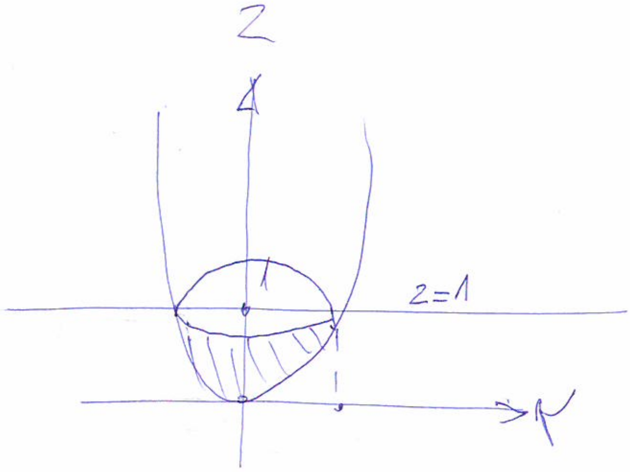
MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: Miro Novković **BROJ INDEKSA:** 54339-2006	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20 20
3. Zadan je trokut s vrhovima $A(-2, 4)$ $B(10, 5)$ i $C(0, -1)$. Izracunati $\oint_{\widehat{ABC}} (x^2 - y)\,dx + \sin(y^3)\,dy$. 20
4. Izračunati integral funkcije $f(x, y) = \dfrac{2}{\sqrt{x^2 + y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20
5. Odrediti integral funkcije $f(x, y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$

   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: 80



(2) $x^2+y^2=5z$ $z\leq 1$

$r^2=5z=1$

$\boxed{r=\sqrt{5\cdot 1}=\sqrt{5}}$

$r\in(0,\sqrt{5})$ $5z=r^2$

$z\in(\frac{r^2}{5},1)$ $z=\frac{r^2}{5}$

$\varphi\in(0,2\pi)$

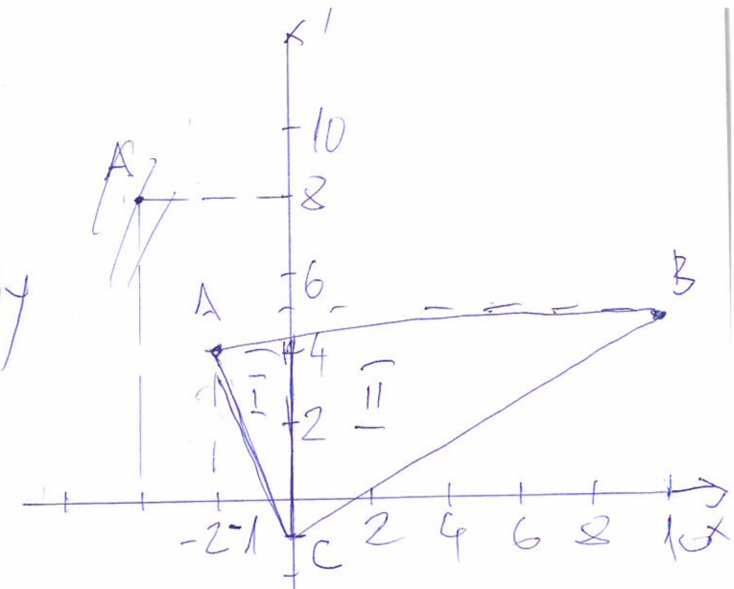
$$V=\int_0^{2\pi}\int_{\frac{r^2}{5}}^{1}\int_0^{\sqrt{5}} r\,dz\,dr\,d\varphi=\int_0^{2\pi}\int_{\frac{r^2}{5}}^{1} rz\Big|_0^{\sqrt{5}}dr\,d\varphi=\int_0^{2\pi}\int_{\frac{r^2}{5}}^{1} r\sqrt{5}\,dr\,d\varphi=\int_0^{2\pi}\sqrt{5}\frac{r^2}{2}\Big|_{\frac{r^2}{5}}^{1}d\varphi$$

$$V=-\int_0^{2\pi}\int_0^{\sqrt{5}}\int_{\frac{r^2}{5}}^{1} r\,dz\,dr\,d\varphi=\int_0^{2\pi}\int_0^{\sqrt{5}} rz\Big|_{\frac{r^2}{5}}^{1}dr\,d\varphi=\int_0^{2\pi}\int_0^{\sqrt{5}}\left(r\cdot(1)-(r)\left(\frac{r^2}{5}\right)\right)dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^{\sqrt{5}} r-\frac{r^3}{5}\,dr\,d\varphi=\int_0^{2\pi}\frac{r^2}{2}-\frac{1}{5}\frac{r^4}{4}\Big|_0^{\sqrt{5}}d\varphi=\int_0^{2\pi}\frac{(\sqrt{5})^2}{2}-\frac{1}{5}\frac{(\sqrt{5})^4}{4}\,d\varphi=\int_0^{2\pi}\left(\frac{5}{2}-\frac{25}{20}\right)$$

$$\int_0^{2\pi}\frac{25}{20}=\frac{5}{4}\quad\int_0^{2\pi}\frac{5}{4}d\varphi=\frac{5}{4}\cdot 2\pi=\frac{5\pi}{2}$$

③ A(-2,4)   B(10,5)   C(0,-1)



AB

$\int (x^2-y)dx \quad \sin(y^3)dy$

$A(\overset{x_1}{-2},\overset{y_1}{4})$

$B(\overset{x_2}{10},\overset{y_2}{5})$

AB... $(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(10-(-2))(y-(4))=(5-4)(x-(-2))$

$12y-48=x+2$

$12y=48+2+x$

$12y=50+x$

$\boxed{y=\frac{50+x}{12}\ AB}$

$A(\overset{x_1}{-2},\overset{y_1}{4})\quad B(\overset{x_2}{10},\overset{y_2}{5})$

$(y-y_1)(x_2-x_1)=(y_2-y_1)(x-x_2)$

$(y-4)(10-(-2))=(5-4)(x-10)$

$(y-4)12=x-10$

$12y-48=x-10$

$12y=38x$

B(10,5)   C(0,1)

BC

GREEN

$\frac{\partial Q}{\partial x}=\frac{y^3}{\partial x}=0$

$\frac{\partial P}{\partial y}=\frac{x^2-y}{\partial y}=-1$

$\iint \frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\,dx\,dy=\boxed{1}$

$\boxed{\iint 1\,dx\,dy}$

$B(\overset{x_1}{10},\overset{y_1}{5})$

$C(\overset{x_2}{0},\overset{y_2}{-1})$

~~BC $y=\frac{3}{5}x$~~

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(0-10)(y-5)=(-1-5)(x-10)$

$-10y+50=-6x+50$

$-10y=-6x$

$y=\frac{-63}{-105}x$: BC

$y=\frac{3}{5}x$

$A(\overset{x_1}{-2},\overset{y_1}{4})\quad C(\overset{x_2}{0},\overset{y_2}{-1})$

$(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(0+2)(y-4)=(-1-4)(x+2)$

$2y-8=-5x-10$

$\boxed{AC \quad 2y=-5x-2 \quad y=\frac{-5x-2}{2}}$

I $\int_{-2}^{0}\int_{\frac{-5x-2}{2}}^{\frac{50+x}{12}} dy\,dx$

$\int_{-2}^{0} \frac{50+x}{12}-\left(\frac{-5x-2}{2}\right)dx$

$\int_{-2}^{0}\frac{50+x+30x+12}{12}dx=\int_{-2}^{0}\frac{31x+62}{12}$

$\frac{31}{12}x+\frac{62}{12}=\frac{31x^2}{122}+\frac{62}{12}x\Big]-\Big[\frac{31(-2)^2}{12}+$

Mirko Mašković

(2) $r(u,v) = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi \\ \frac{r^2}{5} \end{pmatrix} = \begin{pmatrix} u \\ v \\ \frac{u^2+v^2}{5} \end{pmatrix}$

$\vec{n} = \frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} = \begin{pmatrix} 1 \\ 0 \\ \frac{2}{5}u \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 1 \\ \frac{2}{5}v \end{pmatrix} = \begin{pmatrix} -\frac{2}{5}u \\ -\frac{2}{5}v \\ 1 \end{pmatrix} \quad |\vec{n}| = \sqrt{\frac{4}{25}(u^2+v^2)+1}$

