MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B8

IME I PREZIME: MIHOVIL PEDISIĆ **BROJ INDEKSA:** 1M-2-0253-2012

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* ~~10+5~~

2. Riješiti sustav 4 jednadžbe s 4 nepoznanice $\begin{matrix} 5x+4z+2t=3 \\ x-y+2z+t=1 \\ 4x+y+2z=1 \\ x+y+z+t=0 \end{matrix}$ i obavezno provjeri rješenje: 10+5

3. Za funkciju $f(x) = \frac{\sin(2x)}{x}$ odrediti koliko iznosi $f'(\pi)$. ~~8+2~~

4. Za funkciju: $g(x) = \sqrt{x^2+4x+3}$ treba:
   (a) ispitati domenu ~~5~~
   (b) pronaći lokalne ekstreme 11
   (c) ispitati asimptote ~~14~~

5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2+5}{x-4}$ ~~20~~ (graf) 10

6. Izračunati i obavezno uvrštavanjem provjeriti koliko iznosi $\lim_{x\to -1}\left(\frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3}\right)$. ~~8+2~~

Ukupno: 35

2.

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -1 & -3 & -3 & 1 \end{array}\right] \sim$$

$1r \leftrightarrow 4r$; $2r - 1r$; $3r - 4\cdot 1r$; $4r - 5\cdot 1r$; $2r\cdot(-1)$; $4r + 2\cdot 2r$; $4r\cdot(-1)$; $4r \leftrightarrow 2r$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 3 & -1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 3 & -1 \\ 0 & 0 & 7 & 5 & -2 \\ 0 & 0 & -7 & -6 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 3 & -1 \\ 0 & 0 & 7 & 5 & -2 \\ 0 & 0 & 0 & -1 & -1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 3 & -1 \\ 0 & 0 & 7 & 5 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 7 & 0 & -7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \sim$$

$3r + 3\cdot 2r$; $4r - 2\cdot 2r$; $4r + 3r$; $4r\cdot(-1)$; $3r - 5\cdot 4r$; $2r - 3\cdot 4r$; $1r - 4r$; $3r : 7$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$2r - 3\cdot 3r$; $1r - 3r$; $1r - 2r$

$x = 1$
$y = -1$
$z = -1$
$t = 1$

PROVJERA: $5x+4z+2t=3$; $5\cdot 1 + 4\cdot(-1) + 2\cdot 1 = 3$; $5 - 4 + 2 = 3$; $3 = 3$

$x+y+z+t=0$; $1-1-1+1=0$; $0=0$

$x-y+2z+t=1$; $1+1+2\cdot(-1)+1=1$; $1+1=2+1=1$; $1=1$

$4x+y+2z=1$; $4\cdot 1+1+2\cdot(-1)=1$; $4-1-2=1$; $1=1$

✓

3. $f(x) = \frac{\sin(2x)}{x}$

$(\sin(2x))' = \cos(2x) \cdot (2x)'$

$2\cos(2x)$

$f'(x) = \frac{(\sin(2x))' \cdot x - \sin(2x) \cdot x'}{x^2}$

$f'(x) = \frac{2\cos(2x) \cdot x - \sin(2x)}{x^2}$

$f'(x) = \frac{2x\cos(2x) - \sin(2x)}{x^2}$ ✓

$f'(\pi) = \frac{2\pi \cdot \cos(2\pi) - \sin(2\pi)}{\pi^2} = 0,636619772$ ✓

4. $g(x) = \sqrt{x^2+4x+3}$

a) DOMENA

$\mathbb{D}(g(x)) = \mathbb{R}$ ✗

c) asimptote

V.A ✗

H.A $\lim\limits_{x\to\infty} \frac{\sqrt{x^2+4x+3} \cdot \sqrt{x^2+4x+3}}{\sqrt{x^2+4x+3}} = \lim\limits_{x\to\infty} \frac{x^2+4x+3 \,/:x^2}{\sqrt{x^2+4x+3}\,/:x^2} =$

$= \lim\limits_{x\to\infty} \frac{1+\frac{4}{x}+\frac{3}{x^2}}{\frac{1}{x}+\frac{4}{x^2}+\frac{3}{x^6}} = \frac{1}{0} = \infty$ nema H.A.

K.A $k = \frac{f(x)}{x} = \lim\limits_{x\to+\infty} \frac{\sqrt{x^2+4x+3}}{x} \overset{1:x}{=} \lim\limits_{x\to\infty} \frac{1}{1} = 1$

$l = [f(x) - kx] = \lim\limits_{x\to\infty} [\sqrt{x^2+4x+3} - x] = \lim\limits_{x\to+\infty} \left[(\sqrt{x^2+4x+3} - x) \cdot \frac{x}{x}\right]$

$\lim\limits_{x\to+\infty} \frac{(\sqrt{x^2+4x+3} - x)\,x}{x}$?

