MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** Domagoj Nekić **BROJ INDEKSA:** 17-2-0028-2010	Grupa xxoxo POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2=4$, a iznad stošca $x^2+y^2=z^2$. 20 15

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\, ds$. 20

5. Izračunati $\int_{\widehat{K}} y dx + y dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi \vec{\mathbf{j}} + 2\sin\phi \vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

TREBALO JE PISATI φ

Ukupno: 55

(4) $x = \cos 2t$ $\quad t \in [0, 3\pi]$ $\quad \int_S (x+2y)\, ds$

$y = \sin 2t$

$z = t$

$$r(t) = \begin{pmatrix} \cos 2t \\ \sin 2t \\ t \end{pmatrix} = r'(t) = \begin{pmatrix} -\sin 2t \cdot 2 \\ \cos 2t \cdot 2 \\ 1 \end{pmatrix} = r'(t) = \begin{pmatrix} -2\sin 2t \\ 2\cos 2t \\ 1 \end{pmatrix}$$

$$\int_S (x+2y)\, ds = \int_0^{3\pi} (\cos 2t + 2\sin 2t) \cdot \sqrt{5}\, dt$$

$$\|r'(t)\| = \sqrt{(-2\sin 2t)^2 + (2\cos 2t)^2 + 1^2} = \sqrt{4\sin^2 t + 4\cos^2 t + 1} = \sqrt{4(\sin^2 t + \cos^2 t) + 1} = \sqrt{4 \cdot 1 + 1} = \sqrt{5}$$

$$\int_0^{3\pi} \underbrace{(\sqrt{5}\cos 2t + 2\sqrt{5}\sin 2t)}_{= 0} dt = 0 \text{ //} \checkmark$$

① $f'''(t) - f''(t) = \cos(t)$ $\qquad f(0) = f'(0) = f''(0) = 0$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) - \left(s^2F(s) - sf(0) - f'(0)\right) = \frac{s}{s^2+1^2}$$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) - s^2F(s) + sf(0) + f'(0)) = \frac{s}{s^2+1^2}$$

$$s^3F(s) - s^2F(s) = \frac{s}{s^2+1^2}$$

$$F(s)\,(s^3 - s^2) = \frac{s}{s^2+1^2}$$

$$F(s) = \frac{\frac{s}{s^2+1}}{\frac{s^3-s^2}{1}} = \frac{s}{(s^3-s^2)\cdot(s^2+1)} = \frac{s}{s^2(s-1)\cdot(s^2+1)}$$

$$F(s) = \frac{s}{s^2(s-1)\cdot(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s-1)} + \frac{Ds+E}{(s^2+1)} \quad /\cdot\ s^2(s-1)(s^2+1)$$

$$s = As(s-1)\cdot(s^2+1) + B(s-1)(s^2+1) + Cs^2(s^2+1) + (Ds+E)\cdot s^2\cdot(s-1)$$

$$s = (As^2 - As)\cdot(s^2+1) + (Bs - B)(s^2+1) + Cs^4 + Cs^2 + (Ds^3 + Es^2)\cdot(s-1)$$

$$s = As^4 + As^2 - As^3 - As + Bs^3 + Bs - Bs^2 - B + Cs^4 + Cs^2 + Ds^4 - Ds^3 + Es^3 - Es^2$$

$$s = (A+C+D)\cdot s^4 + (-A+B-D+E)\cdot s^3 + (A-B+C-E)\cdot s^2 + (-A+B)\cdot s - B$$

$A+C+D=0$

$-A+B-D+E=0$

$A-B+C-E=0$

$-A+B=1$

$-B=0$

$B=0$

$-A+B=1$

$-A+0=1$

$-A=1 \quad /:(-1)$

$A=-1$

$-A+B-D+E=0$

$A-B+C-E=0$

$1+0-D+E=0$

$-1-0+C-E=0$ $\quad \}+$

$-D+E=0$

$C-E=0$

$-D+C=0$

$-1+1=0$

$D=1$

$C=1$

$+A+B-D+E=0$

$1+0-1+E=0$

$0+E=0$

$E=0$

$$F(s) = -\frac{1}{s} + \frac{0}{s^2} + \frac{1}{s-1} + \frac{1s-0}{(s^2+1)}$$

$$F(s) = -\frac{1}{s} + \frac{1}{s-1} + \frac{1s}{(s^2+1)}$$

$f(t) = -1 + e^t + \sin t$ ✗

PROVJERA: ?

