MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MARKO BAMBIĆ	**BROJ INDEKSA:** 55952-2007

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0,4)$. Izračunati $\int_{\partial K} (3x+3)\,ds$. ~~20~~
2. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(3,3)$. Izračunati $\iint_K (3x+2)\,dx dy$. 20
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 3^2$ za koji vrijedi $z \le 1$. 15
4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2+y^2$, $z = 3$. 15
5. Zadana krivulja Γ s parametrizacijom $x = 3\cos t$, $y = 3\sin t$ i $z = t^2$, $t \in [-1,1]$. Još je zadano $f(x,y,z) = \sqrt{z}$. Izračunati: $\int_\Gamma f\,ds$. Pomoć: kod riješavanja integracije možeš iskoristiti supstituciju $2t+3 \mapsto u$. ~~15~~ 12
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) + y''(t) + y'(t) + y(t) = t + 1, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 47

1. $r=3$ $\quad x^2+(y-4)^2=9 \quad s\in[0,2\pi]$

$T(0,4)$

$$\int_{\partial K}(3x+3)\,ds \qquad r(t)=\begin{pmatrix} r\cos t\\ r\sin t+4\end{pmatrix}=\begin{pmatrix} 3\cos t\\ 3\sin t+4\end{pmatrix}$$

$$r'(t)=\begin{pmatrix} -3\sin t\\ 3\cos t\end{pmatrix} \checkmark$$

$$\|r'(t)\|=\sqrt{9\sin^2t+9\cos^2t}=\sqrt{9(\sin^2t+\cos^2t)}=\sqrt{9}=3 \checkmark$$

$$\int_0^{2\pi}3(3\cos t+3)\,dt = \int_0^{2\pi}(9\cos t+9)\,dt=(9\sin t+9t)\Big|_0^{2\pi}=18\pi$$

×

TOČNO BI BILO

$$\int_0^{2\pi}(3\cdot 3\cos t+3)\cdot 3\,dt$$

2. $r=3$

$T(3,3)$

$$\iint_K (3x+2)\,dx\,dy$$

$x = r\cos\varphi + 3$

$y = r\sin\varphi + 3$

$dx\,dy = r\,dr\,d\varphi$

$r \in [0,3]$

$\varphi \in [0,2\pi]$

$$\iint (3x+2)\,dx\,dy = \int_0^{2\pi}\int_0^{3} [3(r\cos\varphi + 2)]\,r\,dr\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^{3} (3r^2\cos\varphi + 9 + 2)\,r\,dr\,d\varphi = \int_0^{2\pi}\int_0^{3} (3r\cos\varphi + 11)\,r\,dr\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^{3} (3r\cos\varphi + 11)\,r\,dr\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^{3} (3r^2\cos\varphi + 11r)\,dr\,d\varphi = \int_0^{2\pi} \left(\cancel{3}\frac{r^3}{\cancel{3}}\cos\varphi + 11\frac{r^2}{2}\right)\Big|_0^3 d\varphi$$

$$= \int_0^{2\pi} \left(27\cos\varphi + \frac{11}{2}\cdot 9\right) d\varphi = 27\sin\varphi + \frac{99}{2}\varphi \Big|_0^{2\pi} =$$

$$= 27\sin\varphi + \frac{99}{2}\varphi \Big|_0^{2\pi} = 99\pi - (0-0) = 99\pi$$

20

5. $x = 3\cos t$ $\quad t \in [-1, 1]$

$y = 3\sin t$

$z = t^2$ $\quad f(x,y,z) = \sqrt{z}$

$$r(t) = \begin{pmatrix} 3\cos t \\ 3\sin t \\ t^2 \end{pmatrix} \qquad r'(t) = \begin{pmatrix} -3\sin t \\ 3\cos t \\ 2t \end{pmatrix} \checkmark$$

$$\|r'(t)\| = \sqrt{9\sin^2 t + 9\cos^2 t + 4t^2} = \sqrt{9(\sin^2 t + \cos^2 t) + 4t^2} = \sqrt{9 + 4t^2} \checkmark$$

$$\int_{-1}^{1} \left(\sqrt{t^2}\sqrt{9+4t^2}\right) dt = \checkmark \int_{-1}^{1} |t|\sqrt{9+4t^2}\, dt = \left(\frac{1}{12}\sqrt{(9+4t^2)^3}\right)\Bigg|_{-1}^{1} =$$