MIHOVIL PEDISIĆ

5. $h(x) = \frac{x^2+5}{x-4}$

1] DOMENA

$x-4=0$

$x=4$

$D(h(x)) = \langle -\infty, 4\rangle \cup \langle 4, +\infty\rangle$

2] NUL TOCKE

$x^2+5=0$

$x^2=-5$

$x=\sqrt{(-5)}$

$x_1 = -i\sqrt{5}$

$x_2 = i\sqrt{5}$

3] EKSTREMI, RAST-PAD

$h'(x) = \frac{(x^2+5)'\cdot(x-4)-(x^2+5)\cdot(x-4)'}{(x-4)^2}$

$h(x)' = \frac{2x\cdot(x-4)-(x^2+5)}{(x-4)^2} = \frac{2x^2-8x-x^2-5}{(x-4)^2}$

$h(x)' = \frac{x^2-8x-5}{(x-4)^2}$

$x^2-8x-5=0$

$x_{1,2} = \frac{8\pm\sqrt{64+20}}{2}$

$= \frac{8\pm\sqrt{4\cdot 21}}{2} = \frac{8\pm 2\sqrt{21}}{2}$

$x_1 = \frac{8+2\sqrt{21}}{2}$

$x_2 = \frac{8-2\sqrt{21}}{2}$

OVAKO TREBA:

	$-\infty$	$\frac{8-2\sqrt{21}}{2}$	4	$\frac{8+2\sqrt{21}}{2}$	$+\infty$
$f(x)$	+	−	−	+	
$f'(x)$	↗	↘	↘	↗	

	$-\infty$	$\frac{8-2\sqrt{21}}{2}$	4	$\frac{8+2\sqrt{21}}{2}$	$+\infty$
$f(x)$	−	−	+	+	
$f'(x)$	↘	↘	↗	↗	

4] ASIMPTOTE

V.A $\lim_{x\to 4} \frac{x^2+5}{x-4} = \frac{16+5}{0} = +\infty$ $\boxed{x=4}$ V.A.

H.A. - nema jer je stupanj brojnika veći od stupnja nazivnika

K.A. $k = \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\frac{x^2+5}{x-4}}{x} = \lim_{x\to+\infty} \frac{x^2+5}{x^2-4x} \,|:x = 1$

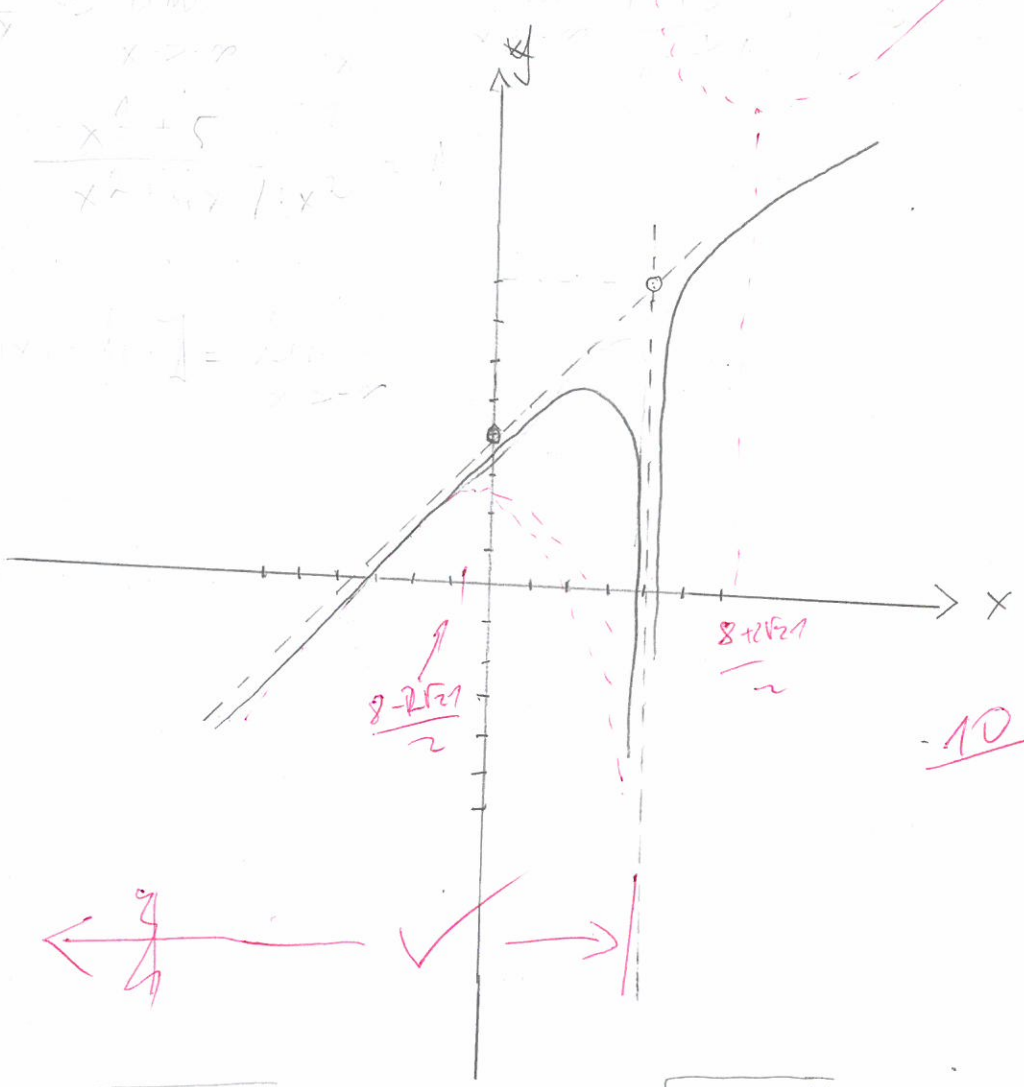
$l = [f(x) - kx] = \lim_{x\to+\infty} \left[\frac{x^2+5}{x-4} - x\right] = \lim_{x\to+\infty} \frac{x^2+5-x^2+4x}{x-4} =$

$\lim_{x\to+\infty} \frac{4x+5}{x-4} \,|:x = 4$

$y = kx + l$

$\boxed{y_1 = x+4}$ lijeva i desna

5. zadatak



6. $\lim_{x \to -1^+} \left(\frac{\sqrt{x^2-8x+17} - 3 + x}{x^2+4x+3} \right) = \lim_{x \to -1} \frac{\sqrt{1+8+17} - 3 - 1}{1-4+3} =$

$= \lim_{x \to -1^+} \frac{\sqrt{16} - 3 - 1}{1-4+3} = \frac{4-3-1}{1-4+3} = \frac{0}{0^+} = +\infty$

$-0,811$
$-0,19$

$\lim_{x \to -1} = -\infty$

x	$\frac{\sqrt{x^2-8x+17} - 3 + x}{x^2+4x+3}$
10	0,085289
100	0,018548
1000	+0,0000070326

OVDJE DVIJE INFORMACIJE SE NE PODUDARAJU, A TREBALE BI.