$f(0) = 0$ ✓ $\quad f'(0) = e^t + \cos t$

$f'(0) = 2$ ✗

VIDI MAGAŠ

DOMAGOJ NEKIĆ BROJ INDEKSA: 17-2-0028-2010

(2) $\iint_{\partial K} F\,ds = \iiint_K \operatorname{div} F\,dx\,dy\,dz$

$T(2,1,0)$

$r = 1$

$x^2+y^2+z^2 = R^2$

$x^2+y^2+z^2 = 1$

$x^2+y^2 = r^2$

$r^2+z^2 = 1$

$z = \pm\sqrt{1-r^2}$

$x = r\cos\varphi + 2$

$y = r\sin\varphi + 1$

$z = z$

$dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$\varphi \in [0, 2\pi]$

$r \in [0, 1]$

$z \in [-\sqrt{1-r^2}, \sqrt{1-r^2}]$

$F\begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ $\operatorname{div} F = \dfrac{\partial(x^2+y^2)}{\partial x} + \dfrac{\partial(z)}{\partial y} + \dfrac{\partial(1)}{\partial z} = 2x$

$\int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} 2(r\cos\varphi+2)\,r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} (2r^2\cos\varphi + 4r)\,dz\,dr\,d\varphi$

$= \int_0^{2\pi}\int_0^1 (2r^2\cos\varphi + 4r)(\sqrt{1-r^2}+\sqrt{1-r^2})\,dr\,d\varphi$

$= \int_0^{2\pi}\int_0^1 (2r^2\cos\varphi + 4r)(2\sqrt{1-r^2})\,dr\,d\varphi$

$= \int_0^{2\pi}\int_0^1 (\underbrace{4r^2\cos\varphi\sqrt{1-r^2}}_{0} + 8r\sqrt{1-r})\,dr\,d\varphi$

$= \int_0^{2\pi}\int_0^1 8r\sqrt{1-r^2}\,dr\,d\varphi = \int_0^{2\pi}\left(-\frac{8}{3}\sqrt{(1-r^2)^3}\right)\Big|_0^1 d\varphi$

$= \int_0^{2\pi} -\frac{8}{3}\left(\sqrt{(1-1)^3} - \sqrt{(1-0)^3}\right) d\varphi$

$= \int_0^{2\pi} -\frac{8}{3}\cdot(-1)\,d\varphi$

$= \int_0^{2\pi} \frac{8}{3}\,d\varphi = \frac{8}{3}\varphi\Big|_0^{2\pi} = \frac{8}{3}\cdot 2\pi = \frac{16}{3}\pi$

20

$\int 8r\sqrt{1-r^2}\,dr = \left\{ \begin{array}{l} 1-r^2 = t \\ -2r\,dr = dt \\ r\,dr = -\frac{1}{2}dt \end{array} \right.$

$\int \overset{4}{\cancel{8}}\sqrt{t}\cdot(-\frac{1}{\cancel{2}}dt)$

$\int -4t^{\frac{1}{2}}\,dt$

$= -4\cdot\frac{2}{3}\sqrt{t^3}$

$= -\frac{8}{3}\sqrt{t^3} = -\frac{8}{3}\sqrt{(1-r^2)^3}$

③ $z > 0$ $x^2+y^2+z^2=4$ $x^2+y^2=z^2$

Nthiz: D

$\varphi \in [0, 2\pi]$ ✓

$r \in [0, \sqrt{2}]$ ✓

$z \in [r, \sqrt{4-r^2}]$ ✓

$x^2+y^2+z^2=R^2$

$R^2=4$

$R=\pm 2$

$x^2+y^2=z^2$

$x^2+y^2=r^2$

$r^2=z^2$

$z=|r|$

$x^2+y^2+z^2=4$

$x^2+y^2=r^2$

$r^2+z^2=4$

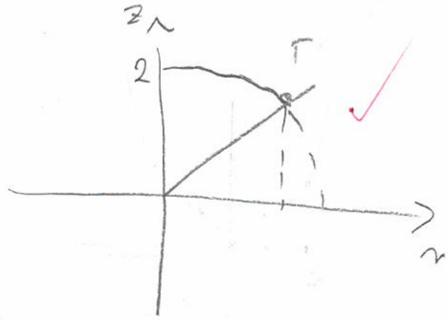
$z=\sqrt{4-r^2}$

z^2+r^2+4 na $r=2$

$2r^2=4$

$r^2=2$

$r=\sqrt{2}$ ✓



1,5

$$V=\int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{2}} r\left(\sqrt{4-r^2}-r\right)dr\,d\varphi$$

$$=\int_0^{2\pi}\int_0^{\sqrt{2}}\left(r\sqrt{4-r^2}-r^2\right)dr\,d\varphi=\int_0^{2\pi}\left(\frac{r^2}{2}\sqrt{4-r^2}-\frac{r^2}{2}\right)\Big|_0^{\sqrt{2}}d\varphi$$

$$=\int_0^{2\pi}\left(\frac{(\sqrt{2})^2}{2}\sqrt{4-(\sqrt{2})^2}-\frac{(\sqrt{2})^2}{2}\right)d\varphi=\int_0^{2\pi}\left(1\sqrt{2}-1\right)d\varphi$$

$$=\left(\sqrt{2}\cdot\varphi-1\varphi\right)\Big|_0^{2\pi}=\sqrt{2}\cdot 2\pi-2\pi=2\pi\sqrt{2}-2\pi$$ ✗

