kosa asimptota

$$\lim_{x \to -\infty} \frac{f(x)}{-x} = \frac{\sqrt{(-x)^2+4(-x)+3}}{-x} = \frac{\sqrt{x^2-4x+3}}{-x} \;/:x \;/:-x = \frac{-1}{-1} = -1 \;//$$

$$\lim_{x \to -\infty} \sqrt{x^2-4x+3} + x \cdot \frac{\sqrt{x^2-4x+3}-x}{\sqrt{x^2-4x+3}-x} = \frac{x^2-4x+3-x^2}{\sqrt{x^2-4x+3}-x} \;/:x$$

$$\lim_{x \to -\infty} \frac{-4}{1-1} = \frac{-4}{0} = \infty \;//$$ Nema lijeve kose asimptote!

BODUJE SE SKICA GRAFA.

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: MARKO ĆULINA **BROJ INDEKSA:** 17-1-0008-2010

USMENI - PROF UGLEŠIĆ

C7

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Među kompleksnim brojevima odrediti rješenja jednadžbe $z^3 = -(-i)^{548}$. *Prikazati rješenja u kompleksnoj ravnini!* 10+5

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} 5x + 4z + 2t &= 3 \\ x - y + 2z + t &= 1 \\ 4x + y + 2z &= 1 \\ x + y + z + t &= 0 \end{aligned}$$

3. Odrediti kada je $\frac{x+1}{\sqrt{x^2-x}} - 1 > 0$. Obavezno uvrštavanjem provjeriti rješenje jednadžbi koje se javljaju tokom rješavanja nejednadžbe. 13+2

4. Za funkciju: $f(x) = \sqrt{x^2+4x+3}$ treba:

   (a) pronaći drugu derivaciju 10

   (b) na temelju ispitivanja toka funkcije skicirati graf 20(graf)

5. Odrediti i provjeriti rješenje $\lim_{x\to+\infty} \left(\frac{n}{n-4}\right)^n =$ 8+2

6. Grafički odrediti rješenja nejednadžbe: $\ln(x-1) < \arccos x$. *Provjeriti uvrštavanjem!* 10+5

Ukupno: 16

MARKO ĆULINA

2/

$$\left|\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right| \rightarrow \left|\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right| \begin{array}{l} \\ R2-5R1 \\ R3-4R1 \\ R4-R1 \end{array} \left|\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right| \begin{array}{l} \\ \\ \\ R2:\frac{1}{5} \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & -\frac{2}{5} \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right| \begin{array}{l} R1+R2 \\ \\ R3-5R2 \\ R4-2R2 \end{array} \left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & -6/5 & -3/5 & -2/5 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 7/5 & 6/5 & -1/5 \end{array}\right| \rightarrow$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & -6/5 & -3/5 & -2/5 \\ 0 & 0 & 7/5 & 6/5 & -1/5 \\ 0 & 0 & 0 & -1 & -1 \end{array}\right| R3:\frac{5}{7} \left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & -6/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 6/7 & -1/7 \\ 0 & 0 & 0 & -1 & -1 \end{array}\right| \begin{array}{l} R1-\frac{4}{5}R3 \\ R2+\frac{6}{5}R3 \\ \\ \\ \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & -2/7 & 5/7 \\ 0 & 1 & 0 & 3/7 & -4/7 \\ 0 & 0 & 1 & 6/7 & -1/7 \\ 0 & 0 & 0 & -1 & -1 \end{array}\right| R4\cdot(-1) \left|\begin{array}{cccc|c} 1 & 0 & 0 & -2/7 & 5/7 \\ 0 & 1 & 0 & 3/7 & -4/7 \\ 0 & 0 & 1 & 6/7 & -1/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right| \begin{array}{l} R1+\frac{2}{7}R4 \\ R2-\frac{3}{7}R4 \\ R3-\frac{6}{7}R4 \\ \\ \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right| \left(\begin{array}{c} x \\ y \\ z \\ t \end{array}\right) \quad \text{PROVJERA} \quad \left|\begin{array}{cccc} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right| \left(\begin{array}{c} 1 \\ -1 \\ -1 \\ 1 \end{array}\right) = \left|\begin{array}{l} 5\cdot 1+0\cdot(-1)+4\cdot(-1)+2\cdot 1=3 \\ 1\cdot 1+(-1)\cdot(-1)+2\cdot(-1)+1\cdot 1=1 \\ 4\cdot 1+1\cdot(-1)+2\cdot(-1)+0\cdot 1=1 \\ 1\cdot 1+1\cdot(-1)+1\cdot(-1)+1\cdot 1=0 \end{array}\right|$$