MIHOVIL PEDISIĆ

1. $\left|\frac{z}{2}\right|^2 = z+1+i$

$\frac{x^2+y^2}{4} = x+yi+1+i \quad 4$

$z = \frac{x^2+y^2}{4} - 1 - i$

$z = \frac{x^2+y^2-4-4i}{4} \quad /\cdot 4$

$4z = x^2+y^2-4-4i \quad /\sqrt{\ }$

$2\sqrt{z} = \sqrt{x^2+y^2} - \sqrt{4-4i}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇊

IME I PREZIME: Luka Žilić **BROJ INDEKSA:** 17-2-0208-2012

B8

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* — 10+5

2. Riješiti sustav 4 jednadžbe s 4 nepoznanice $\begin{array}{l} 5x + 4z + 2t = 3 \\ x - y + 2z + t = 1 \\ 4x + y + 2z = 1 \\ x + y + z + t = 0 \end{array}$ i obavezno provjeri rješenje: — 10+5

3. Za funkciju $f(x) = \frac{\sin(2x)}{x}$ odrediti koliko iznosi $f'(\pi)$. — 8+2

4. Za funkciju: $g(x) = \sqrt{x^2 + 4x + 3}$ treba:
   (a) ispitati domenu — 5
   (b) pronaći lokalne ekstreme — 11
   (c) ispitati asimptote — 14

5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2 + 5}{x - 4}$ — 20 (graf)

6. Izračunati i obavezno uvrštavanjem provjeriti koliko iznosi $\lim_{x \to -1} \left(\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$. — 8+2

Ukupno: 35

1. $\left|\frac{z}{2}\right|^2 = z + 1 + i$

$\frac{|z|^2}{4} = z + 1 + i \;/\cdot 4$

$|z|^2 = 4z + 4 + 4i$

$\left(\sqrt{x^2 + y^2}\right)^2 = 4x + 4yi + 4 + 4i$

$x^2 + y^2 = 4x + 4yi + 4 + 4i$

Re: $x^2 + y^2 = 4x + 4$

Im: $0 = 4y + 4$

$-4y = 4 \;/\cdot \left(-\frac{1}{4}\right)$

$y = -1$ ✓

$x^2 + y^2 = 4x + 4$

$x^2 + (-1)^2 = 4x + 4$

$x^2 + 1 = 4x + 4$

$x^2 - 4x - 3 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

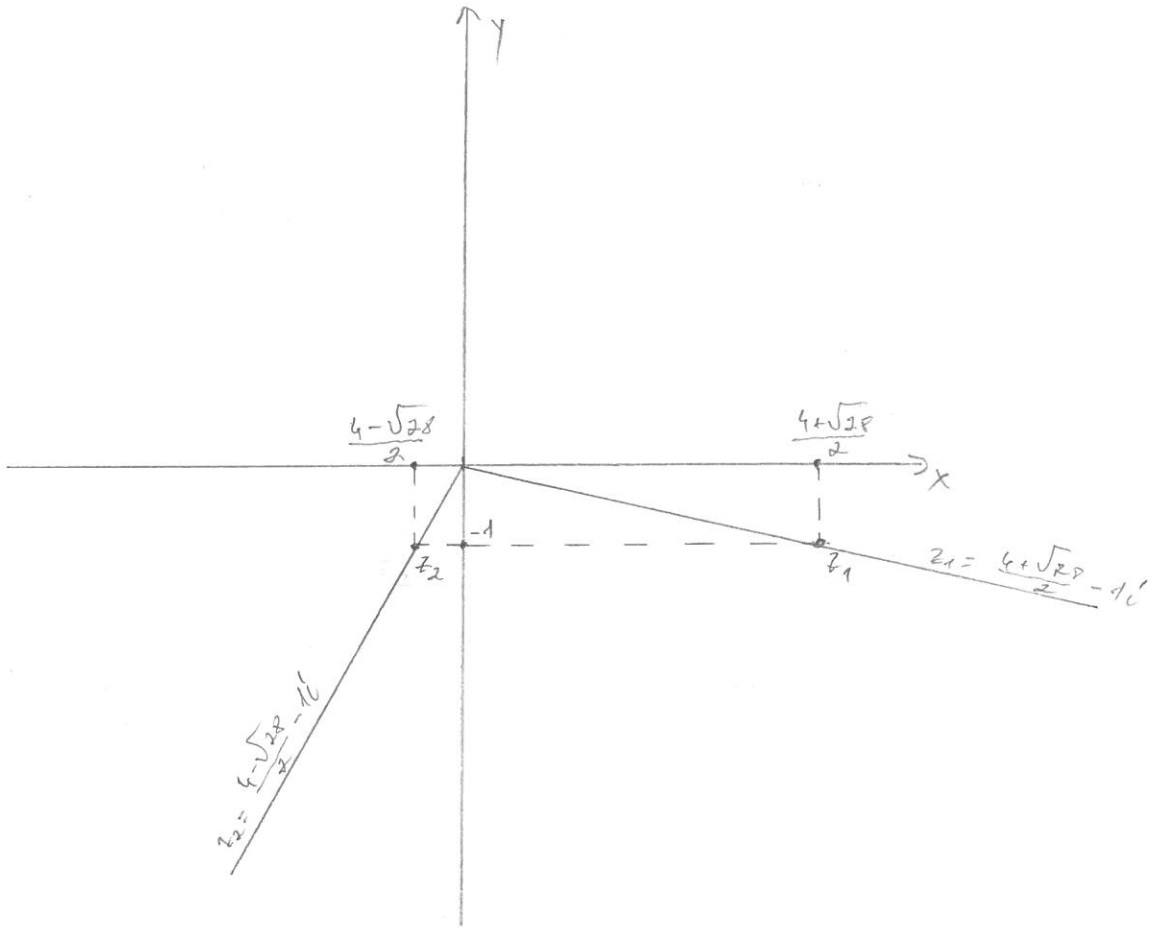
$x_{1,2} = \frac{4 \pm \sqrt{16 + 12}}{2}$