DOMAGOJ NEKIĆ BROJ INDEKSA: 17-2-0028-2010

(5) $\int_{\overset{\curvearrowright}{K}} y\,dx + y\,dy \qquad \overset{\curvearrowright}{K} \quad r(\varphi) = 2\cos\varphi \vec{j} + 2\sin\varphi \vec{k}$

$$r(t) = \begin{pmatrix} 0 \\ 2\cos t \\ 2\sin t \end{pmatrix} \begin{matrix} x \\ y \\ z \end{matrix} \Rightarrow r'(t) = \begin{pmatrix} 0 \\ -2\sin t \\ 2\cos t \end{pmatrix} \begin{matrix} x' \\ y' \\ z' \end{matrix}$$

$$\|r'(t)\| = \sqrt{(-2\sin t)^2 + (2\cos t)^2} = \sqrt{4\sin^2 t + 4\cos^2 t} = \sqrt{4\cdot 1} = 2 \checkmark$$

$$\int_{\overset{\curvearrowright}{K}} y\,dx + y\,dy = \int_0^{2\pi} 2\cdot(2\sin t)\,dt = \int_0^{2\pi} (4\sin t)\,dt$$

$$= 4\int_0^{2\pi} \sin t\,dt = -4\cos t \Big|_0^{2\pi} = (-4\cos\cdot 2\pi) - (-4\cos\cdot 0)$$

$$= -4 + 4 = 0 //$$

$$F = \begin{pmatrix} y \\ y \\ 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2\cos t \\ 2\cos t \\ 0 \end{pmatrix}$$

PREKO DEFINICIJE BI BILO

$$\int_{\overset{\curvearrowright}{K}} F = \int_0^{2\pi} \begin{pmatrix} 2\cos t \\ 2\cos t \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -2\sin t \\ 2\cos t \end{pmatrix} dt = -4\int_0^{2\pi} \cos t \sin t\,dt = 0$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: MARIN MAGAŠ **BROJ INDEKSA:** 17-2-0061-2010	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2 = 4$, a iznad stošca $x^2+y^2 = z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\,ds$. 20

5. Izračunati $\int_{\widehat{K}} y\,dx + y\,dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\mathbf{j} + 2\sin\phi\mathbf{k}$. Koristiti Stokesovu formulu. 20

Ukupno: 40

1. $s^3 F(s) - s^2 \cancel{f(0)} - s\cancel{f'(0)} - \cancel{f''(0)} - s^2 F(s) + s\cancel{f(0)} + \cancel{f'(0)} = \frac{s}{s^2+1}$

$$s^3F(s) - s^2F(s) = \frac{s}{s^2+1}$$

$$F(s)(s^3 - s^2) = \frac{s}{s^2+1} \Rightarrow F(s) = \frac{\frac{s}{s^2+1}}{\frac{s^3-s^2}{1}}$$

$$F(s) = \frac{s}{(s^2+1)(s^3-s^2)} = \frac{s}{s^2(s-1)(s^2+1)}$$

$$= \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s-1} + \frac{Ds+E}{s^2+1} = A(s-1)(s^2+1) + Bs(s-1)(s^2+1)$$

$$+ C(s^2(s^2+1)) + (Ds+E)s^2(s-1) = A(s^3+s-s^2-1) + Bs(s^3+s-s^2-1)$$

$$+ C(s^4+s^2) + (Ds+E)(s^3-s^2) = As^3 + As - As^2 - A + Bs^4 + Bs^2$$

$$- Bs^3 - Bs + Cs^4 + Cs^2 + Ds^4 - Ds^3 + Es^3 - Es^2$$

$0 = B + C + D \Rightarrow 1 = C + D$ $\qquad$ $0 = C + E$

$0 = A - B - D + E \Rightarrow -1 = E - D$ $\qquad$ $1 = C - E \;/+$

$0 = -A + B + C - E$ $\qquad$ $1 = C - E$ $\qquad$ $1 = 2C$

$1 = A - B \Rightarrow \boxed{B = -1}$ $\qquad$ $\boxed{D = \frac{1}{2}}$ $\qquad$ $\boxed{C = \frac{1}{2}}$ $\qquad$ $\boxed{E = -\frac{1}{2}}$

$0 = -A \Rightarrow \boxed{A = 0}$

$\Rightarrow$

$$0 - \frac{1}{s} + \frac{1}{2} \cdot \frac{1}{s-1} + \frac{\frac{1}{2}s - \frac{1}{2}}{s^2+1} = -1 + \frac{1}{2}e^t + \frac{1}{2}\left(\frac{s-1}{s^2+1}\right) \nearrow \frac{2s}{s^2+1} - \frac{1}{s^2+1}$$

$$= \underbrace{-1 + \frac{1}{2}e^t + \frac{1}{2}(\cos t - \sin t)}_{= f(t)} \,//$$

PROVJERA:

$f(0) = -1 + \frac{1}{2} + \frac{1}{2} = 0$ ✓ $f'(t) = \frac{1}{2}e^t + \frac{1}{2}(-\sin t - \cos t)$

$f'(0) = \frac{1}{2} - \frac{1}{2} = 0$ ✓ $f''(t) = \frac{1}{2}e^t + \frac{1}{2}(\sin t - \cos t)$