$P = \iint \|\vec{n}\| = \int_0^{\sqrt{5}}\int_0^{2\pi} \sqrt{\frac{4}{25}r^2+1}\; r\, d\varphi\, dr = 2\pi \left[\frac{25}{2\cdot 4} \frac{\left(\frac{4}{25}r^2+1\right)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\sqrt{5}}$ 20

$= \frac{25\pi}{6}\left(\left(\frac{9}{5}\right)^{\frac{3}{2}} - 1\right) = \frac{25\pi}{6}\sqrt{2}$

(3) $\oint (x^2-y)\,dx + \sin(y^3)\,dy =$

$P(x,y) = x^2 - y$

$Q(x,y) = y^3$

$\iint_{ABC} \frac{\partial}{\partial x}\sin(y^3) - \frac{\partial(x^2-y)}{\partial y}$

(4) $\varphi = \left(0, \frac{3\pi}{2}\right)$

$r \in (0,2)$

$\int_0^{\frac{3\pi}{2}} \frac{2}{r} \cdot r\, dr\, d\varphi = 2 \cdot 2 \cdot \frac{3\pi}{2} = \boxed{6\pi}$

$x^2+y^2 = r$

$\frac{2}{\sqrt{r^2}}$

$\int_0^{\frac{3\pi}{2}}\int_0^{2} \frac{2}{\sqrt{x^2+y^2}}\, r\, dr\, d\varphi$

Miro Novković

1) $f'''(t) + f''(t) = \sin(t)$ $f'(0) = 0$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + s^2F(s) - sf(0) - f'(0) = \frac{1}{s^2+1} =$$

$$(s^3+s^2)F(s) = s^2+s+1+\frac{1}{s^2+1}$$

$$F(s) = \frac{s^2+s+1}{s^2(s+1)} + \frac{1}{s^2(s+1)(s^2+1)}$$

$$\frac{s^2+s+1(s^2+1)+1}{s^2(s+1)(s^2+1)} = \frac{s^4+s^2}{} = $$

$$F(s)\ \frac{s^4+s^2+s^3+s+s^2+2}{s^2(s+1)(s^2+1)} =$$

$$F(s)\ s^4+s^3+2s^2+s+2 = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1}$$

$$= A(s+1)(s^2+1) + Bs(s+1)(s^2+1) + Cs^2(s^2+1) + (Ds+E)s^2(s+1)$$

$$= As^3 + As + As^2 + A + Bs(s^3+s+s^2+1) + Cs^4 + Cs^2 + (Ds+E)(s^3+s^2)$$

$$s^4+s^3+2s^2+s+2 = As^3+As+As^2+A+Bs^4+Bs^2+Bs^3+Bs+Cs^4+Cs^2+Ds^4+Ds^3+Es^3+Es^2$$

$D = \frac{-1}{2}$

$A+B+C+E=0$

$1+C+E=0$ $B=-1$

$C+E=-1$

$E=\frac{1}{2}$

$B+C+D=1$

$A+B+D+E=1$

$A+B+C+E=2$

$A+B=1$ $B=1-2$

$A=2$

$C=\boxed{\frac{3}{2}}$

$A+B+D+E=1$

$-1+2+D+E=1$

$D+E=1+1=2$

$D+E=0$

$A+B+C+E=2$

$1+C+E=2$

$C+E=1 = E=1-C$

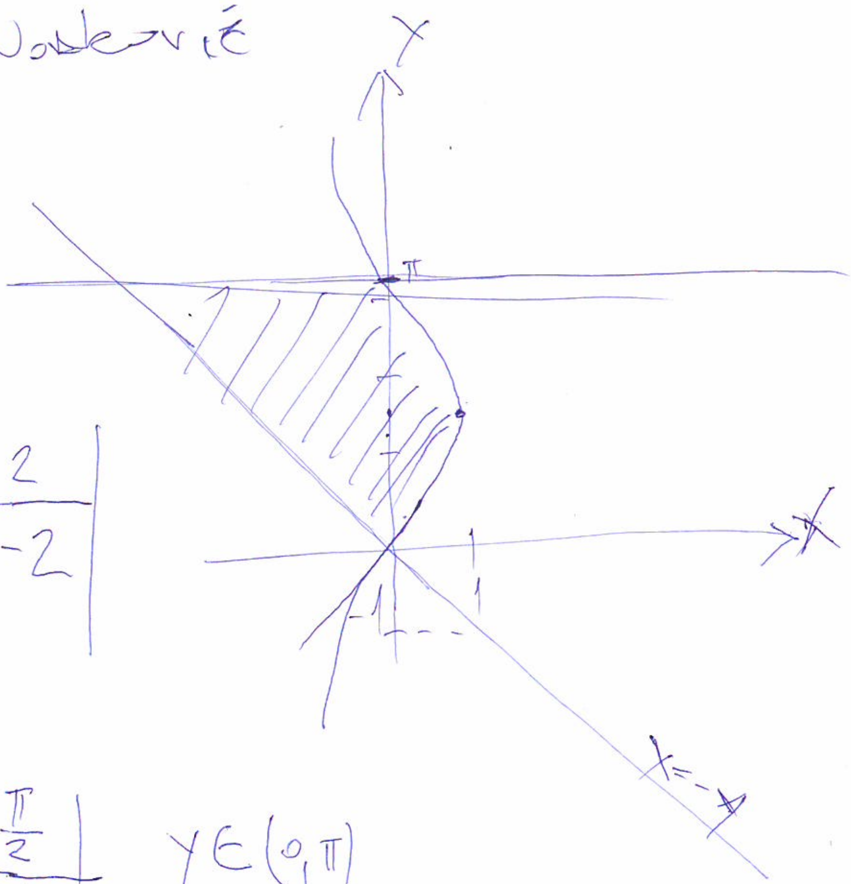
NAJJAK BNA DRUGOM KLJU

Miro Novaković

(5) $f(x,y) = -y$ $\quad x = \sin y$

$y = -x$

$y = \pi$



x	0	1	2
$y=-x$	0	-1	-2

y	$\frac{3\pi}{4}$	π 0	$\frac{\pi}{2}$
$x=\sin y$		0	1

$y \in (0, \pi)$

$x \in (-y, \sin y)$

2x

3x3=

x=2

4·4=16

4^2

$$\int_0^\pi \int_{-y}^{\sin y} -y \, dx\, dy = \int_0^\pi -yx \Big|_{-y}^{\sin y} dy = \int_0^\pi -y(\sin y) - [-y(-y)]\, dy$$

$$= \int_0^\pi [-y\sin y - y^2]\, dy \qquad y+y$$

$$= \int_0^\pi \frac{-y^2}{2}(-\cos y) - \frac{y^3}{3}\, dy$$

$$-\frac{\pi^2}{2}(-\overset{-1}{\cos\pi}) - \frac{(\pi)^3}{2} =$$

$$-\frac{\pi^2}{2} - \frac{\pi}{2}$$

Miro Novković

(5) $\int_0^{\pi}\int_{-y}^{\sin y} -y \, dx\,dy$

$= -\int_0^{\pi} y\sin y + y^2 \, dy = \left[\frac{y^3}{3} - t\cos t + \sin t\right]_0^{\pi}$

$\boxed{= \frac{\pi^3}{3} + \pi}$

(1) $\underset{=1}{s^3F(s) - s^2 f(0)} - \underset{=0}{s f'(0)} - \underset{=1}{f''(0)} + s^2F(s) - \underset{=1}{s f(0)} - \underset{=0}{f'(0)} = \frac{1}{s^2+1}$

$(s^3+s^2)F(s) = s^2+s+1 + \frac{1}{s^2+1} \quad F(s) = \frac{s^2+s+1}{s^2(s+1)} + \frac{1}{s^2(s+1)(s^2+1)}$

$F(s) = \frac{2}{s^2} - \frac{1}{s} + \frac{3}{2}\frac{1}{s+1} + \frac{\frac{1}{2} + \frac{1}{2}s}{1+s^2}$ NA DRUGOM PAPIRU