$\underline{12}$ ✗

$$= \frac{1}{12}\left(13\sqrt{13} - 13\sqrt{13}\right) = 0$$

$$\begin{pmatrix} 9+4t^2 = u \\ 8t\,dt = du \\ t\,dt = \frac{1}{8}du \end{pmatrix}$$

$$\frac{1}{8}\int \sqrt{u}\, du = \frac{1}{8}\int u^{\frac{1}{2}}\, du = \frac{1}{8}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{8}\cdot\frac{2}{3}u^{\frac{3}{2}} = \frac{2}{24}\sqrt{(9+4t^2)^3} =$$

$$= \frac{1}{2}\sqrt{(9+4t^2)^3}$$

MARKO BAMBIĆ 54952-2007

4. $z=x^2+y^2 \quad z=3$

r	0	1	-1	2	-2
$z=r^2$	0	1	1	4	4

$z=r^2$

$r^2=z$

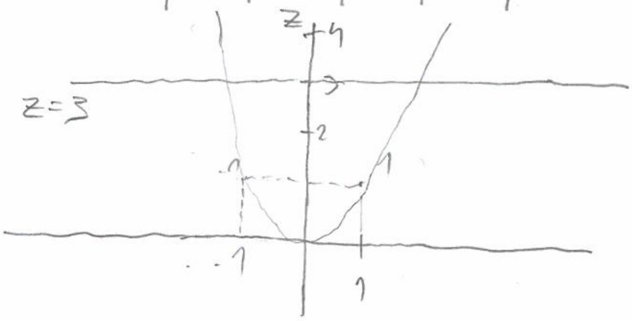
$r^3=3$

$r=\sqrt{3}$

$r\in[0,\sqrt{3}]$ ✓

$\varphi=[0,2\pi)$ ✓

$z\in[r^2,3]$ ✓



$V=\int_0^{2\pi}\int_0^{\sqrt{3}}\int_{r^2}^{3} r\,dz\,dr\,d\varphi$ ✓

$$V=\int_0^{2\pi}\int_0^{\sqrt{3}} r(3-r^2)\,dr\,d\varphi=\int_0^{2\pi}\int_0^{\sqrt{3}}(3r-r^3)\,dr\,d\varphi=\int_0^{2\pi}\left(3\frac{r^2}{2}-\frac{r^4}{4}\right)\Bigg|_0^{\sqrt{3}}d\varphi=\int_0^{2\pi}\left(3\cdot\frac{3}{2}-\frac{9}{4}\right)d\varphi=$$

$$=\int_0^{2\pi}\left(\frac{9}{2}-\frac{9}{4}\right)d\varphi=\int_0^{2\pi}\frac{9}{4}\,d\varphi=\frac{9}{4}\varphi\Big|_0^{2\pi}=\frac{9}{\cancel{4}}\cdot\cancel{2}\pi=\frac{9}{2}\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: DANIJEL SORIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0,4)$. Izračunati $\int_{\partial K} (3x+3)\,\mathrm{d}s$. ~~20~~
2. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(3,3)$. Izračunati $\iint_K (3x+2)\,\mathrm{d}x\mathrm{d}y$. ~~20~~
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 3^2$ za koji vrijedi $z \leq 1$. 15
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6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) + y''(t) + y'(t) + y(t) = t+1, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 30

2.

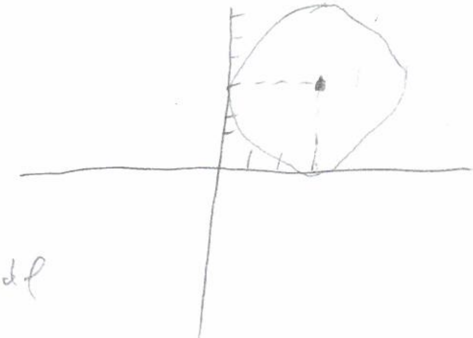
$T[3,3]$ $\qquad \iint_K (3x+2)\,dxdy$

$r=3$

$x = r\cos\varphi + 3$

$y = r\sin\varphi + 3$

$dxdy = r\,dr\,d\varphi$



$$\int_0^{2\pi}\int_0^3 (3\cdot(r\cos\varphi+3)+2)\, r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^3 (3r\cos\varphi+5)\,r\,dr\,d\varphi = \int_0^{2\pi}\int_0^3 (3r^2\cos\varphi + 5r)\,dr\,d\varphi$$

$$\int_0^3\left[\int_0^{2\pi} 3r^2\cos\varphi\,d\varphi + \int_0^{2\pi} 5r\,d\varphi\right]dr = \int_0^3\left[\underbrace{3r^2\cdot(-\sin\varphi)\Big|_0^{2\pi}}_{0} + 5r\cdot\varphi\Big|_0^{2\pi}\right]dr$$