$f''(0) = \frac{1}{2} - \frac{1}{2} = 0$ ✓ $f'''(t) = \frac{1}{2}e^t + \frac{1}{2}(\sin t + \cos t)$

$f'''(t) - f''(t) = \cos t$ ✓

4. $x = \cos 2t \quad y = \sin 2t \quad z = t \quad t \in [0, 3\pi) \quad \int (x + 2y)\,ds$

$$r\begin{pmatrix} \cos 2t \\ \sin 2t \\ t \end{pmatrix} = \begin{pmatrix} -2\sin 2t \\ 2\cos 2t \\ 1 \end{pmatrix} \quad \|r'\| = \sqrt{\underbrace{(-2\sin 2t)^2 + (2\cos 2t)^2}_{4} + 1^2} = \sqrt{5}$$

$$\int_0^{3\pi} (\cos 2t + 2\sin 2t) \cdot \sqrt{5}\,dt$$

$$\underbrace{\sqrt{5}\int_0^{3\pi} \cos 2t\,dt}_{\mathrm{I}} + \underbrace{2\sqrt{5}\int_0^{3\pi} \sin 2t\,dt}_{\mathrm{II}}$$

I $\left| \begin{array}{l} 2t = u \\ 2dt = du \\ dt = \frac{du}{2} \end{array} \right| \Rightarrow \sqrt{5}\int_0^{3\pi} \cos u \frac{du}{2} = \frac{\sqrt{5}}{2} \sin 2t \Big|_0^{3\pi} = \emptyset$ ✓ $\quad \emptyset \,//$

II $\left| \begin{array}{l} 2t = u \\ 2dt = du \\ dt = \frac{du}{2} \end{array} \right| \Rightarrow 2\sqrt{5}\int_0^{3\pi} \sin \frac{du}{2} = \sqrt{5}\int_0^{3\pi} \sin u\,du = -\sqrt{5} \cdot \cos 2t \Big|_0^{3\pi} = \emptyset$ ✓

3. $z > 0 \quad x^2 + y^2 + z^2 = 4 \quad x^2 + y^2 = z^2$

$\varphi \in [0, 2\pi]$ $\quad r^2 + t^2 = 4 \quad r^2 = t^2$

$r \in [t, \sqrt{4 - t^2}]$ $\quad r = \sqrt{4 - t^2} \quad r = t$

$t \in [0, 2 \quad]$

$\Rightarrow$

NAKIN MAGAS

3. $\int_0^{2\pi} d\varphi \int_0^{2} dt \int_t^{\sqrt{4-t^2}} r\,dr = \int_0^{2\pi} d\varphi \int_0^{2} (4-2t^2)\,dt$

VIDI NEKIĆ

$= \frac{16}{3}\int_0^{2\pi} d\varphi = \frac{16}{3}\cdot 2\pi = \frac{32}{3}\pi$

2. $\iint F \cdot dS \quad \begin{pmatrix} x^2+y^2 \\ t \\ \lambda \end{pmatrix}$ $\quad r=1 \quad T(2,1,0)$

$\to 2x = 2\cos\varphi$

$\iiint 2\cos\varphi \, r\,dr\,d\varphi\,dt$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

Grupa xxoxo
POPUNJAVA NASTAVNIK
Broj ↓ bodova

IME I PREZIME: STIPE DUŠEVIĆ

BROJ INDEKSA: 17-2-0051-2010

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2, 1, 0)$, a koji je orijentiran vanjskom normalom. 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2 + y^2 + z^2 = 4$, a iznad stošca $x^2 + y^2 = z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x + 2y)\, ds$. 20

5. Izračunati $\int_{\widehat{K}} y dx + y dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi \vec{\mathbf{j}} + 2\sin\phi \vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: 0

1. $f'''(t) - f''(t) = \cos(t), \ f(0) = f'(0) = f''(0) = 0$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** Macin Smolić **BROJ INDEKSA:** 55376/2007	Grupa XXOXO POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t)\,, \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint\limits_{\partial K} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2, 1, 0)$, a koji je orijentiran vanjskom normalom. 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2 = 4$, a iznad stošca $x^2 + y^2 = z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x + 2y)\,\mathrm{d}s$. 20

5. Izračunati $\int_{\widehat{K}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: ∅

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

Grupa xxoxo

POPUNJAVA NASTAVNIK

IME I PREZIME: LOURE MACOLA

BROJ INDEKSA: JG194-2008

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint\limits_{\partial K} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2, 1, 0)$, a koji je orijentiran vanjskom normalom. 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2 + y^2 + z^2 = 4$, a iznad stošca $x^2 + y^2 = z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x + 2y)\,\mathrm{d}s$. 20

5. Izračunati $\int_{\widehat{K}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno:

3

$z \geq 0 \quad x^2 + y^2 + z^2 = 4$

$x^2 + y^2 = z^2$

1 $f'''(t) - f''(t) = \cos(t) \qquad f(0) = f'(0) = f''(0) = 0$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** Luka Peroš **BROJ INDEKSA:** 02184	Grupa xxoxo POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2 + y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2, 1, 0)$, a koji je orijentiran vanjskom normalom. 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2 + y^2 + z^2 = 4$, a iznad stošca $x^2 + y^2 = z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x + 2y)\,ds$. 20