$x_{1,2} = \frac{4 \pm \sqrt{28}}{2}$

$x_1 = \frac{4 + \sqrt{28}}{2}$, $x_2 = \frac{4 - \sqrt{28}}{2}$

$z_1 = \frac{4 + \sqrt{28}}{2} - 1i$ ✓

$z_2 = \frac{4 - \sqrt{28}}{2} - 1i$ ✓



2.

$$\begin{aligned} 5x + 4z + 2t &= 3 \\ x - y + 2z + t &= 1 \\ 4x + y + 2z &= 1 \\ x + y + z + t &= 0 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \overset{4R \leftrightarrow 1R}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right] \begin{array}{l} 2R-1R \\ 3R-4R \\ 4R-5R \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] \cdot(-1) \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & 3 & 2 & 4 & -1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] \overset{2R+3R}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 3 & 2 & 4 & -1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] 1R-2R$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 3 & 2 & 4 & -1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right] \begin{array}{l} 3R-3\cdot 2R \\ 4R+5\cdot 2R \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & -7 & -8 & -1 \\ 0 & 0 & 14 & 17 & 3 \end{array}\right] 4R+2\cdot 3R$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & -7 & -8 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\begin{aligned} x - 2z - 3t &= 0 \\ y + 3z + 4t &= 0 \\ -7z - 8t &= -1 \\ t &= 1 \end{aligned}$$

$-7z - 8 \cdot 1 = -1$

$-7z - 8 = -1$

$-7z = 7 \mid \cdot \left(-\frac{1}{7}\right)$

$z = -1$ //

$y + 3 \cdot (-1) + 4 \cdot 1 = 0$

$y - 3 + 4 = 0$

$y = -4 + 3$

$y = -1$ //

$x - 2 \cdot (-1) - 3 \cdot 1 = 0$

$x + 2 - 3 = 0$

$x = 3 - 2$

$x = 1$ //

PROVJERA:

$5x + 4z + 2t = 3$

$5 \cdot 1 + 4 \cdot (-1) + 2 \cdot 1 = 3$

$5 - 4 + 2 = 3$

$3 = 3$ //

$x - y + 2z + t = 1$

$1 - (-1) + 2 \cdot (-1) + 1 = 1$

$2 - 2 + 1 = 1$

$1 = 1$ //

$4x + y + 2z = 1$

$4 \cdot 1 - 1 + 2 \cdot (-1) = 1$

$4 - 1 - 2 = 1$

$1 = 1$ //

$x + y + z + t = 0$

$1 - 1 - 1 + 1 = 0$

$0 = 0$ //

✓

3.

$$f(x) = \frac{\sin(2x)}{x}$$

$$f'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2} = \frac{-\cos(2x) \cdot 2 \cdot x - \sin(2x) \cdot 1}{x^2}$$ ✗

$$= \frac{-\cos(2x) \cdot 2x - \sin(2x)}{x^2}$$

$$f'(\pi) = \frac{-\cos(2\pi) \cdot 2\pi - \sin(2\pi)}{\pi^2} = -0,64$$

$f'(\pi) = -0,64$ //

4. $g(x) = \sqrt{x^2+4x+3}$

a) DOMENA:

$x^2+4x+3 \geq 0$

$a=1,\ b=4,\ c=3$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{-4 \pm \sqrt{16-12}}{2}$

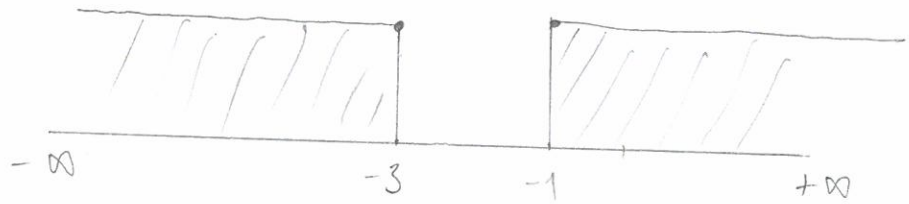
$x_{1,2} = \frac{-4 \pm \sqrt{4}}{2}$

$x_1 = \frac{-4+2}{2} = \frac{-2}{2} = -1$

$x_2 = \frac{-4-2}{2} = \frac{-6}{2} = -3$

	$-\infty$		-3		-1		$+\infty$
x^2+4x+3		$+$	0	$-$	0	$+$	

$D(f) = \langle -\infty, -3] \cup [-1, +\infty \rangle$ ✓



c) ASIMPTOTE:

V.A.:

$\lim_{x \to -3^-} \sqrt{x^2+4x+3} = \sqrt{(-3)^2+4\cdot(-3)+3} = 0^+$

$\lim_{x \to -1^+} \sqrt{x^2+4x+3} = \sqrt{(-1)^2+4\cdot(-1)+3} = 0^-$

$\lim_{x \to \infty} \sqrt{x^2+4x+3} = \sqrt{\infty+\infty+3} = \sqrt{\infty} = \infty$

TREBALO JE TRAŽITI KOSE ASIMPTOTE !!!