$$= \int_0^3 10r\pi\,dr = 10\pi\cdot\int_0^3 r\,dr = 10\pi\cdot\frac{r^2}{2}\Big|_0^3 = 10\pi\cdot\frac{1}{2}\cdot r^2\Big|_0^3 = 5\pi\cdot 9$$

$$= 45\pi$$

1. $r=3$ $\quad \int_{\partial K} (3x+3)\,ds$

$T[0,4]$

$$x = r\cos t \quad (r = 3)$$
$$y = r\sin t + 4 \quad = \begin{pmatrix} 3\cos t \\ 3\sin t \end{pmatrix} + 4$$

$$r' = \begin{pmatrix} -3\sin t \\ 3\cos t \end{pmatrix}$$

$$\|r'(t)\| = \sqrt{(-3\sin t)^2 + (3\cos t)^2} = \sqrt{9\sin^2 t + 9\cos^2 t} = \sqrt{9} = 3$$

$$\int_0^{2\pi} (3\cdot(3\cos t + 3))\cdot 3\,dt = \int_0^{2\pi} \underline{9\cos t + 9\,dt} = 9\int_0^{2\pi} \cos t + 9\int_0^{2\pi} dt$$

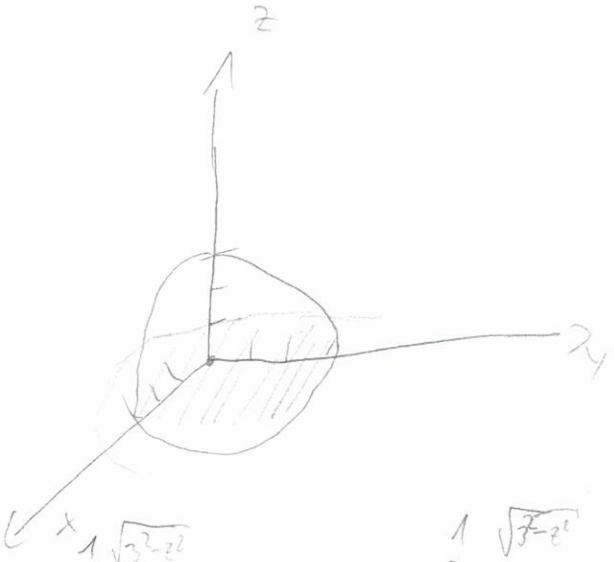
$$= 9\cdot(-\sin t)\Big|_0^{2\pi} + 9\cdot t\Big|_0^{2\pi} = 9\cdot 2\pi = 18\pi$$
$$(-\sin t)\Big|_0^{2\pi} = 0$$

3. $x^2+y^2+z^2 = 3^2$

$z \le 1$

$x = r\cos\varphi$
$y = r\sin\varphi$
$z = z$
$dxdy = r\,dr\,d\varphi$
$dz = dz$

$x^2+z^2 = 3^2$
$x = \sqrt{3^2-z^2}$



$$\int_0^{2\pi}\int_{-3}^{1}\int_0^{\sqrt{3^2-z^2}} r\,dr\,d\varphi\,dz \quad dr\,dz\,d\varphi$$

$$\int_{-3}^{1}\int_0^{\sqrt{3^2-z^2}} r\,dr\,dz \int_0^{2\pi} d\varphi = \int_{-3}^{1}\int_0^{\sqrt{3^2-z^2}} r\,dr\,dz \cdot \varphi\Big|_0^{2\pi} = \int_{-3}^{1}\int_0^{\sqrt{3^2-z^2}} r\,dr\,dz \cdot 2\pi = 2\pi\int_{-3}^{1} dz \int_0^{\sqrt{3^2-z^2}} r\,dr$$

$$= 2\pi\int_{-3}^{1} dz \cdot \frac{r^2}{2}\Big|_0^{\sqrt{3^2-z^2}} = 2\pi\int_{-3}^{1} dz \left(\frac{\left(\sqrt{3^2-z^2}\right)^2}{2}\right) = 2\pi\cdot\int_{-3}^{1} dz \cdot \frac{3^2-z^2}{2} = 2\pi\cdot\int_{-3}^{1} \frac{9-z^2}{2}\,dz$$

$$2\pi\cdot\frac{1}{2}\int_{-3}^{1} 9 - z^2\,dz = \pi\cdot\int_{-3}^{1} 9\,dz - \pi\int_{-3}^{1} z^2\,dz =$$