RJEŠENJE

4) $f(x) = \sqrt{x^2+4x+3}$

$f'(x) = (x^2+4x+3)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x^2+4x+3)^{\frac{-3}{2}} \cdot (x^2+4x+3)'$ ×

$f'(x) = \frac{1}{2}(x^2+4x+3)^{\frac{-3}{2}} \cdot (2x+4)$ ×

$f'' = \left(\frac{1}{2}(x^2+4x+3)^{\frac{-3}{2}}\right)' \cdot (2x+4) + \left(\frac{1}{2}(x^2+4x+3)^{\frac{-3}{2}}\right) \cdot (2x+4)'$

$f'' = \left(\frac{3}{4}(x^2+4x+3)^{\frac{-5}{2}} \cdot (x^2+4x+3)'\right) + \left(\frac{1}{2}(x^2+4x+3)^{\frac{-3}{2}} \cdot (2)\right)$

$f'' = \left(\frac{3}{4}(x^2+4x+3)^{\frac{-5}{2}} \cdot (2x+4)\right) + \frac{2}{2}(x^2+4x+3)^{\frac{-3}{2}}$

$f'' \quad \frac{3}{4}(x^2+4x+3)^{\frac{-5}{2}} \cdot (2x+4) + (x^2+4x+3)^{\frac{-3}{2}}$

4

b) $f(x) = x^2 + 4x + 3$ $\quad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$

$x^2 + 4x + 3 \geq 0$ $\quad x_{1,2} = \frac{-4 \pm 2}{2}$ $\quad x_1 = \frac{-4+2}{2} = \frac{-2}{2} = -1$

$a = 1$
$b = 4$
$c = 3$

$x_2 = \frac{-4-2}{2} = \frac{-6}{2} = -3$

$D(f) = \langle -\infty, -3 \rangle \cup \langle -1, +\infty \rangle$

ASIMPTOTE

$\lim_{x \to -1} \sqrt{x^2 + 4x + 3} = \sqrt{(-1)^2 + 4 \cdot (-1) + 3} = 0$

$\lim_{x \to -3} \sqrt{x^2 + 4x + 3} = \sqrt{(-3)^2 + 4 \cdot (-3) + 3} = 0$

HORIZONTALNA

$\lim_{x \to +\infty} \sqrt{x^2 + 4x + 3} = \lim_{x \to \infty} \sqrt{x^2 + 4x + 3} : x^2 = \lim_{x \to \infty} = 1$

$\lim_{x \to -1^-} \sqrt{x^2 + 4x + 3} = \lim_{x \to \infty} \sqrt{x^2 + 4x + 3} : x^2 = \lim_{x \to \infty} 1$

1 - HORIZONTALNA ASIMPTOTA

NULTOČKE

NA OSI $y = 0$

$x^2 + 4x + 3 = 0$

$x_1 = -1$
$x_2 = -3$

$N_1(-1, 0)$
$N_2(-3, 0)$

NA OSI $x = 0$

$y = 0^2 + 4 \cdot 0 + 3$

$y = 3$

$N_3(0, 3)$

GRAF ?