D.H.A.:

$\lim_{x \to -\infty} \frac{\sqrt{x^2+4x+3}}{1} = \frac{\sqrt{x^2+4x+3}}{1} \cdot \frac{\sqrt{x^2+4x+3}}{\sqrt{x^2+4x+3}} = \frac{x^2+16x^2+9}{\sqrt{x^2+4x+3}} \quad /:x = \frac{\frac{x}{x}+\frac{16x}{x}+\frac{9}{x}}{\sqrt{\frac{x}{x}+4+\frac{3}{x}}} =$

$= \frac{0}{2} = 0 //$

NEMAMO H. ASIMPTOTA

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ B8

IME I PREZIME: JOSIP PREDOVAN **BROJ INDEKSA:** 17-1-0126-2012

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

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3. Za funkciju $f(x) = \frac{\sin(2x)}{x}$ odrediti koliko iznosi $f'(\pi)$. 8+2

4. Za funkciju: $g(x) = \sqrt{x^2 + 4x + 3}$ treba:
   (a) ispitati domenu 5
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5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2 + 5}{x - 4}$ 20 (graf)

6. Izračunati i obavezno uvrštavanjem provjeriti koliko iznosi $\lim_{x \to -1} \left(\frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$. 8+2

Ukupno: 28

JOSIP PREDOVAN

MAT 1

③ $f(x) = \frac{\sin(2x)}{x}$

$f'(x) = \frac{(\sin(2x))' \cdot x - \sin 2x \cdot x'}{x^2}$

$f'(x) = \frac{\cos(2x) \cdot (2x)' \cdot x - \sin 2x \cdot 1}{x^2}$

$f'(x) = \frac{\cos(2x) \cdot 2 \cdot x - \sin 2x}{x^2}$

$f'(x) = \frac{\cos(2x) \cdot 2x - \sin 2x}{x^2}$ ✓ $f'(\pi) = ?$

④

$g(x) = \sqrt{x^2 + 4x + 3}$

① DOMENA

$x^2 + 4x + 3 \geq 0$

$a = 1$
$b = 4$
$c = 3$

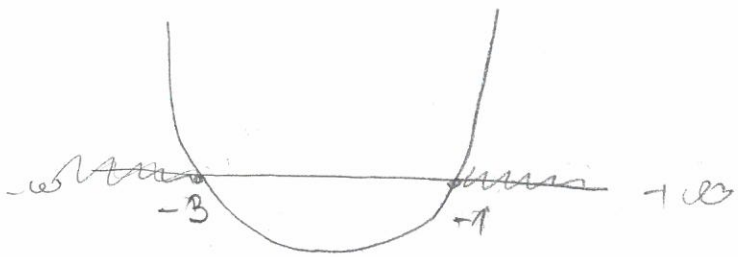
$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2 \cdot a}$

$x_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$

$x_{1,2} = \frac{-4 \pm 2}{2}$

$x_1 = -1$

$x_2 = -3$

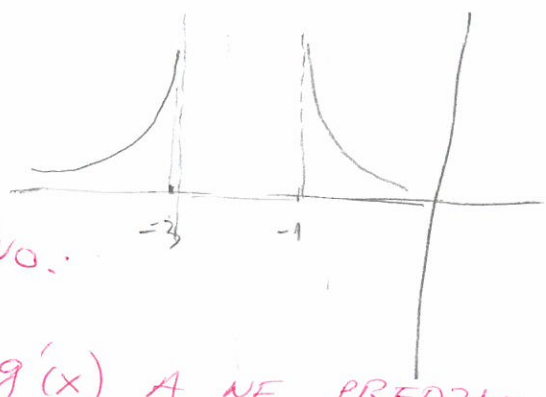


$Df \langle -\infty, -3] \cup [-1, +\infty \rangle$ ✓

⑤

	$-\infty$		-3		-1		$+\infty$
		-4			0		
$g'(x)$		$+$		N/P		$+$	
		↗				↗	
			LOK MAX			LOK MIN	

POGREŠNO..



TREBA ISPITATI PREDZNAK OD $g'(x)$, A NE PREDZNAK $g(x)$

c) ASIMPTOTE:

$x_1 = -1$

$x_2 = -3$

$$y(x) = \sqrt{x^2+4x+3}$$

V.A

$$\lim_{x \to (-1)^-} \sqrt{x^2+4x+3} = \lim_{x \to (-1)^-} \sqrt{(-1)^2+4\cdot(-1)+3} = 0^-$$

$$\lim_{x \to (-1)^+} \sqrt{x^2+4x+3} = \lim_{x \to (-1)^+} \sqrt{(-1)^2+4\cdot(-1)+3} = 0^+$$

$$\lim_{x \to (-3)^-} \sqrt{x^2+4x+3} = \lim_{x \to (-3)^-} \sqrt{(-3)^2+4\cdot(-3)+3} = 0^+$$

$$\lim_{x \to (-3)^+} \sqrt{x^2+4x+3} = \lim_{x \to (-3)^+} \sqrt{(-3)^2+4(-3)+3} = 0^-$$