$f(t) = 2t - 1 + \frac{3}{2}e^{-t} + \frac{1}{2}(\cos t - \sin t)$ 20

$f(0) = 2\cdot(0) - 1 + \frac{3}{2}e^{0} + \frac{1}{2}(\cos 0 - \sin 0)$

$\frac{1}{2}(1) \qquad \frac{3}{2} + \frac{1}{2} = -$

$f(0) = -2 + \frac{3}{2}(1) + \frac{1}{2}$

$f(0) = -2 + 2 = 0$

$f'(t) = 1 - t\left(\frac{3}{2}\right)e + \frac{1}{2}(-\sin t + \cos t) \frac{1}{2}$

$f'(0) = 1 -$

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   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. — 20

Ukupno: 60

(1.) $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ $f(0) = f''(0) = 1$

$$s^3 F(s) - s^2 \overset{1}{\cancel{f(0)}} - s \overset{0}{\cancel{f'(0)}} - \overset{1}{f''(0)} + s^2 F(s) - s\overset{1}{\cancel{f(0)}} - \overset{0}{f'(0)} = \frac{1}{s^2+1}$$

$$s^3 F(s) - s^2 - 1 + s^2 F(s) - s = \frac{1}{s^2+1}$$

$$F(s)\left(s^3 + s^2\right) = \frac{1}{s^2+1} + s^2 + s + 1$$

$$F(s) \cdot s^2 (s+1) = \frac{1 + s^2(s^2+1) + s\cdot(s^2+1) + s^2 + 1}{s^2+1} \cdot \frac{1}{s^2(s+1)}$$

$$F(s) = \frac{1 + s^4 + s^2 + s^3 + s + s^2 + 1}{s^2(s+1)(s^2+1)} = \frac{s^4 + s^3 + 2s^2 + s + 2}{s^2(s+1)(s^2+1)}$$

$\downarrow s_1 = 0$ $\downarrow s_2 = -1$

$$\frac{s^4 + s^3 + 2s^2 + s + 2}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1} \quad \Big/ \cdot s^2(s+1)(s^2+1)$$

$$s^4 + s^3 + 2s^2 + s + 2 = A \cdot s(s+1)(s^2+1) + B(s+1)(s^2+1) + C s^2(s^2+1) + (Ds+E)s^2(s+1)$$

→ za $s_1 = 0$ → $2 = B$ → $B = 2$

→ za $s_2 = -1$ → $3 = 2C$ → $C = \frac{3}{2}$

$$s^4 + s^3 + 2s^2 + s + 2 = As^4 + As^2 + As^3 + As + Bs^3 + Bs + Bs^2 + B + Cs^4 + Cs^2 + Ds^4 + Es^3 + Ds^3 + Es^2$$

→ uz s^4 : $1 = A + C + D$ → $D = 1 - (\underbrace{A + C}_{=\frac{1}{2}})$ → $D = \frac{1}{2}$

→ uz s^3 : $1 = A + B + E + D$

→ uz s^2 : $2 = A + B + C + E$ → $E = 2 - (\underbrace{A + B + C}_{=\frac{5}{2}})$ → $E = -\frac{1}{2}$

→ uz s : $1 = A + B$ → $A = 1 - B$ → $A = -1$

(→ uz s : $2 = B$)

1. ... nastavak

$$F(s) = -1\cdot\frac{1}{s} + 2\cdot\frac{1}{s^2} + \frac{3}{2}\cdot\frac{1}{s+1} + \frac{\frac{1}{2}s - \frac{1}{2}}{s^2+1}$$

$$= -1\cdot\frac{1}{s} + 2\cdot\frac{1}{s^2} + \frac{3}{2}\cdot\frac{1}{s+1} + \frac{1}{2}\cdot\frac{s}{s^2+1} - \frac{1}{2}\cdot\frac{1}{s^2+1}$$

$$f(t) = -1 + 2t + \frac{3}{2}e^{-t} + \frac{1}{2}\cos(t) - \frac{1}{2}\sin(t) \quad //$$

4. $f(x,y) = \frac{2}{\sqrt{x^2+y^2}}$, $S(0,0)$

$\varphi \in [0, \frac{3\pi}{2}]$ $\qquad x = r\cos\varphi$

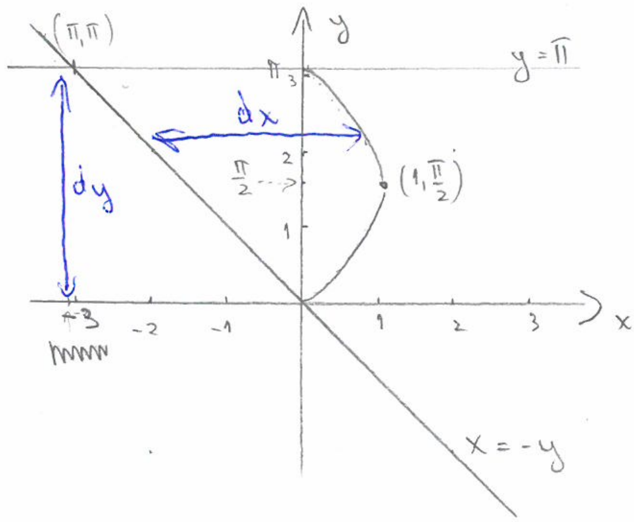
$r \in [0, 2]$ $\qquad y = r\sin\varphi$

$$\int_0^{\frac{3\pi}{2}}\int_0^2 \frac{2}{\sqrt{\underbrace{r^2\cos^2\varphi + r^2\sin^2\varphi}_{r^2\cdot(\underbrace{\cos^2\varphi+\sin^2\varphi}_{1})}}}\cdot r\,dr\,d\varphi = 2\int_0^{\frac{3\pi}{2}}\int_0^2 \frac{1}{\cancel{\sqrt{r^2}}}\cancel{r}\,dr\,d\varphi =$$

$$= 2\int_0^{\frac{3\pi}{2}} d\varphi \cdot \int_0^2 dr = 2\cdot[\varphi]_0^{\frac{3\pi}{2}}\cdot[r]_0^2 = 3\pi\cdot 2 = 6\pi \quad //$$

5. $f(x,y) = -y$

$X \ldots \begin{cases} x = \sin y \\ y = -x \Rightarrow x = -y \\ y = \pi \end{cases}$



$y = \frac{\pi}{2} \Rightarrow x = 1$

$y = \pi \Rightarrow x = 0$

$y \in [0,\pi]$

$x \in [-y, \sin y]$

$$\int_0^{\pi}\int_{-y}^{\sin y} -y\,dx\,dy = \int_0^{\pi} -y[x]_{-y}^{\sin y}\,dy = -\int_0^{\pi} y(\sin y + y)\,dy =$$

$$= -\int_0^{\pi} y\sin y\,dy - \int_0^{\pi} y^2\,dy =$$

$$\left\{\begin{matrix} y = u /' & \sin y\,dy = dv /\int \\ dy = du & -\cos y = v \end{matrix}\right\}$$