6) $\ln(x-1) < \arccos(x)$

MARKO ČULINA

$\ln x - 1 < 0$

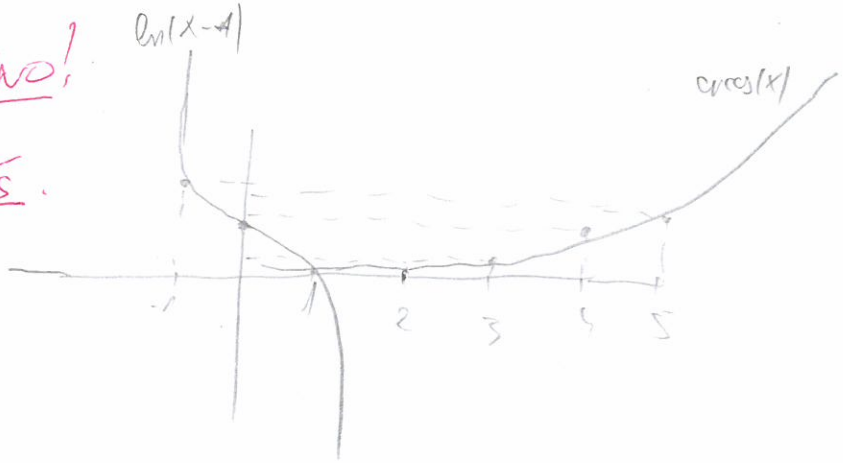
$\arccos x < 0$

x	0	1	2	3	4	5
$\ln(x-1)$	/	/	0	0.69	1.1	1.4

x	-2	-1	0	1	2	3	4
$\arccos x$	$\emptyset$	π	1.6	0	/		

POGREŠNO!

VIDI RADAS.



3) $\frac{x+1}{\sqrt{x^2-x}} - 1 > 0$

OVO VRIJEDI ZA SVE $x > 1$ X

$x^2 - x \geq 0$

$x(x-1) \geq 0$

$x = 0 \quad x - 1 = 0$

$x \geq 1$

PROVJERA UVRŠTAVANJE (-1)

$\frac{-1+1}{\sqrt{-1^2+1}} - 1 = -1$ (REZULTAT NE VRIJEDI)

UVRŠTAVANJE-2

$\frac{2+1}{\sqrt{2^2-2}} - 1 = 1,12$ (REZULTAT VRIJEDI)

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ C7

IME I PREZIME: Franko Babarić **BROJ INDEKSA:** 0269070613

POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 15

2. $\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right] \begin{array}{l} II - I \\ III - 4I \\ IV - 5I \end{array}$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] \begin{array}{l} \\ II - III \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] \begin{array}{l} \\ III + 3II \\ IV + 5II \\ \\ \end{array} \sim$

$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 7 & 8 & 1 \\ 0 & 0 & 14 & 17 & 3 \end{array}\right] \begin{array}{l} \\ \\ \\ IV - 2III \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 7 & 8 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} I - IV \\ II - 4IV \\ III - 8IV \\ \\ \end{array}$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 7 & 0 & -7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} I - III \\ II - 3III \\ \\ \\ \end{array}$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} I - II \\ \\ \\ \\ \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$ PROVJERA ✓

$x+y+z+t=0$
$1-1-1+1=0$
$0=0$ //✓

$4x+4y+2z=1$
$4-1-2+0=1$
$1=1$ //✓

$x-y-z+t=1$
$1+1-2+1=1$
$1=1$ //✓

$5x+4z+2t=3$
$5-4+2=3$
$3=3$ //✓

1. $z^3 = -(-i)^{5+8}$

$z^3 = -(-\sqrt{-1})^2$

$z^3 = -(+1)$

$z^3 = -1$

$(x+iy)^3 = -1$ ✗

5. $\lim_{x \to \infty} \left(\frac{n}{n-4}\right)^n = \left(\frac{\infty}{\infty - 4}\right)^\infty = \frac{\infty}{\infty}$ ✗

3. $\frac{x+1}{\sqrt{x^2-x}} - 1 > 0$

$\frac{x+1}{\sqrt{x^2-x}} - 1 = 0 \quad / \cdot \sqrt{x^2-x}$

$x+1 - \sqrt{x^2-x} = 0 \quad /^2$ ✗

[illegible]

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇊ C7

IME I PREZIME: Hrnjez Dijana **BROJ INDEKSA:** 0269068730

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Među kompleksnim brojevima odrediti rješenja jednadžbe $z^3 = -(-i)^{548}$. *Prikazati rješenja u kompleksnoj ravnini!* 10+5

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje: 10+5

$$\begin{aligned} 5x + 4z + 2t &= 3 \\ x - y + 2z + t &= 1 \\ 4x + y + 2z &= 1 \\ x + y + z + t &= 0 \end{aligned}$$