5. Izračunati $\int_{\widehat{K}} y dx + y dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: ∅

1. $f'''(t) - f''(t) = \cos(t) \quad , \quad f(0) = f'(0) = f''(0) = 0$

$s^3F(s) - \cancel{s^2 f(0)} - \cancel{s f'(0)} - \cancel{f''(0)} - s^2 F(s) - \cancel{s f(0)} - \cancel{f'(0)} = \dfrac{s}{s^2+1}$

$F(s)(s^3 - s^2) = \dfrac{s}{s^2+1} \quad \Big| : (s^3 - s^2)$

$F(s) = \dfrac{\frac{s}{s^2+1}}{\frac{s^3-s^2}{1}} = \dfrac{s}{s^5+s^3-s^4+s^2} = \dfrac{s}{s^5-s^4+s^3+s^2} = \dfrac{s}{s^2(s^3-s^2+s+1)} = \dfrac{\cancel{s}}{(s^2+1)s^{\cancel{2}}(s-1)}$

$\dfrac{1}{(s^2+1)s(s-1)} = \dfrac{As+B}{(s^2+1)} + \dfrac{C}{s} + \dfrac{D}{s-1} \quad \Big| \cdot (s^2+1)s(s-1)$

$1 = As + B(s^2 - s) + C(s^2+1)(s-1) + D(s^2+1)s$

$1 = As^3 - As^2 + Bs^2 - Bs + Cs^3 - Cs^2 + Cs - C + Ds^3 + Ds$

$1 = s^3(A+C+D) + s^2(-A+B-C) + s(-B+C+D) - C$

$1 = -B + \frac{1}{2}$

$\boxed{B = -\frac{1}{2}}$

$0 = A+C+D$
$0 = -A+B-C$
$0 = -B+C+D$
$\boxed{C = -1}$

$0 = A - 1 + D$
$0 = -A + B + 1$
$0 = -B - 1 + D$

$1 = \cancel{A} + D$
$-1 = -\cancel{A} + B$
$1 = -B + D$

$0 = \cancel{B} + D$
$1 = -\cancel{B} + D$
$1 = 2D$
$\boxed{D = \frac{1}{2}}$

$1 = A + \frac{1}{2}$
$\boxed{A = \frac{1}{2}}$

$$F(s) = \frac{\frac{1}{2} - \frac{1}{2}}{(s^2+1)} + \frac{-1}{s} + \frac{\frac{1}{2}}{s-1}$$

$$F(s) = 0 + \frac{-1}{s} + \frac{1}{2} \cdot \frac{1}{s-1}$$

$$F(t) = -1 + \frac{1}{2} e^{t}$$

PROVJERA:

$$f(0) = -1 + \frac{1}{2} e^{0} = -\frac{1}{2} \neq 0$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa XXOXO
IME I PREZIME: BUTERIN ŠIME **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2 = 4$, a iznad stošca $x^2+y^2 = z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\,ds$. 20

5. Izračunati $\int_{\widehat{K}} y dx + y dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: 0

(1) $f'''(t) - f''(t) = \cos(t)$, $f(0) = f'(0) = f''(0) = 0$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) - s^2F(s) - sf(0) - f'(0) = \frac{s}{s^2+4}$$

$$s^3F(s) - s^2F(s) = \frac{s}{s^2+1}$$

$$s^3F(s) - s^2F(s) = \frac{s}{s^2+1}$$

$2 \cdot 3 = 6$

$$\frac{s}{s^2+1} = \frac{s^3F(s)}{\frac{s^2F(s)}{1}}$$

0

$$\frac{s}{s^2+1} = \frac{A}{s} +$$

Kružna uzvojnica (spirala) S

BUTERIN ŠIME

$x = \cos 2t$

$y = \sin 2t$

$z = t$

$t \in [0, 3\pi]$

$x' = x^2 \quad 2x^2$

$r(t) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} \cos 2t \\ \sin 2t \\ t \end{pmatrix} \qquad r'(t) = \begin{pmatrix} -\sin 2t \\ \cos 2t \\ t' \end{pmatrix} \qquad \| r(t) \| = \sqrt{(-\sin 2t)^2 + (\cos^2 t) + (t)^2}$

$\| r(t) \| = \sqrt{(\sin 4t) + (\cos^4 t) + (t^2)} = \sqrt{1} = 1$

$\int_S (x+2y)\, ds = \int_0^{3\pi} (\cos 2t + 2 \cdot \sin 2t)\, dt = \int_0^{3\pi} (\cos 2t + 2 \sin 4t)\, dt$

$= \sin 2t - 2 \cos 4t \Big|_0^{3\pi}$

$(\sin 2 \cdot 3\pi - 2 \cos 4 \cdot 3\pi) - (\sin 2 \cdot 0 - 2 \cos 4 \cdot 0)$