$$u\cdot v - \int v\,du = -y\cos y + \int \cos y\,dy =$$
$$= -y\cos y + \sin y$$

$$= -[-y\cos y + \sin y]_0^{\pi} - \left[\frac{y^3}{3}\right]_0^{\pi}$$

$$= -(-\pi\cdot(-1) + 0) - \frac{1}{3}\pi^3 =$$

$$= -\pi - \frac{1}{3}\pi^3 \approx -13{,}48 \quad //$$

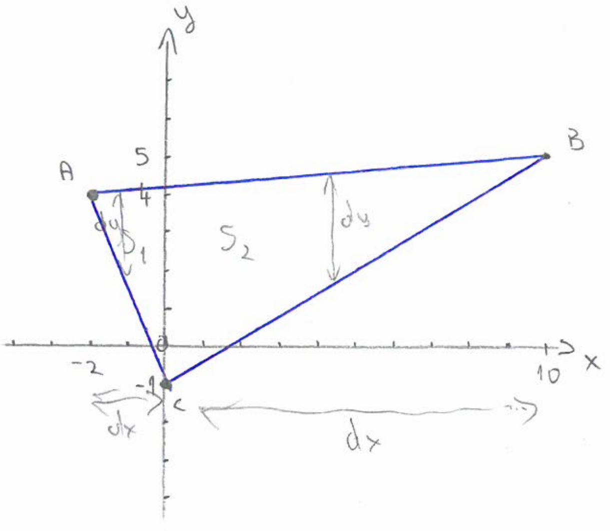
Kristijan Paleka 57308-2009

3. A $(-2,4)$

B $(10,5)$

C $(0,-1)$

$$\oint (x^2-y)\,dx + \sin(y^3)\,dy$$



$$\oint P\,dx + Q\,dy = \iint \frac{dQ}{dx} - \frac{dP}{dy}\,dx\,dy$$

$P = (x^2 - y)$

$Q = \sin(y^3)$

AB : ... $x = 12y - 50$ ✗

AC : ... $x = -\frac{2}{5}y - \frac{2}{5}$

BC : ... $x = \frac{5}{3}y + \frac{5}{3}$

S_1 : $x \in [-2, 0]$ $y \in [AC, AB]$

S_2 : $x \in [0, 10]$ $y \in [BC, AB]$

$$I_1 = \int_{-2}^{0}\int_{-\frac{2}{5}y-\frac{2}{5}}^{12y-50} \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dx\,dy = \int_{-2}^{0}\int_{-\frac{2}{5}y-\frac{2}{5}}^{12y-50} (0 - -1)\,dx\,dy =$$

$$= \int_{-2}^{0} [x]_{-\frac{2}{5}y-\frac{2}{5}}^{12y-50}\,dy = \int_{-2}^{0}\left(12y - 50 + \frac{2}{5}y + \frac{2}{5}\right) dy = \dots = -124$$

$$I_2 = \int_{0}^{10}\int_{\frac{5}{3}y+\frac{5}{3}}^{12y-50} \frac{dQ}{dx} - \frac{dP}{dy}\,dx\,dy = \int_{0}^{10}\int_{\frac{5}{3}y+\frac{5}{3}}^{12y-50} 1\,dx\,dy = \int_{0}^{10}\left(12y - 50 - \frac{5}{3}y - \frac{5}{3}\right) dy$$

$$= \left[\frac{31}{3}\frac{y^2}{2} - \frac{155}{3}y\right]_0^{10} = 0$$

$$I_{ukupni} = I_1 + I_2 = -124$$ //

LAPLACE PROVJERA :

$$f(t) = -1 + 2t + \frac{3}{2}e^{-t} + \frac{1}{2}\cos t - \frac{1}{2}\sin t$$

$$f'(t) = 2 - \frac{3}{2}e^{-t} - \frac{1}{2}\sin t - \frac{1}{2}\cos t$$

$$f''(t) = \frac{3}{2}e^{-t} - \frac{1}{2}\cos t + \frac{1}{2}\sin t$$

$$f'''(t) = -\frac{3}{2}e^{-t} + \frac{1}{2}\sin t + \frac{1}{2}\cos t$$

$$f''(0) = \frac{3}{2} - \frac{1}{2} + 0 = 1$$

$$f'''(0) = -\frac{3}{2} + \frac{1}{2} = -1$$

$$\sin(0) = 0$$

$$f'''(0) + f''(0) = \sin(0)$$

$$-1 + 1 = 0$$

$$0 = 0 \quad \checkmark$$

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IME I PREZIME: ANTONIO VUJATOVIĆ **BROJ INDEKSA:** 17-1-0011-2010	POPUNJAVA NASTAVNIK Broj ↓ bodova

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   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: 40

4.

$$f(x,y) = \frac{2}{\sqrt{x^2+y^2}} \qquad \varphi \in \left[0, \frac{3\pi}{2}\right] \qquad x = r\cos\varphi$$

$$r \in [0,2] \qquad y = r\sin\varphi$$

$$= \frac{2}{\sqrt{(r\cos\varphi)^2 + (r\sin\varphi)^2}}$$

$$= \frac{2}{\sqrt{r^2\cos^2\varphi + r^2\sin^2\varphi}}$$

$$= \frac{2}{\sqrt{r^2\underbrace{(\sin^2\varphi + \cos^2\varphi)}_{1}}} = \frac{2}{\sqrt{r^2}} = \frac{2}{r}$$

$$= \int_0^{3\pi/2} d\varphi \int_0^2 \frac{2}{r}\, r\,dr\,d\varphi$$

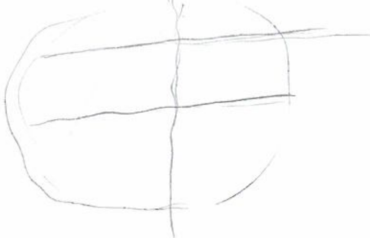
$$= 2\int_0^{3\pi/2} d\varphi \int_0^2 dr = 2\int_0^{3\pi/2} d\varphi \cdot (r)\Big|_0^2 = 2\int_0^{3\pi/2} 2$$

$$= 2 \cdot 2\varphi \Big|_0^{3\pi/2}$$

$$= 2 \cdot \cancel{2} \cdot \frac{3\pi}{\cancel{2}}$$

$$= 6\pi$$

② $x^2+y^2=5z \quad z\le 1 \quad \varphi\in[0,2\pi]$



$r^2=5z \;/\sqrt{\ }$ $\quad r\in[0,\sqrt{5z}]$

$r=\sqrt{5z} \quad z\in[0,1]$

$$\int_0^{2\pi} d\varphi \int_0^1 dz \int_0^{\sqrt{5z}} r\,dr = \int_0^{2\pi} d\varphi \int_0^1 dz \cdot \frac{r^2}{2}\Big|_0^{\sqrt{5z}}$$

$$= \int_0^{2\pi} d\varphi \int_0^1 (\sqrt{5z})^2\,dz = \frac{1}{2}\int_0^{2\pi} d\varphi \int_0^1 5z = \frac{1}{2}\int_0^{2\pi} d\varphi \cdot 5z\Big|_0^1$$

$$= \frac{1}{2}\int_0^{2\pi} 5 = \frac{1}{2}\cdot 5\varphi\Big|_0^{2\pi}$$

$$= \frac{1}{2}\cdot 5\cdot 2\pi$$

$$= 5\pi$$

① $f'''(t)+f''(t)=\sin(t) \quad f'(0)=0$

$f''(0)=1$

$f(0)=1$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + s^2F(s) - sf(0) - f'(0) = \frac{1}{s^2+1}$$

$$F(s)=(s^3+s^2) = \frac{1}{s^2+1} + \frac{s^2}{1} + \frac{1}{1} + \frac{s}{1}$$

$$= \frac{1+s^2(s^2+1)+(s^2+1)+s(s^2+1)}{s^2+1}$$

s^3+s

$s^2(s+1)$

$$= \frac{1+s^4+s^2+s^2+1+s^3+s}{s^2+1}$$

$$= \frac{s^4+s^3+2s^2+s+2}{s^2+1} \quad /:(s^3+s^2)$$

$$= \frac{s^4+s^3+2s^2+s+2}{(s^2+1)(s^3+s^2)}$$

NASTAVAK →

$$= \frac{s^4+s^3+2s^2+s+2}{s^2(s+1)(s^2+1)}$$

ANTONIO VUJATOVIĆ

$$= \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1}$$

$$A(s^3+s+s^2+1) + B(s^4+s^2+s^3+s) + C(s^4+s^2) + (Ds+E)(s^3+s^2)$$

$$As^3 + As + As^2 + A + Bs^4 + Bs^2 + Bs^3 + Bs + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2$$

$$s^4+s^3+2s^2+s+2 = \quad 1 = B+C+D$$

$$1 = A+B+D+E$$

$$2 = A+B+C+E$$

$$1 = A+B$$

$$2 = A \Rightarrow$$

$A=2$

$B=-1$

$D=\frac{1}{2}$

$C=\frac{3}{2}$

$E=-\frac{1}{2}$

$$1 = C+E \Rightarrow 1-C=E$$

$$0 = D+E$$

$$2 = C+D$$

$$0 = D+1-C$$

$$2 = D+C$$

$$\left.\begin{aligned} -1 &= D-C \\ 2 &= D+C \end{aligned}\right| +$$

$$1 = 2D$$

$$D=\frac{1}{2}$$

$$2 = C+D$$

$$2 = C+\frac{1}{2}$$

$$C=\frac{3}{2}$$

$$F(s) = 2\cdot\frac{1}{s^2} - \frac{1}{s} + \frac{3}{2}\cdot\frac{1}{s+1} + \frac{1}{2}\cdot\frac{s}{s^2+1} - \frac{1}{2}\cdot\frac{1}{s^2+1}$$