3. Odrediti kada je $\frac{x+1}{\sqrt{x^2-x}} - 1 > 0$. Obavezno uvrštavanjem provjeriti rješenje jednadžbi koje se javljaju tokom rješavanja nejednadžbe. 13+2

4. Za funkciju: $f(x) = \sqrt{x^2+4x+3}$ treba:

   (a) pronaći drugu derivaciju 10

   (b) na temelju ispitivanja toka funkcije skicirati graf 20(graf)

5. Odrediti i provjeriti rješenje $\lim_{x\to+\infty}\left(\frac{n}{n-4}\right)^n =$ 8+2

6. Grafički odrediti rješenja nejednadžbe: $\ln(x-1) < \arccos x$. *Provjeriti uvrštavanjem!* 10+5

Ukupno: 0

$\lim_{x\to 0}\left(\frac{n}{n-4}\right)$

2. $\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right] \begin{matrix} \cdot(-1) \\ \\ \cdot(-4) \\ \cdot(-5) \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] \begin{matrix} \\ \cdot(3) \\ \cdot(5) \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -4 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} & -3 & \frac{11}{2} \end{array}\right] \begin{matrix} \\ \\ \cdot(-1) \\ \cdot(1) \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & -4 & \frac{5}{2} \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \cdot\left(-\frac{1}{2}\right)$

$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 8 & -5 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$

$t = 3$

$z + 8t = -5$

$z = -5 - 24$

$z = -29$

$y + \frac{1}{2}z = \frac{1}{2}$

$y = \frac{1}{2} + \frac{29}{2}$

$y = 15$

$A = \begin{bmatrix} 1 \\ 15 \\ -29 \\ 3 \end{bmatrix}$

$x + 15 - 29 + 3 = 0$

$x = 11$

$5x + y - 12z = 1$

$44 + 15 + 58 =$

$59 - 58 = 1$

$1 = 1$

5. $\lim_{x\to\infty}\left(\frac{n}{n-4}\right)^n = \lim_{x\to\infty}\left(\frac{n}{4}+\frac{n}{n-4}\right) = \lim_{n\to\infty}\left(1+\frac{n}{\left(\frac{1}{n-4}\right)^{\frac{1}{4}}}\right)^4 = e^4$ ✗

3 $\frac{x+1}{\sqrt{x^2+x}} - 1 > 0$

vyjeti: 1) $\sqrt{x^2+x} \geq 0 \,/^2$ 2) $\frac{x+1}{\sqrt{x^2+x}} \neq 0 \,/\cdot\sqrt{x^2+x}$

$x^2+x \geq 0$

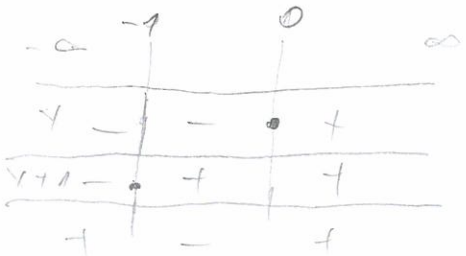
$x+1 \neq 0$

$x \neq -1$

$x^2+x=0$

$x(x+1)=0$

$x=0 \quad x=-1$



$D_f = \langle -\infty, -1\rangle \cup \langle 0, +\infty\rangle$ ✗

4. $f(x) = \sqrt{x^2+4x+3}^2$

$= x^2+4x+3$

$f'(x) = 2x+4$ ✗

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IME I PREZIME: MARTIN ŠOŠA **BROJ INDEKSA:** 17-1-0097-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Među kompleksnim brojevima odrediti rješenja jednadžbe $z^3 = -(-i)^{548}$. *Prikazati rješenja u kompleksnoj ravnini!* 10+5
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6. Grafički odrediti rješenja nejednadžbe: $\ln(x-1) < \arccos x$. *Provjeriti uvrštavanjem!* 10+5

Ukupno: 0

2.