NEMA V.A ✓

3 * 3/5

H.A

$$\lim_{x \to +\infty} \sqrt{x^2+4x+3} \,/ :x = \lim_{x \to \infty} \sqrt{\frac{x^2}{x^2}+\frac{4x}{x^2}+\frac{3}{x^2}} = \sqrt{1} = 1 \;//$$ ✗

$$\lim_{x \to -\infty} \sqrt{x^2+4x+3} = \lim_{x \to \infty} \sqrt{(-x)^2+4\cdot(-x)+3} = \lim_{x \to \infty} \sqrt{x^2-4x+3} \,/ : x$$

$$= \lim_{x \to \infty} \sqrt{\frac{x^2}{x^2}-\frac{4x}{x^2}+\frac{3}{x^2}} = \sqrt{1} = 1 \;//$$

IMA H.A. ✗

KOSE NEMA

JOSIP PREDOVAN

MAT 1

① $\left|\frac{z}{2}\right|^2 = 2+1\cdot i$

$x=4$
$y=2$

$\left|\frac{z}{2}\right|^2 = 2+1\cdot i \;/\cdot 2$

$|z|^2 = 2\cdot(2+1\cdot i)$

$|z|^2 = 4+2i$

$k=0$

$\operatorname{ctg}\varphi = \frac{y}{x} = \frac{2}{4} = 0{,}46°$

$|w| = \sqrt{x^2+y^2}$

$|w| = \sqrt{4^2+2^2}$

$|w| = 2\sqrt{5}$

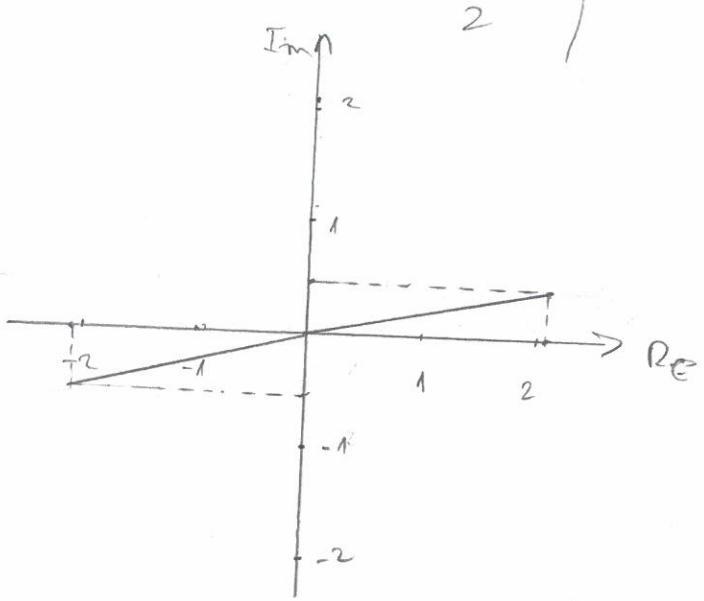
$z_1 = \sqrt{2\sqrt{5}}\cdot\left(\cos\frac{0{,}46+2\cdot 0\cdot\pi}{2} + i\sin\frac{0{,}46+2\cdot 0\cdot\pi}{2}\right)$

$z_1 = 2{,}06 + 0{,}48i$

$k=1$

$z_2 = \sqrt{2\sqrt{5}}\cdot\left(\cos\frac{0{,}46+2\cdot 1\cdot\pi}{2} + i\sin\frac{0{,}46+2\cdot 1\cdot\pi}{2}\right)$

$z_2 = -2{,}06 - 0{,}48i$



② $5x + 4z + 2t = 3$

$x - y + 2z + t = 1$

$4x + y + 2z = 1$

$x + y + z + t = 0$

$$\left|\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right| \quad \left|\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right| \quad \begin{array}{l} \text{II} - \text{I} \\ \text{III} - 4\cdot\text{I} \\ \text{IV} - 5\cdot\text{I} \end{array} \left|\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right| \quad \text{II} - \text{III}$$

$$\left|\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right| \quad \begin{array}{l} \text{I} - \text{II} \\ \text{III} + 3\cdot\text{II} \\ \text{IV} + 5\cdot\text{II} \end{array} \left|\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 7 & 8 & 1 \\ 0 & 0 & 14 & 17 & 3 \end{array}\right| \quad :7 \left|\begin{array}{cccc|c} 1 & 0 & -2 & -3 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 14 & 17 & 3 \end{array}\right| \quad \begin{array}{l} \text{I} + 2\cdot\text{III} \\ \text{II} - 3\cdot\text{III} \\ \text{IV} - 14\cdot\text{III} \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{5}{7} & \frac{2}{7} \\ 0 & 1 & 0 & \frac{4}{7} & -\frac{3}{7} \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & 1 \end{array}\right| \quad \begin{array}{l} \text{I} + \frac{5}{7}\cdot\text{IV} \\ \text{II} - \frac{4}{7}\cdot\text{IV} \\ \text{III} - \frac{8}{7}\cdot\text{IV} \end{array} \left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right|$$

$x = 1$

$y = -1$

$z = -1$

$t = 1$

PROVJERA UVRŠTAVANJEM :

① $5\cdot(1) + 4\cdot(-1) + 2\cdot 1 = 3$

$3 = 3$

②

$1 - (-1) + 2\cdot(-1) + 1 = 1$

$1 = 1$

③ $4\cdot 1 + (-1) + 2\cdot(-1) = 1$

$1 = 1$

✓

④ $1 + (-1) + (-1) + 1 = 0$

$0 = 0$

$h(x) = \frac{x^2+5}{x-4}$

$Df: \mathbb{R} \setminus \{4\}$

1° DOMENA

$x-4 \neq 0$

$x \neq 4$

2° GLOBALNA SVOJSTVA

$f(-x) = \frac{-(x^2)+5}{-x-4} = \left(\frac{x^2+5}{-x-4}\right)$

NITI PARNA NITI NEPARNA

NEPERIODIČNA

3° ASIMPTOTE

V.A

$\lim_{x \to 4^-} \frac{x^2+5}{x-4} = \frac{4^2+5}{4-4} = \frac{21}{0^-} = -\infty$