$$F(t) = 2t - 1 + \frac{3}{2}\cdot e^{-t} + \frac{1}{2}\cdot\cos(t) - \frac{1}{2}\cdot\sin(t)$$

$$F'(t) = 2 - \frac{3}{2}\cdot e^{-t} - \frac{1}{2}\sin(t) - \frac{1}{2}\cdot\cos(t)$$

$$F''(t) = \frac{3}{2}\cdot e^{-t} - \frac{1}{2}\cdot\cos(t) + \frac{1}{2}\sin(t)$$

$$F'''(t) = -\frac{3}{2}e^{-t} + \frac{1}{2}\sin t + \frac{1}{2}\cos t$$

P

ANTONIO VUJATOVIĆ

PROVJERA: $f(0)=1$ ✓

$f'(0)=0$ ✓

$f''(0)=1$ ✓

$f'''(t)+f''(t)=\left(\cancel{-\frac{3}{2}e^{-t}}+\frac{1}{2}\sin t+\cancel{\frac{1}{2}\cos t}\right)+\left(\cancel{\frac{3}{2}e^{-t}}-\cancel{\frac{1}{2}\cos t}+\frac{1}{2}\sin t\right)=\sin t$ ✓

(3.)

$A(-2,4)$

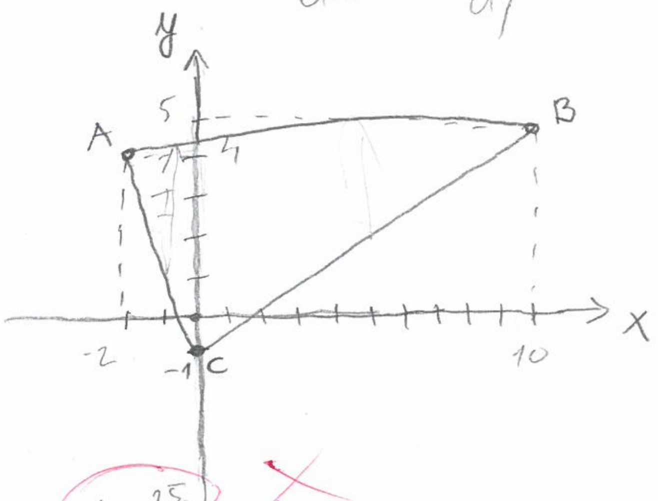
$B(10,5)$

$C(0,-1)$

$$\oint_{ABC}(x^2-y)dx+\sin(y^3)dy$$

$$\frac{\partial Q}{dx}=0 \qquad \frac{\partial P}{dy}=-1$$

$$\frac{\partial Q}{dx}-\frac{\partial P}{dy}=0+1=1 \checkmark$$



AB: $y-y_1=\frac{y_2-y_1}{x_2-x_1}\cdot(x-x_1)$

$y-4=\frac{5-4}{10+2}\cdot(x+2)$

$y-4=\frac{1}{12}x+\frac{1}{6}$ ✓

$y=\frac{1}{2}x+\frac{25}{6}$ ✓ ✗

AC: $y-4=\frac{-1-4}{0+2}\cdot(x+2)$

$y-4=-\frac{5}{2}x-5$

$y=-\frac{5}{2}x-1$

BC: $y-5=\frac{-1-5}{0-10}\cdot(x-5)$

$y-5=\frac{6}{10}x-3$

$y=\frac{3}{5}x+2$

I

$\frac{1}{2}x+\frac{25}{6}$ ✗

$$\int_{-2}^{0}dx\int_{-\frac{5}{2}x-1}^{\frac{1}{2}x+\frac{25}{6}}dy=\int_{-2}^{0}dx\cdot\left(\frac{1}{2}x+\frac{25}{6}+\frac{5}{2}x+1\right)$$

$$=\left(\frac{1}{2}\frac{x^2}{2}+\frac{25}{6}x+\frac{5}{2}\frac{x^2}{2}+x\right)\Big|_{-2}^{0}$$

$$=\frac{13}{3}$$

→

ANTONIO VUJATOVIĆ

$\underline{II}$

$$= \int_0^{10} dx \int_{\frac{3}{5}x+2}^{\frac{1}{2}x+\frac{25}{6}} dy = \int_0^{10} dx \cdot (y) \Big|_{\frac{3}{5}x+2}^{\frac{1}{2}x+\frac{25}{6}}$$

$$= \int_0^{10} \left(\frac{1}{2}x + \frac{25}{6} - \frac{3}{5}x - 2\right) dx = \left(\frac{1}{2}\cdot\frac{x^2}{2} + \frac{25}{6}x - \frac{3}{5}\frac{x^2}{2} - 2x\right) \Big|_0^{10}$$

$$= \left(\frac{1}{2}\cdot 50 + \frac{25}{6}\cdot 10 - \frac{3}{5}\cdot 50 - 2\cdot 10\right.$$

$$= \frac{50}{3}$$

$$\underline{I} + \underline{II} = \frac{13}{3} + \frac{50}{3} = 21$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: KRISTINA POŽARINA **BROJ INDEKSA:** 1720021 2010

Grupa xxoxx
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20

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Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: 20

(2) $x^2 + y^2 = 5z \quad z \leq 1$

$x = r\cos\varphi$
$y = r\sin\varphi$
$z = z$
$dx\,dy = r\,dr\,dz\,d\varphi$

$r^2\cos^2\varphi + r^2\sin^2\varphi = 5z$

$r^2 \underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1} = 5z$

$r^2 = 5z \;/\sqrt{}$

$r = \sqrt{5z}$

$\varphi \in [0, 2\pi]$
$r \in [0, \sqrt{5z}]$
$z \in [0, 1]$

$$\int_0^{2\pi}\int_0^1\int_0^{\sqrt{5z}} r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_0^1 \frac{r^2}{2}\Big|_0^{\sqrt{5z}} dz\,d\varphi = \int_0^{2\pi}\int_0^1 \frac{(\sqrt{5z})^2}{2}\,dz\,d\varphi = \int_0^{2\pi}\int_0^1 \frac{5z}{2}\,dz\,d\varphi =$$

$$= \frac{5}{2}\int_0^{2\pi} d\varphi \frac{z^2}{2}\Big|_0^1 = \frac{5}{2}\int_0^{2\pi} \frac{1}{2}\,d\varphi = \frac{5}{4}\,\varphi\Big|_0^{2\pi} = \frac{5}{4}\cdot 2\pi = \frac{5\pi}{2}$$

KRISTINA POŽARINA

$$s^3F(s) - s^2f(0) - s\cancel{f'(0)}^{0} - f''(0) + s^2F(s) - sf(0) - \cancel{f'(0)}^{0} = \frac{1}{s^2+1}$$

$$s^3F(s) + s^2F(s) - s^2 - 1 - s = \frac{1}{s^2+1}$$

$$F(s)\left(s^3+s^2\right) - s^2 - 1 - s = \frac{1}{s^2+1}$$

$$F(s)\left(s^3+s^2\right) = \frac{1}{s^2+1} + s^2 + s + 1$$

$$F(s)\left(s^3+s^2\right) = \frac{1+s^4+s^2+s^3+s+s^2+1}{s^2+1} = \frac{s^4+s^3+2s^2+s+2}{s^2+1} \Big/ :\left(s^3+s^2\right)$$

$$F(s) = \frac{s^4+s^3+2s^2+2+s}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{(s^2+1)}$$

s^2+s

$$s^4+s^3+2s^2+s+2 = As(s^3+s+s^2+1) + B(s^3+s+s^2+1) + Cs^2(s^2+1) + Ds(s(s+1)) + Es^2(s+$$

$$\boxed{s^4}+s^3+2s^2+s+2 = \boxed{As^4}+As^2+As^3+As+Bs^3+Bs+Bs^2+B+\boxed{Cs^4}+Cs^2+Ds^3+Ds^2+Es^3+Es$$

$\boxed{B=2}$

$1 = A + B$

$1 = A + 2$

$1 - 2 = A$

$\boxed{A=-1}$

$2 = A + B + C + D + E$

$1 = A + B + D + E$

$1 = A + C$

$1 + 1 = C$

$2 = C$

$\boxed{C=2}$

$2 = -1 + 2 + 2 + D + E$

$2 + 1 - 2 - 2 = D + E$

$-1 = D + E$

$1 = -1 + 2 + D + E$

$1 + 1 - 2 = D + E$

$0 = D + E$

$D = 0$

$E = 0$

$$F = \frac{-1}{s} + \frac{2}{s^2} + \frac{2}{s+1}$$

$f = -1 + 2t + 2e^{-t}$ × PROVJERA?