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ -4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R1\cdot(-5)+R2 \\ R1\cdot(-4)+R3 \\ R1\cdot(-1)+R4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 1 & -1 & 0 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & -1 & -4 & 2 \\ 0 & 0 & -1 & -3 & 3 \end{array}\right] \begin{array}{l} R2+R1 \\ R2\cdot(-5)+R3 \\ R2\cdot(-5)+R4 \end{array} \quad \begin{array}{l} /(-1) \sim \\ R3\cdot(-1)+R1 \\ R3+R2 \\ R3+R4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & 4 & -3 \\ 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & -7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R4\cdot(3)+R1 \\ R4\cdot(-4)+R2 \\ R4\cdot(-4)+R3 \end{array} \quad \Rightarrow$$

$x = 5$
$y = -7$
$z = -6$
$t = 1$ ✗

$5x + 4z + 2t = 3$

$5 \cdot 5 + 4 \cdot (-6) + 2 \cdot 1 = 3$

$25 + (-24) + 2 = 3$

$25 - 24 + 2 = 3$

$1 + 2 = 3$ //

$x - y + 2z + t = 1$

$5 - (-7) + 2(-6) + 1 = 1$

$12 + (-12) + 1 = 1$

$1 = 1$ //

9. $\dfrac{x+1}{\sqrt{x^2-x}} - 1 > 0$

$x^2 - x \geq 0$

$x(x-1) \geq 0 \quad / : x$

$x - 1 \geq 0$

$x \geq 1$

$\sqrt{x^2-x} \neq 0 \qquad x+1 \neq 0$

za $x = 1 \Rightarrow \sqrt{x^2-x} = \sqrt{1-1} = \sqrt{0} = 0$

a $\sqrt{x^2-x} \neq 0$ znači $x \neq 1$

ili $x > 1$

$x + 1 = \sqrt{x^2-x}$

$x - \sqrt{x^2-x} = -1$

$x - (x^2-x)^{\frac{1}{2}} = -1$ ✗ JAKO POGREŠNO

$x - (x - x^2)^{\frac{1}{2}} = -1$

$-x^{\frac{1}{2}} = -1 \; / ^2$

$x = 1$

$x = -1$

$\sqrt{-1^2 - (-1)}$

$\sqrt{1+1}$

$\sqrt{2}$

$\dfrac{x+1}{\sqrt{x^2-x}} - 1 > 0$

za $\langle 1, +\infty \rangle$

VIDI KNEŽEVIĆ ✗

MARTIN ŠOŠA ⇒ 17-1-0097-20M

4. $f(x) = \sqrt{x^2+4x+3}$

b) $x^2+4x+3 \geq 0$

$a = +1$ ∪

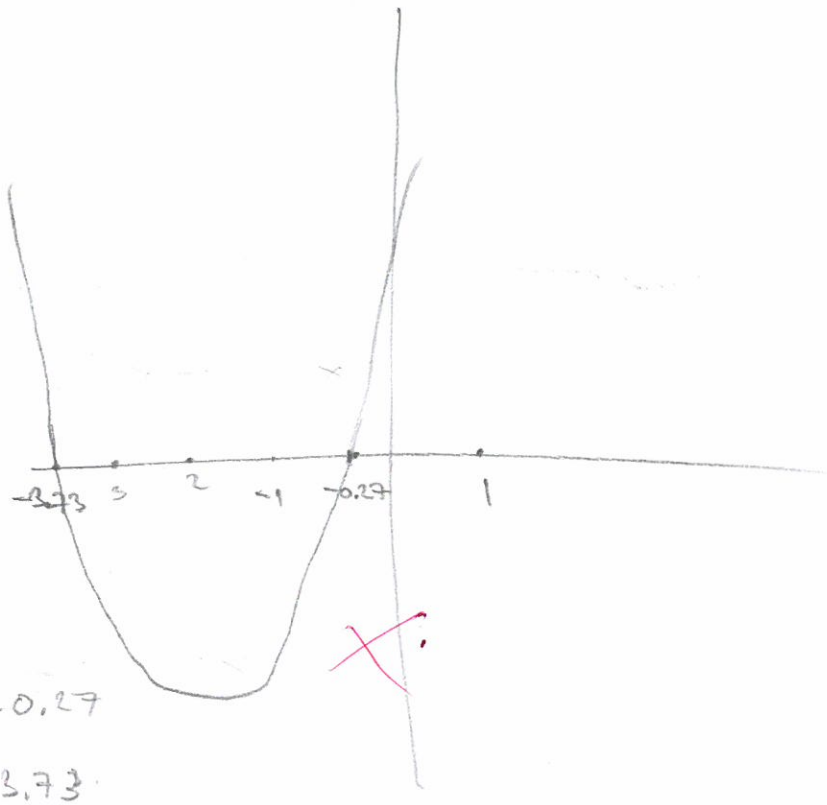
$x_{1,2} = \frac{-b \pm \sqrt{4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4 \cdot 1 \cdot 3}}{2}$