$\sin 6\pi - 2 \cos 12\pi = 0 - 2 \cdot 1 = 2$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: IVAN ŠIKIĆ **BROJ INDEKSA:** 17-1-0014-2010	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint\limits_{\partial K} \mathbf{F}\cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2, 1, 0)$, a koji je orijentiran vanjskom normalom. ~~20~~

x y z

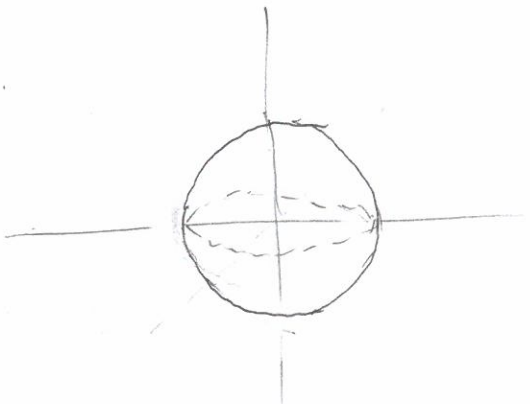
3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2 = 4$, a iznad stošca $x^2+y^2 = z^2$. ~~20~~

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\,\mathrm{d}s$. ~~20~~

5. Izračunati $\int_{\widehat{K}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: ~~Ø~~

2.



$x = x^2+y^2$

$z = z$

$z = 1$

$x^2+y^2+z^2=1$

$(x^2+y^2)^2+z^2+1=1$

$(x^2+y^2)^2+z^2=0$

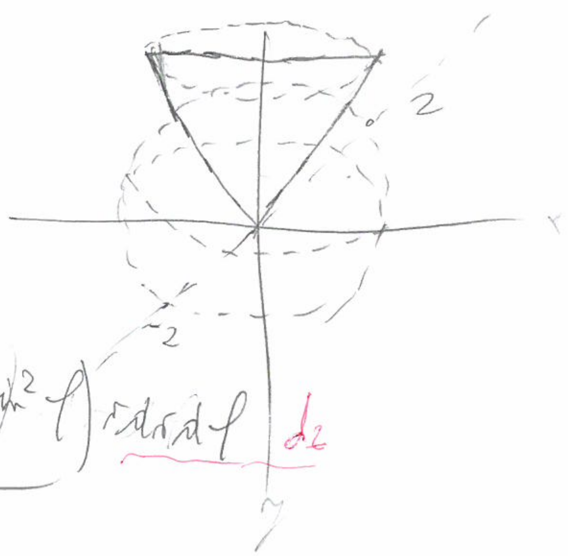
Prema teoremu o divergenciji ~~DIV = 0~~

3.

$z>0$

$x^2+y^2+z^2=4$

$x^2+y^2=z^2$



$$\int_0^{2\pi}\int_{-2}^{2}\int_{z}^{\sqrt{4-z^2}} \underbrace{(\cos^2\varphi+\sin^2\varphi)}_{1}\, r\,dr\,d\varphi \quad dz$$

$\cos^2\varphi+\sin^2\varphi = r^2$

$r = z$

$x = \cos\varphi$

$y = \sin\varphi$

$z = r$

4.

$x = \cos 2t$

$y = \sin 2t$

$z = t$

$(\cos 2t)^2 + (\sin 2t)^2 = 1$

$$\int_0^{3\pi}\int_{-1}^{1} (x+2y)\, dx\, dy = \int_0^{3\pi} (1+2) = 9\pi$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: [handwritten name] **BROJ INDEKSA:** 57649	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t)\,, \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F}\cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. ~~20~~

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2 = 4$, a iznad stošca $x^2+y^2 = z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\,ds$. ~~20~~

5. Izračunati $\int_{\widehat{K}} y\,dx + y\,dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: ~~0~~

$T(2,1,0) \quad k=1$

2. $\cancel{\operatorname{div} F}$ -

$F\begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix} \quad \operatorname{div} F = \begin{pmatrix} 2x \\ 0 \\ 0 \end{pmatrix} = \operatorname{div} F = 2x$

$x^2+y^2 = r^2 \quad z = r$
$r^2 = x^2+y^2 \quad z = \pm\sqrt{r}$
$r^2 = z \quad z = 1$
$r = \sqrt{z}$
$r = 0$

$x = r\cos\varphi + 2$
$y = r\sin\varphi + 1$
$z = z$

$\varphi \in [0, 2\pi] \quad k[0,1] \quad z \in [0,1]$

$\iint_{\partial K} F\,ds = \iiint \operatorname{div} F$

$\int_0^{2\pi}\int_0^1\int_0^1 2x\, r\,dr\,dz\,d\varphi$

$\cancel{2x \cdot \int_0^1\int_0^1}$

$2 \cdot \int_0^{2\pi}\int_0^1\int_0^1 2\cdot(\cos\varphi + 2)\, r\,dr\,d\varphi\,dz$