$\lim_{x \to 4^+} \frac{x^2+5}{x-4} = \frac{4^2+5}{4-4} = \frac{21}{0^+} = +\infty$

H.A

$\lim_{x \to +\infty} \frac{x^2+5}{x-4} \; / : x^2 = \lim_{x \to \infty} \frac{\frac{x^2}{x^2}+\frac{5}{x^2}}{\frac{x}{x^2}-\frac{4}{x^2}} = \frac{1}{0} = 0$

$\lim_{x \to -\infty} \frac{(-x)^2+5}{-x-4} \; / : x^2 = \lim_{x \to \infty} \frac{\frac{x^2}{x^2}+\frac{5}{x^2}}{-\frac{x}{x^2}-\frac{4}{x^2}} = \frac{1}{0} = 0$

KOSA

$k = \lim_{x \to \infty} \frac{f(x)}{x} = \left(\frac{\frac{x^2+5}{x-4}}{\frac{x}{1}}\right) = \frac{x^2+5}{x^2-4x} \; / : x^2 = \frac{\frac{x^2}{x^2}+\frac{5}{x^2}}{\frac{x^2}{x^2}-\frac{4x}{x^2}} = \frac{1}{1} = 1$

NEMA H.A

$l = \lim_{x \to \infty} (f(x) - k \cdot x) = \lim_{x \to \infty} \frac{x^2+5}{x-4} - x = \lim_{x \to \infty} \frac{\frac{x^2}{x^2}+\frac{5}{x^2}}{\frac{x}{x^2}-\frac{4}{x^2}} - \frac{x}{x^2} = \frac{1}{1} - 0$

$= 1$

BODUJE SE GRAF!!!

④ SJECIŠTA S KOORDINATNIM OSIMA

$h(x) = 0$

$h(0) = \frac{0^2+5}{0-5} = -1$

$\frac{x^2+5}{x-4} = 0$ NIJE

⑤ DERIVACIJE

$h'(x) = \frac{x^2+5}{x-4} = \frac{(x^2+5)' \cdot (x-4) - (x^2+5) \cdot (x-4)'}{(x-4)^2}$

$= \frac{2x \cdot (x-4) - (x^2+5) \cdot 1}{(x-4)^2}$

$= \frac{2x^2-8x-x^2+5}{(x-4)^2}$

JOSIP PREDOVAN

MAT 1

⑥ $\lim_{x\to(-1)} \left(\frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3} \right) = \frac{0}{0}$ L'H =

$\lim_{x\to(-1)} \frac{\left(\sqrt{x^2-8x+7}-3+x\right)'}{(x^2+4x+3)'} = \frac{\left((x^2-8x+7)^{\frac{1}{2}}-3+x\right)'}{2x+4} =$

$\lim_{x\to(-1)} = \frac{\frac{1}{2}\cdot(x^2-8x+7)^{-\frac{1}{2}}\cdot(x^2-8x+7)' + 1}{2x+4}$

$\lim_{x\to(-1)} \frac{\frac{1}{2}\cdot(x^2-8x+7)^{-\frac{1}{2}}\cdot(2x-8)+1}{2x+4} = \frac{x\cdot\frac{1}{\sqrt{x^2-8x+7}}-1}{2x+4}$ ✓ ✓

= ... JOŠ JE TREBALO SAMO UVRSTITI $x=-1$

= KRAJ

⑤ $\lim_{x\to-1} \left(\frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3} \right) \Big|:x^2 \Big|:x^2 = \lim_{x\to(-1)} \frac{\sqrt{\frac{x^2}{x^4}-\frac{8x}{x^4}+\frac{7}{x^4}}-\frac{3}{x^2}+\frac{x}{x^2}}{\frac{x^2}{x^2}+\frac{4x}{x^2}+\frac{3}{x^2}}$

$= \lim_{x\to(-1)} \frac{0}{1} = 0$

POGREŠNO!

OVAKO BI IŠLO KADA BI BIO $\lim_{x\to+\infty}$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ B8

IME I PREZIME: MAHAĐIĆ FRANE **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunaj sve kompleksne brojeve z takve da $\left|\frac{z}{2}\right|^2 = z + 1 + i$. *Prikaži ih u kompleksnoj ravnini!* 10+5

2. Riješiti sustav 4 jednadžbe s 4 nepoznanice $\begin{array}{l} 5x+4z+2t=3 \\ x-y+2z+t=1 \\ 4x+y+2z=1 \\ x+y+z+t=0 \end{array}$ i obavezno provjeri rješenje: 10+5

$$\left[\begin{array}{ccccc} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

3. Za funkciju $f(x) = \frac{\sin(2x)}{x}$ odrediti koliko iznosi $f'(\pi)$. 8+2

4. Za funkciju: $g(x) = \sqrt{x^2+4x+3}$ treba:
   (a) ispitati domenu 5
   (b) pronaći lokalne ekstreme 11
   (c) ispitati asimptote 14

5. Na temelju ispitivanja toka skicirati graf funkcije e $h(x) = \frac{x^2+5}{x-4}$ 20 (graf)

6. Izračunati i obavezno uvrštavanjem provjeriti koliko iznosi $\lim_{x\to -1}\left(\frac{\sqrt{x^2-8x+7}-3+x}{x^2+4x+3}\right)$. 8+2