DANIJEL SOKIC

$$= \pi \cdot \int_{-3}^{1} 9\,dz - \pi \int_{-3}^{1} z^2\,dz = \pi \cdot 9 \cdot z \Big|_{-3}^{1} - \pi \cdot \frac{z^3}{3} \Big|_{-3}^{1}$$

$$= 9\pi \cdot (1+3) - \pi \cdot \frac{1}{3} \cdot \left(1^3 - (-3)^3\right) = 9\pi \cdot 4 - \frac{\pi}{3} \cdot (1+27)$$

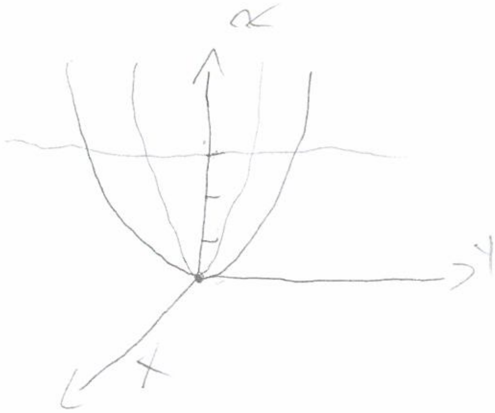
$$= 36\pi - \frac{28\pi}{3} = \frac{108\pi - 28\pi}{3} = \frac{80}{3}\pi$$

4.

$z = x^2 + y^2$ $\quad x = r\cos\varphi \quad dx\,dy = r\,dr\,d\varphi$

$z = 3$ $\quad y = r\sin\varphi \quad dz = dz$

$z = z$



$$\int_0^{2\pi} \int_0^{3} \int_0^{\sqrt{z}} r\,dr\,d\varphi\,dz =$$

$$\int_0^3 dz \int_0^{\sqrt{z}} r\,dr \int_0^{2\pi} d\varphi = \int_0^3 dz \int_0^{\sqrt{z}} r\,dr \cdot \varphi \Big|_0^{2\pi} = \int_0^3 dz \int_0^{\sqrt{z}} r\,dr \cdot 2\pi$$

$$= 2\pi \cdot \int_0^3 dz \cdot \int_0^{\sqrt{z}} r\,dr = 2\pi \cdot \int_0^3 dz \cdot \frac{r^2}{2} \Big|_0^{\sqrt{z}} = 2\pi \cdot \int_0^3 dz \cdot \frac{(\sqrt{z})^2}{2} = 2\pi \cdot \frac{1}{2} \int_0^3 dz \cdot z$$

$$\pi \cdot \int_0^3 z\,dz = \pi \cdot \frac{z^2}{2} \Big|_0^3 = \pi \cdot \frac{9}{2} = \frac{9}{2}\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MANDICA ERCEG **BROJ INDEKSA:** 55146-2007

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0,4)$. Izračunati $\int_{\partial K} (3x+3)\,ds$. 20
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Ukupno: 15

1.) $r = 3$

$T(0,4)$

$x^2 + (y-4)^2 = 9$ ⇐ JEDNADŽBA KRUŽNICE U $T(0,4)$

$\varphi \in [0, 2\pi)$

$\int_{\partial K} (3x+3)\,ds$

$r(t) = \begin{pmatrix} r\cos t \\ r\sin t + 4 \end{pmatrix} = \begin{pmatrix} 3\cos t \\ 3\sin t + 4 \end{pmatrix}$ ✓

$r^2(t) = \begin{pmatrix} -3\cos t \\ 3\sin t \end{pmatrix}$ ✗

$\|r^2(t)\| = \sqrt{9\sin^2 t + 9\cos t} =$

$\int_0^{2\pi} 3(5\cos t + 9)\,dt = \int_0^{2\pi} (9\cos t + 9)\,dt = (9\sin t + 9t)\Big|_0^{2\pi}$ ✗

$= 18\pi$

2.)