$\int_0^{2\pi}\int_0^1\int_0^1 2\cos\varphi + 4r\,dr\,d\varphi\,dz$

$\int_0^{2\pi}\int_0^1 2\cos\varphi\frac{r^2}{2} + 4\frac{r^2}{2}\Big|_0^1 dz\,d\varphi$

$\int_0^{2\pi}\int_0^1 \cos\varphi r^2 + 2r^2\Big|_0^1 dz\,d\varphi$

$\int_0^{2\pi}\int_0^1 \cos\varphi + 2\,dz\,d\varphi$

$\int_0^{2\pi}\int \cos\varphi z + 2z\Big|_0^1$

$$\int_0^{2\pi} \cos\varphi + 2 \, d\varphi$$

$$-\sin\varphi + 2\varphi \Big|_0^{2\pi}$$

$$-\sin 2\pi + 2 \cdot 2\pi - \left(-\sin 0 + 2 \cdot 0\right)$$

$$= -0 + 4\pi + 0 + 0$$

$$= 4\pi$$

4. $x = \cos 2t$ $y = \sin 2t$ $z = t$

$t \in [0, 3\pi]$

Frane Zemić

$\int_c (x+2y)\,ds$

$x = \cos 0$ $y = \sin 0$ $z = 0$

$x = \cos 6\pi$ $y = \sin 6\pi$ $z = 3\pi$

$y = 0$ $z = 0$

$x = 1$ $z = 3\pi$

$x = 1$ $y = 0$

$\int_0^1\int_0^0\int_0^{3\pi} (x+2y)\,dx\,dy\,dz$

$\partial_x f = \int x+2y = \frac{x^2}{2} + 2xy$

$\partial_y f = x+2y \Rightarrow \cancel{x}+2y + C(y) = \cancel{x}+2y = \int 2y = \frac{2y^2}{2} = y^2$

$$\int_1^1 \int_0^0 \int_0^{3\pi} x+2y \,dx\,dy\,dz$$

$$3\pi \cdot \int_1^1 xy + \frac{2y^2}{2} \Big|_0^0 \,dx$$

$$3\pi \cdot \int_1^1 0 + 0 \,dx$$

$$= 0$$

5. $\cancel{\int_k y\,dx + y\,dy}$

4. $\cancel{\int_s (dx + 2y)}$

~~5.~~

5. $\int_k y\,dx + y\,dy$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: MARKO DANILOVIĆ **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(\overset{x}{2}, \overset{y}{1}, \overset{z}{0})$, a koji je orijentiran vanjskom normalom. 20

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2=4$, a iznad stošca $x^2+y^2=z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\, ds$. 20

5. Izračunati $\int_{\widehat{K}} y dx + y dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi \vec{\mathbf{j}} + 2\sin\phi \vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: 0

①. $f'''(t) - f''(t) = \cos(t)$ $\quad f(0) = f'(0) = f''(0) = 0$

$T = (\overset{x_0}{2}, \overset{y_0}{1}, \overset{z_0}{0})$

②. $\iint_{\partial K} F \cdot dS$ gdje je $F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ $\quad r = 1 \quad T(\overset{x_0}{2}, \overset{y_0}{1}, 0)$

$$r(t) = \begin{pmatrix} r\cos t + x_0 \\ r\sin t + y_0 \end{pmatrix} = \begin{pmatrix} 1\cos t + 2 \\ 1\sin t + 1 \end{pmatrix}' = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$$

$$\|r\| = \sqrt{\cos t^2 + \sin^2}$$

$$\|r\| = 1$$

$$\iint_{\partial K} F \cdot ds$$

$$F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix} \Rightarrow F\begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$$

⑨ – $x = \cos t \qquad t \in [0, 3\pi) \qquad \int_s (x + 2y)\, ds$

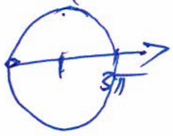
$y = \sin 2t$

$z = t$

$\cancel{t = \in [0, 3\pi]}$

$r \qquad \int_s (\cos t + 2(\sin 2t))\, ds$

$r = \frac{3\pi}{2}$

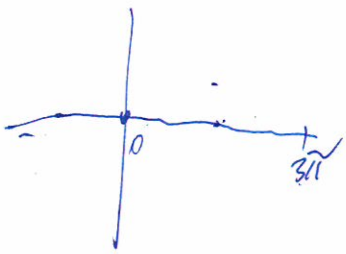


⑤ $\int_s (\cos t + 2\sin \cancel{2} t)\, d\cancel{s}$

$= \cancel{\int_s \sin + 2 \,\text{arc}\cos}$

$= \cancel{\sin + 2 \arccos}$

$r = \frac{3\pi}{2} \qquad T =$



$r'(\cancel{t}) = \left(\frac{3\pi}{2} \right.$

$$r(t) = \begin{pmatrix} \frac{3\pi}{2} \cos t + 0 \\ \frac{3\pi}{2} \sin 2t + 3\pi \end{pmatrix}' = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$$

$$\|r\| = \sqrt{\left(\frac{3\pi}{2}\cos t\right)^2 + \left(\frac{3\pi}{2}\sin 2t + 3\pi\right)^2}$$

$$= \sqrt{\left(\frac{3\pi}{2}\right)^2 \cos^2 t + \sin 2^2 t + (3\pi)^2}$$

$$\sqrt{\left(\frac{3\pi}{2}\right)^2 \cos^2 t + \sin 2^2 t + 9\pi^2}$$

$$\sqrt{\frac{9\pi}{4}\left(\cos^2 t + \sin 2^2 t + 9\pi^2\right.}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxo
IME I PREZIME: Jure Pavić **BROJ INDEKSA:** 51884	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) - f''(t) = \cos(t), \quad f(0) = f'(0) = f''(0) = 0.$$