Ukupno: 8

5) $h(x) = \frac{x^2+5}{x-4}$

1° Domena

$x-4 \neq 0$

$x \neq 4$

$D_f = \mathbb{R}\setminus\{4\}$

$h(x) = \frac{x^2+5}{x-4}$

$h'(x) = \frac{(x^2+5)'\cdot(x-4) - (x^2+5)\cdot(x-4)'}{(x-4)^2}$

$h'(x) = \frac{2x\cdot(x-4) - x^2+5\cdot 1}{(x-4)^2}$

$h'(x) = \frac{2x^2-4-x^2+5}{(x-4)^2}$

$h''(x) = \frac{(2x^2-4-x^2+5)'\cdot(x-4)^2 - 2x^2-4-x^2+5\cdot((x-4)^2)'}{(x-4)^4}$

$h''(x) = \frac{4x-2x\cdot(x-4)^2 - 2x-4-x^2+5\cdot 2x}{(x-4)^4}$

BODUJE SE GRAF!!!

3/ $f(x) = \frac{\sin(2x)}{x}$

$f'(x) = \frac{(\sin(2x))' \cdot x - \sin(2x) \cdot (x)'}{x^2}$

$f'(x) = \frac{\cos(2x) \cdot 2 \cdot x - \sin(2x) \cdot 1}{x^2}$

$f'(x) = \frac{\cos(2x) \cdot x - \sin(2x)}{x^2}$ ✓

$f'(\pi) = ?$

$f(\pi) = \frac{\sin(2\pi)}{\pi}$

$f'(\pi) = \frac{(\sin(2\pi))' \cdot \pi - \sin(2\pi) \cdot (\pi)'}{\pi^2}$

$f'(\pi) = \frac{\cos(2\pi) \cdot \pi - \sin(2\pi) \cdot \pi}{\pi^2}$

$f'(x) = \frac{(\sin(2x))' \cdot 2x \cdot x - \sin(2x) \cdot x}{x^2}$

$f'(x) = \frac{2\sin 2x \cdot x - \sin(2x) \cdot 1}{x^2}$

4) $f(x) = \sqrt{x^2+4x+3}$

a) domena

UVJET

$x^2+4x+3 \geq 0$

$a=1 \Rightarrow \cup$

$a=1 \quad b=4 \quad c=3$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{-4 \pm \sqrt{4^2-4 \cdot 1 \cdot 3}}{2 \cdot 1}$

$x_{1,2} = \frac{-4 \pm \sqrt{4}}{2}$

$x_{1,2} = \frac{-4 \pm 2}{2}$

$x_1 = \frac{-4+2}{2} = -1$

$x_2 = \frac{-4-2}{2} = \frac{-6}{2} = -3$

$x_1 = -1$

$x_2 = -3$

$Df = \mathbb{R} \setminus \{-3, 1\}$ ✗

c) ASIMPTOTE

$\lim_{x \to -3} \sqrt{x^2+4x+3} = \lim_{x \to -3} \sqrt{(-3)^2+4 \cdot (-3)+3} = 0$

VERTIKALNE ASIMPTOTE VEKA ✓

$\lim_{x \to -1} \sqrt{x^2+4x+3} = \lim_{x \to -1} \sqrt{(-1)^2+4 \cdot (-1)+3} = 0$

HORIZONTALNA ASIMPTOTA

$\lim_{x \to \pm\infty} \sqrt{x^2+4x+3} = \lim_{x \to \pm\infty} \sqrt{x^2+4x+3} \cdot \frac{\sqrt{x^2+4x+3}}{\sqrt{x^2+4x+3}}$

$= \frac{x^2+4x+3}{\sqrt{x^2+4x+3}} \Big/ \cdot x \Big/ \cdot x^2 = 1$ ✗

2

2) $\begin{bmatrix} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{bmatrix} \begin{matrix} /\cdot(-1) \\ \cdot(-4) \\ \cdot(-5) \end{matrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{bmatrix} /:(-2) \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{bmatrix} \begin{matrix} \\ /\cdot(-1) \\ /\cdot(3) \\ /\cdot 5 \end{matrix}$

$\sim \begin{bmatrix} 1 & 0 & \frac{3}{2} & 1 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{7}{2} & -4 & -\frac{1}{2} \\ 0 & 0 & -\frac{7}{2} & -3 & \frac{1}{2} \end{bmatrix} /\cdot\left(-\frac{2}{7}\right) \sim \begin{bmatrix} 1 & 0 & \frac{3}{2} & 1 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & -\frac{7}{2} & -3 & \frac{1}{2} \end{bmatrix} /\cdot\left(+\frac{7}{2}\right) /\cdot\left(-\frac{3}{2}\right) /\cdot\left(+\frac{7}{2}\right)$

$\sim \begin{bmatrix} 1 & 0 & 0 & -\frac{5}{7} & \frac{2}{7} \\ 0 & 1 & 0 & \frac{4}{7} & \frac{4}{7} \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} /\cdot\left(-\frac{8}{7}\right) /\cdot\left(-\frac{4}{7}\right) /\cdot\left(+\frac{5}{7}\right) \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

$5x+4z+2t=3$

$5\cdot 1+4\cdot(-1)+2\cdot 1=3$

$x=1$

$y=0$

$z=-1$

$t=1$

PROVJERA.

1) IZRAČUNAJ SVE KOMPLEKSNE BROJEVE z TAKVE DA
$\left|\frac{z}{2}\right|^2 = z + 1 + i$. (PRIKAŽI U KOMPLEKSNOJ RAVNINI)