$= \frac{-4 \pm \sqrt{12}}{2}$

$x_1 = \frac{-4+\sqrt{12}}{2} = -0{,}27$

$x_2 = \frac{-4-\sqrt{12}}{2} = -3{,}73$



a) $f'(x) = (x^2+4x+3)^{\frac{1}{2}}$

$= \frac{1}{2}(x^2+4x+3)^{-\frac{1}{2}} + (2x+4)^{\frac{1}{2}}$ ×

$f'(x) = \frac{1}{2\sqrt{x^2+4x+3}} + \sqrt{2x+4}$

$f''(x) = \frac{1' \cdot 2\sqrt{x^2+4x+3} - 1 \cdot (2\sqrt{x^2+4x+3})'}{(2\sqrt{x^2+4x+3})^2} + \frac{1}{2\sqrt{2x+4}} \cdot \sqrt{2}$

$= \frac{2\sqrt{x^2+4x+3} - 1 \cdot \frac{1}{2\sqrt{x^2+4x+3}}}{2(x^2+4x+3)} + \frac{1}{2\sqrt{2x+4}} \cdot \sqrt{2}$

$= \frac{2\sqrt{x^2+4x+3} - \frac{1}{2\sqrt{x^2+4x+3}}}{2x^2+8x+6} + \frac{1}{2\sqrt{2x+4}} \cdot \sqrt{2}$

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IME I PREZIME: MARKO MILOLOVIĆ **BROJ INDEKSA:** 17-2-0146-2011

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 0

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$$\left[\begin{array}{cccc|c} 1 & 0 & \tfrac{3}{2} & 1 & 1 \\ 0 & 1 & -\tfrac{1}{2} & 0 & 0 \\ 0 & 0 & \tfrac{5}{2} & -3 & -3 \\ 0 & 0 & 5 & -3 & -2 \end{array}\right] \cdot\tfrac{2}{5} \sim \left[\begin{array}{cccc|c} 1 & 0 & \tfrac{3}{2} & 1 & 1 \\ 0 & 1 & -\tfrac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & -\tfrac{6}{5} & -\tfrac{6}{5} \\ 0 & 0 & 5 & -3 & -2 \end{array}\right] \begin{matrix} \\ \\ \cdot(-\tfrac{3}{2}) \\ \cdot(\tfrac{1}{2}) \\ \cdot(-\tfrac{5}{2}) \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \tfrac{9}{5} & \tfrac{9}{5} \\ 0 & 1 & 0 & & \\ 0 & 0 & 1 & -\tfrac{6}{5} & -\tfrac{6}{5} \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

LEMA RJEŠENJA

4. $f(x) = \sqrt{x^2+4x+3}$

$$f'(x) = \frac{1}{2\sqrt{x^2+4x+3}} + 2x+4$$

$$f'(x) = \frac{2x+4}{2\sqrt{x^2+4x+3}}$$

$$f''(x) = \frac{2\left(2\sqrt{x^2+4x+3}\right) - 2x+4\left(2x+4\right)}{\left(2\sqrt{x^2+4x+3}\right)^2}$$ ✗

$$f''(x) = \frac{4\sqrt{x^2+4x+3} - 4x^2-8x+8x+16}{\left(2\sqrt{x^2+4x+3}\right)^2}$$

$$f''(x) = \frac{4\sqrt{x^2+4x+3} - 4x^2+16}{\left(2\sqrt{x^2+4x+3}\right)^2}$$ //