$r = 3$

$T(3,3)$

$$\iint_K (3x+2)\,dz\,dy$$

$x = r\cos\varphi + 2$ ×

$z = r\sin\varphi + 3$

$x - = r\,dr\,\varphi$

$r \in [0,3]$

$z \in [0, 2\pi]$

$$\int (3x+2)\,dz\,dy = \int_0^{2\pi}\int_0^3 [3(r\cos + 3)]\,dz\,dy$$ ×

$$\int_0^{2\pi}\int_0^3 (3r\cos\varphi + 3 + 2)\,r\,dz\,dy = \int_0^{2\pi}\int_0^3 \left(\cancel{3}\frac{r^2}{3}\cos\varphi + 11\frac{r^2}{2}\right)\Big|_0^3 d\varphi$$ ×

$$\cancel{\int_0^{2\pi}\int (3r\cos\varphi + 3 + 2)\,r\,dz\,dy \Big|_0^{2\pi} 6r\cos + 6 + 4 = 5.90884}$$

×

3.) $x^2+y^2+z^2=3^2 \qquad z \leq 1$

$x^2+y^2+z^2=3$

$x^2+y^2+z^2=9$

$r^2+z^2=9$

$z=9-r^2$

$z==\sqrt{9-r2}$ ✓

$z \in [1-\sqrt{9-r^2}]$ ✗

$r \in [0,3]$ ✓

$f \in [0,2\pi]$ ✓

$$\int_0^{2\pi}\int_0^{3}\int^{1} r\,dz\,dr\,df$$

r	0	2	-2
$z^2=\sqrt{9-r2}$	3	$-\sqrt{3}$	$-\sqrt{3}$

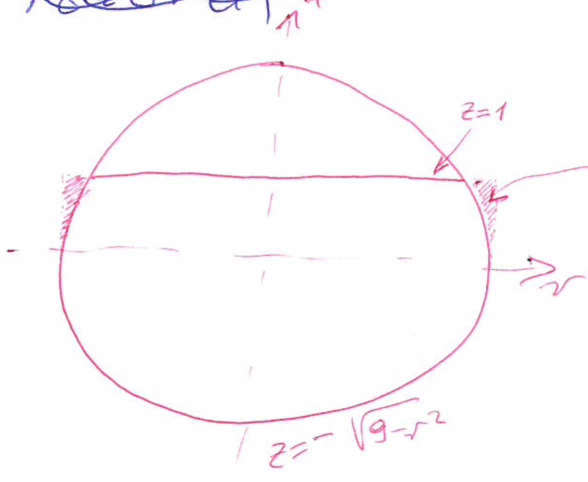
$$\int_0^{2\pi}\int_0^{3} r\left(1+\sqrt{9-r^2}\right)dx\,df$$

$$=\int_0^{2\pi}\int_0^{3}\left(r\sqrt{9-r^2}+r\right)dx\,df$$

$$=\int_0^{2\pi}\left(-\frac{1}{3}\sqrt{9-r^2}\right)^2+\frac{1}{2}r^2\Big|_0^3\,df$$

$$\int_0^{2\pi}\frac{27}{2}df=$$

$$\frac{27}{2}\cdot 2\pi=27\pi$$



VI STE POVIŠE POTREBNOG JOŠ DOMETNULI VOLUMEN KOJI PROIZLAZI IZ OVOG DIJELIĆA.

VIDI SORIĆ

MANDICA ERCEG 55776-2004.

4.

$z=x^2+y^2,\ z=3$

$r\ [0, \sqrt{3}]$

$\varphi\ [0, 2\pi]$

$z\ [r^2, 3]$

$$\int_0^{2\pi}\int_0^{\sqrt{3}}\int_{r^2}^{3} r\,dz\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{3}} z\Big|_{r^2}^{3} r\,dr\,d\varphi =$$

$$\int_0^{2\pi}\int_0^{\sqrt{3}} r(3-r^2)\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{3}}(3r-r^3)\,dr\,d\varphi$$

$$=\int_0^{2\pi}\left(3\frac{r^2}{2}-\frac{r^4}{4}\right)\Big|_0^{\sqrt{3}} d\varphi = \int_0^{2\pi}\left(3\cdot\frac{3}{2}-\frac{8}{4}\right)d\varphi =$$

$$\int_0^{2\pi}\left(\frac{9}{4}-\frac{8}{4}\right)d\varphi$$

$$\int_0^{2\pi}\left(\frac{9}{4}d\varphi-\frac{8}{4}\right)\Big|_0^{2\pi} d\varphi = \frac{9}{4}\cdot 2\pi$$

$$=\frac{9}{2}\pi$$

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POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: Andrija Ribić

BROJ INDEKSA: 57688-2009

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3.	Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2=3^2$ za koji vrijedi $z \leq 1$.	15
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6.	Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $$y'''(t)+y''(t)+y'(t)+y(t)=t+1,\quad y(0)=1,\ y'(0)=0,\ y''(0)=1.$$	15

Ukupno: $\varnothing$