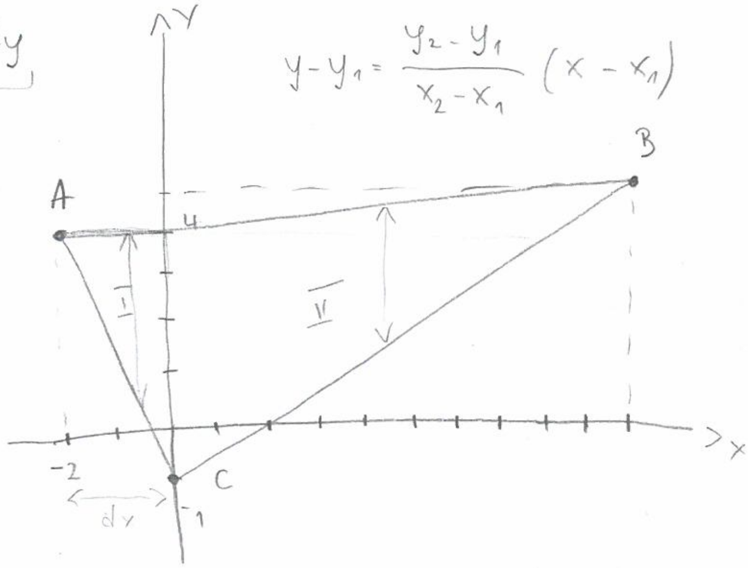
KRISTINA POŽAKINA

(3) $A(-2,4)$

$B(10,5)$ $\qquad \oint_{ABC} \underbrace{(x^2-y)}_{P}dx + \underbrace{\sin(y^3)}_{Q}dy$

$C(0,-1)$

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$



$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$$

$$\frac{\partial Q}{\partial x} = \sin(y^3) = 0$$

$$\frac{\partial P}{\partial y} = (x^2-y) = -1$$

AB: $y-4 = \frac{1}{12}(x+2)$

$y-4 = \frac{1}{12}x+6$

$y-4-6 = \frac{1}{12}x$

$y-10 = \frac{1}{12}x$

$y = \frac{1}{2}x+10$ ✗

I. $\displaystyle\int_{-2}^{0}\int_{\frac52 x+9}^{\frac12 x+10} -\,dy\,dx = \int_{-2}^{0} -y\Big|_{\frac52 x+9}^{\frac12 x+10} dx = \int_{-2}^{0} -\left(\frac12 x+10-\frac52 x-9\right)dx =$

$\displaystyle = -\int_{-2}^{0}(-2x-1)\,dx = -\left(-2\frac{x^2}{2}-x\right)\Big|_{-2}^{0} = (x^2+x)\Big|_{-2}^{0}$

$= 0-(4+2) = 0-4-2 = -6$

AC: $y-4 = \frac{-1-4}{0+2}(x+2)$

$y-4 = \frac52 x+5$

$y-4-5 = \frac52 x$

$y = \frac52 x+9$

II. $\displaystyle\int_{0}^{10}\int_{\frac12 x+10}^{\frac35 x+1} -\,dy\,dx = \int_{0}^{10} -y\Big|_{\frac12 x+10}^{\frac35 x+1} dx = -\int_{0}^{10}\left(\frac35 x+1-\frac12 x-10\right)dx =$

$\displaystyle = -\int_{0}^{10}\left(\frac{1}{10}x-9\right)dx = \left(-\frac{1}{10}\frac{x^2}{2}+9x\right)\Big|_{0}^{10} =$

$\displaystyle = \left(-\frac{x^2}{20}+9x\right)\Big|_{0}^{10} = -\frac{100}{20}+90 = -5+90 = 85$

BC: $y-5 = \frac{-1-5}{0-10}(x-10)$

$y-5 = \frac{6}{10}x-6$

$y-5+6 = \frac35 x$

$y = \frac35 x+1$

I. + II. $= -6+85 = 79$ //

KRISTINA POŽARINA

4.

$$f(x,y) = \frac{2}{\sqrt{x^2+y^2}} \qquad \varphi \in \left[0, \frac{3\pi}{2}\right] \qquad r = 2$$

$$r \in [0, 2]$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$dx\,dy = r\,dr\,d\varphi$

$$\frac{2}{\sqrt{r^2\cos^2\varphi + r^2\sin^2\varphi}} = \frac{2}{\sqrt{r^2\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1}}} = \frac{2}{\sqrt{r^2}} = \frac{2}{r}$$

$$\int_0^{\frac{3\pi}{2}}\int_0^2 \frac{2}{r}\, r\,dr\,d\varphi = \int_0^{\frac{3\pi}{2}} d\varphi\; 2\int_0^2 \frac{1}{\cancel{r}}\cancel{r}\,dr = \int_0^{\frac{3\pi}{2}} d\varphi\; 4 = 4\varphi\Big|_0^{\frac{3\pi}{2}} = \cancel{4}^{2}\cdot\frac{3\pi}{\cancel{2}_{1}} = 6\pi$$

✓

5. $f(x,y) = -y$

$$X\left[\begin{array}{l} x = \sin y \\ y = -x \\ y = \pi \end{array}\right]$$

$$\int y\sin y\,dy = \int y\,dy\int \sin y\,dy = \frac{y^2}{2}\cdot(-\cos y) = \frac{-\cos y\cdot y^2}{2}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: Toma Medić **BROJ INDEKSA:** 17-2-0052	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20
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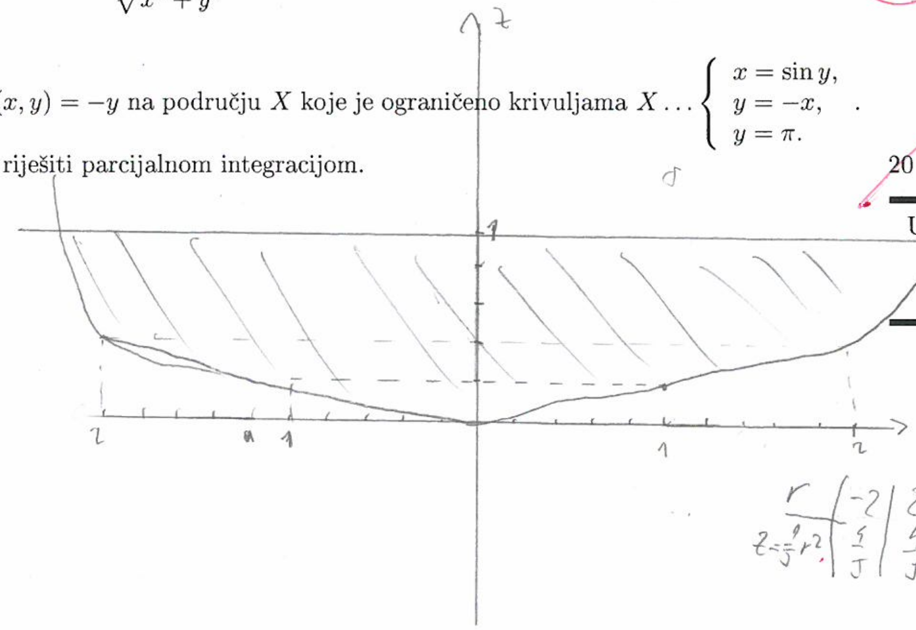
   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: 20