2. Izračunati $\iint\limits_{\partial K} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 1 s centrom u točki $T(2,1,0)$, a koji je orijentiran vanjskom normalom. ~~20~~

3. Prijelazom na cilindrične koordinate izračunati volumen dijela prostora $z > 0$ ispod kugle $x^2+y^2+z^2 = 4$, a iznad stošca $x^2+y^2 = z^2$. 20

4. Zadana je kruzna uzvojnica (spirala) S s jednadzbama $x = \cos 2t$, $y = \sin 2t$ i $z = t$ za $t \in [0, 3\pi]$. Izračunati $\int_S (x+2y)\,\mathrm{d}s$. 20

5. Izračunati $\int_{\widehat{K}} y\,dx + y\,dy$ gdje je $\widehat{K}$ krivulja dana parametrizacijom $r(\varphi) = 2\cos\varphi\vec{\mathbf{j}} + 2\sin\phi\vec{\mathbf{k}}$. Koristiti Stokesovu formulu. 20

Ukupno: Ø

1.) $f'''(t) - f''(t) = \cos(t)$

$f(0) = f'(0) = f''(0) = 0$

$f'''(t) - f''(t) = \cos t(t) = f'''(0) - f'(0) = \cos(0)$

$= f'''(0) - f''(0) = \cos(0)$

$f''' - f'' = \cos(0)$

$\cos(0) - \sin(0) = \cos(0)$

$\cos(0) + \cos(0) = \sin(0)$

$1 + 1 = 0$

$= 2$

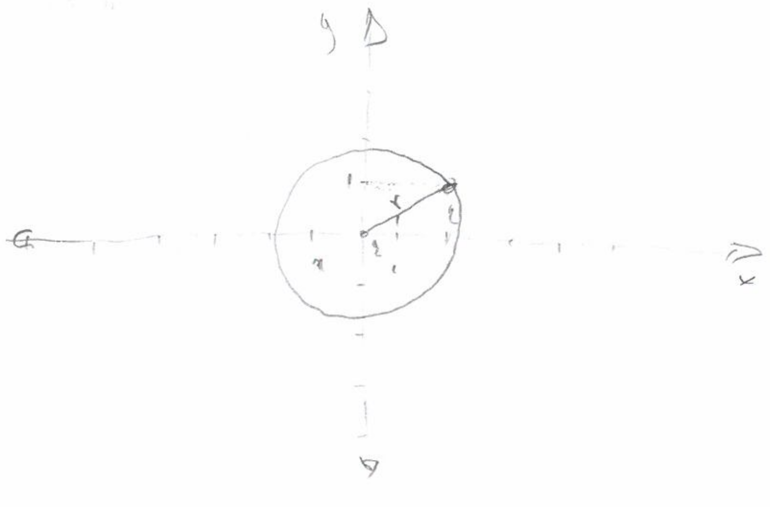
$= 1$

2.) $\iint\limits_{\partial K} F \cdot ds$

$F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix}$

$r = 1$

$T(2,1,0)$



$$\iint_{2k} F \cdot ds \qquad T(2,1,0)$$

$$x = r\cos\varphi + 2$$

$$y = r\sin\varphi + 1$$

$$F = \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix} \qquad dx\,dy = r\,dr\,d\varphi$$

$$\iint_{2k} \begin{pmatrix} x^2+y^2 \\ z \\ 1 \end{pmatrix} \cdot ds = \iint_{2k} \begin{pmatrix} 2^2+1^2 \\ 0 \\ 1 \end{pmatrix} \cdot ds = \iint_{2k} \begin{pmatrix} (r\cos\varphi+2)^2 + (r\sin\varphi+1)^2 \\ 0 \\ 1 \end{pmatrix} \cdot ds$$

$$= \iint_{2k} \left((r\cos\varphi + 4) + (r\sin\varphi + 1) \right) \cdot ds$$

$$= \iint_{2k} \left(r\cos\varphi \sin\varphi + r\cos\varphi + 4r\sin\varphi + 4 \right) \cdot ds$$

$$= \iint_{2k} \left(r\cos\varphi\sin\varphi + 4r\cos\varphi\sin\varphi + 4 \right) \cdot ds$$

$$= \iint_{2k} \left(5r\cos\varphi\sin\varphi + 4 \right) \cdot ds$$

$$= \iint_{2k} \left(5r\,dx\,dy + 4 \right) \cdot ds$$