2.) $x^2 + y^2 = 5z$

$r^2 = 5z$

$z = \frac{1}{5} r^2$



r	-2	2	0	1	-1
$z = \frac{1}{5}r^2$	$\frac{4}{5}$	$\frac{4}{5}$	0	$\frac{1}{5}$	$\frac{1}{5}$

$z \in [\frac{1}{5} r^2, 1]$

$x^2 + y^2 = 5z$

$r^2 = 5$

$r = \sqrt{5}$

$r \in [0, \sqrt{5}]$

$\varphi \in [0, 2\pi]$

VIDI NOVAKOVIĆ

$$O = \int_0^{2\pi}\int_0^{\sqrt{5}}\int_{\frac{1}{5}r^2}^{1} r\,dz\,dr\,d\varphi = 2\pi \int_0^{\sqrt{5}} r z \Big|_{\frac{1}{5}r^2}^{1} dr$$

$$= 2\pi \int_0^{\sqrt{5}} r - \frac{1}{5} r^3\,dr = 2\pi \left(\frac{r^2}{2} - \frac{1}{5} \cdot \frac{r^4}{4} \right) \Big|_0^{\sqrt{5}} = \frac{1}{5}\pi \left(r^2 - \frac{r^4}{4} \right) \Big|_0^{\sqrt{5}} =$$

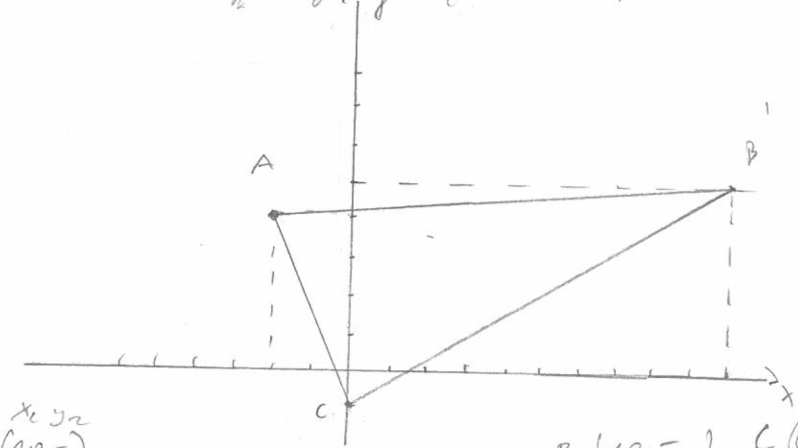
$$= \frac{1}{5}\pi \left[(\sqrt{5})^2 - \frac{(\sqrt{5})^4}{4} \right] = \frac{1}{5}\pi \left[5 - \frac{25}{4} \right] = \frac{1}{5}\pi \left[\frac{20-25}{4} \right] = \frac{1}{5}\pi \left[-\frac{5}{4} \right]$$

$$= -\frac{1}{4}\pi$$

3.) $A(-2,4)$ $B(10,5)$, $C(0,-1)$

$$\oint_{ABC} (x^2-y)\,dx + \sin(y^3)\,dy$$

$$\int_{C} P(x,y)\,dx + Q(x,y)\,dy = \iint_{D} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$



$\quad x_1\ y_1 \qquad x_2\ y_2$

$A(-2,4)$ $B(10,5)$

$\overline{AB}\ldots\ (x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(10+2)(y-4)=(5-4)(x+2)$

$12y-40=x+2$

$12y=x+4$

$y=\frac{1}{12}x+\frac{7}{2}$

$B(10,5)$ $C(0,-1)$

$\overline{BC}\ldots\ (x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(0-10)(y-5)=(-1-5)(x-10)$

$-10y+50=-6x+60$

$-10y=-6x\ 10$

$y=\frac{3}{5}x-1$

$A(-2,4)$ $C(0,-1)$

$\overline{AC}\ldots\ (x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$(0+2)(y-4)=(-1-4)(x+2)$

$2y-8=-5x-10$

$2y=-5x-2$

$y=-\frac{5}{2}x+1$

$-\frac{1}{12}=-1$

$\overline{AB}\ldots\ y=\frac{1}{12}x+\frac{7}{2}$

$-\frac{1}{12}x=-y+\frac{7}{2}\ /\cdot\frac{1}{12}$

$x=12y-42$

$\overline{AC}\ldots\ y=-\frac{5}{2}x+1$

$\frac{5}{2}x=-y+1$

$x=-\frac{2}{5}y-\frac{2}{5}$

$\overline{BC}\ldots\ y=\frac{3}{5}x-1$

$-\frac{3}{5}x=-y-1\ /\cdot\frac{5}{3}$

$x=\frac{5}{3}y+\frac{5}{3}$

$dx\,dy$

3.) NASTAVAK

Toma Mušić

$$P(x,y) = x^2 - y$$

$$Q(x,y) = \sin(y^3)\,dy$$

$$\frac{\partial(Q)}{\partial x} - \frac{\partial(P)}{\partial y} = \frac{\partial(\sin(y^3))}{\partial x} - \frac{\partial(x^2 - y)}{\partial y}$$

$$= 0 - 1 = 1$$

$$\int_{-2}^{0} \int_{-\frac{5}{2}x+1}^{\frac{1}{12}x+\frac{7}{2}} 1\,dy\,dx - \int_{0}^{10} \int_{\frac{3}{5}x-1}^{\frac{1}{12}x+\frac{7}{2}} 1\,dy\,dx =$$

$$\int_{-2}^{0} y \Big|_{-\frac{5}{2}x+1}^{\frac{1}{12}x+\frac{7}{2}} dx - \int_{0}^{10} y \Big|_{\frac{3}{5}x-1}^{\frac{1}{12}x+\frac{7}{2}} dx =$$

Toma Medić

4.) $r = 2$

$r \in [0, 2]$

$\varphi \in [0, \frac{3\pi}{2}]$

$x = r\cos\varphi$

$y = r\sin\varphi$

$dx\,dy = r\,dr\,d\varphi$

$r = \begin{pmatrix} 2\cos\varphi \\ 2\sin\varphi \end{pmatrix}$

$r' = \begin{pmatrix} -2\sin\varphi \\ 2\cos\varphi \end{pmatrix}$

$\| r'(t) \| = \sqrt{(-2\sin\varphi)^2 + (2\cos\varphi)^2}$

$= \sqrt{4\sin^2\varphi + 4\cos^2\varphi}$

$= \sqrt{4} = 2$

$\underline{I(0,0)}$

$= \int_0^{\frac{3\pi}{2}} \int_0^2 \frac{2}{\sqrt{x^2+y^2}}\, r\,dr\,d\varphi =$

$= \int_0^{\frac{3\pi}{2}} \int_0^2 \frac{2}{\cancel{r}} \cancel{r}\,dr\,d\varphi =$

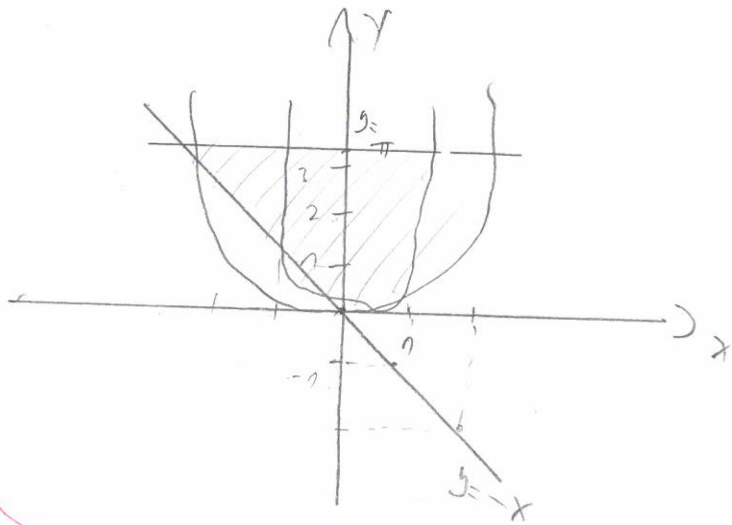
$= \int_0^{\frac{3\pi}{2}} 2 \cdot 2 \, d\varphi = \int_0^{\frac{3\pi}{2}} 4\,d\varphi = 4\varphi \Big|_0^{\frac{3\pi}{2}} = 4 \frac{3\pi}{2} = 6\pi$ ✓

~~$y = x$~~

x	0	1	2
$y=-x$	0	-1	-2

x	0	
$\sin y$	0	

5.)



$y \in [-x, \pi]$

$x \in [-1, 1]$

$\int_{-1}^{1} \int_{-x}^{\pi} -y \, dy\, dx = - \int_{-1}^{1} \frac{y^2}{2} \Big|_{-x}^{\pi} dx$ ✗

$= -\int_{-1}^{1} \left(\frac{\pi^2}{2} - \frac{x^2}{2} \right) dx =$

$= -4{,}9348 \int_{-1}^{1} - \frac{x^2}{\cancel{2}}\, dx$

$= -2{,}9348 \cdot \int_{-1}^{1} -x^2\, dx$

$= -2{,}9348 \left(-\frac{x^3}{3} \right) \Big|_{-1}^{1} = -2{,}9348 \cdot \left[-\frac{1}{3} + \frac{1}{3} \right] = 0$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa XXOXX
IME I PREZIME: Luka Brkavac **BROJ INDEKSA:** 12-2-0022-2010	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20

3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{\widehat{ABC}} (x^2 - y)\,\mathrm{d}x + \sin(y^3)\,\mathrm{d}y$. 20

4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20

5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$.

   Pomoć: $\int y \sin y \mathrm{d}y$ može se riješiti parcijalnom integracijom. 20

Ukupno:

$\varnothing$

① $y'''(t) + y''(t) = \text{sim}(t) \quad /\mathcal{L} \qquad y'(0)=0 \;;\; y(0) = y'' = 1$

$$s^3Y(s) - s^2\underset{1}{y(0)} - \underset{0}{sy'(0)} - \underset{1}{y''(0)} + s^2Y(s) - \underset{1}{sy(0)} - \underset{0}{y'(0)} = \frac{1}{s^2+1}$$

$$s^3Y(s) - s^2 - 1 + s^2Y(s) - s = \frac{1}{s^2+1}$$

$$Y(s)(s^3+s^2) = \frac{1}{s^2+1} + s^2 + 1 + s \Rightarrow \frac{1 + s^2(s^2+1) + 1(s^2+1) + s(s^2+1)}{s^2+1}$$

$$= \frac{1 + s^4 + s^2 + s^2 + 1 + s^3 + s}{s^2+1} = \frac{s^4+s^3+2s^2+s+2}{s^2+1}$$

$$Y(s) = \frac{s^4+s^3+2s^2+s+2}{s^2+1(s^3+s^2)} = \frac{s^4+s^3+2s^2+s+2}{s^2(s+1)(s^2+1)}$$

$$Y(s) = \frac{s^4+s^3+2s^2+s+2}{s^2(s+1)(s^2+1)}$$

$$= \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(s+1)} + \frac{Ds+E}{(s^2+1)} \quad / s^2(s+1)(s^2+1)$$

$$s^4+s^3+2s^2+s+2 = A(s+1)(s^2+1) + Bs(s+1)(s^2+1) + Cs^2(s^2+1) + s^2(Ds+E)(s+1)$$

$s=0 \Rightarrow A(0+1)(0+1) = 2$

$A = 2$

$s=-1 \Rightarrow C(1+1) = 2$

$2C = 2$

$C = 1$

Luka Bilavcu

$s^4 + s^3 + 2s^2 + s + 2 = 2s^3 + 2s + 2s^2 + 2 + Bs^4 + Bs^2 + Bs^3 + Bs + s^4 + s^2$
$+ Ds^4 + Ds^3 + Es^3 + Es^2$

$B + 1 + D = 1$

$2 + B + D + E = 1$

$2 + 1 + E = 2$

$2 + B = 1$

$B = 1 - 2$

$\boxed{B = -1}$ $\boxed{D = 1}$

$\boxed{A = 2}$ $\boxed{E - 1}$

$\boxed{C = 1}$

$$y(s) = \frac{2}{s^2} - \frac{1}{s} + \frac{1}{(s+1)} + \frac{s-1}{s^2+1} \Big/ \mathcal{L}^{-1}$$

$$= 2t - 1 + e^{-t} + \frac{s}{s^2+1} - \frac{1}{s^2+1}$$

$$y(t) = 2t - 1 + e^{-t} + \cos t - \sin t$$

PROVJERA?

$-1 + 1 + D = 1$

$3 + E = 2$

$E = -1$

(2.) $x^2+y^2=5z \quad , z \leq 1 \qquad x^2+y^2=r^2$

Luka Bekavac

$$\varphi\,[0, 2\pi]$$

$$r\,[0, \sqrt{5}\,]$$

$$z\left[\frac{1}{5}r^2, 1\right]$$

VIDI NOVAKOVIĆ

$$\int_0^{2\pi} d\varphi \int_0^{\sqrt{5}} r\,dr \int_{\frac{1}{5}r^2}^{1} dz = \int_0^{2\pi} d\varphi \int_0^{\sqrt{5}} \left(1 - \frac{1}{5}r^2\right) r\,dr$$

$$\int_0^{2\pi} d\varphi \int_0^{\sqrt{5}} \left(r - \frac{\cancel{5}\,1}{5} r^3\right) dr = \int_0^{2\pi} \left(\frac{r^2}{2} - \frac{1}{5}\frac{r^4}{4}\right)\Bigg|_0^{\sqrt{5}} d\varphi$$

$$\int_0^{2\pi} \left(\frac{r^2}{2} - \frac{r^4}{20}\right)\Bigg|_0^{\sqrt{5}} d\varphi = \int_0^{2\pi} \left(\frac{5}{2} - \frac{\cancel{25}}{\cancel{20}}\right) d\varphi$$

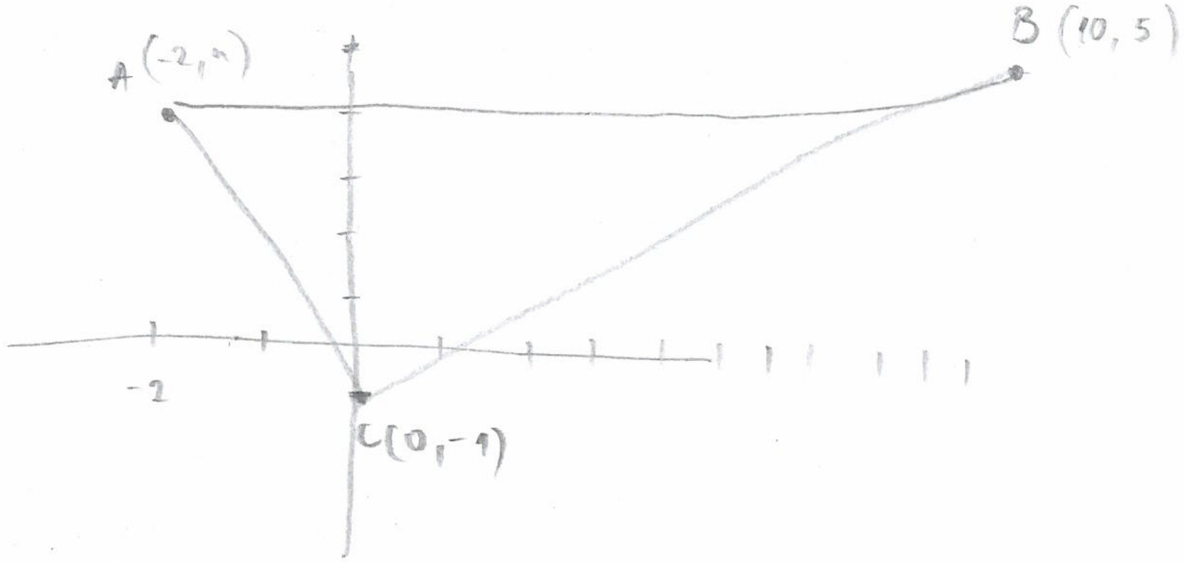
$$\int_0^{2\pi} \left(\frac{5}{2} - \frac{5}{4}\right) d\varphi = \frac{5}{4}\varphi \Big|_0^{2\pi} = \frac{5}{2}\pi$$

3.

Luka Bilavac

A(-2, 4) B(10, 5) C(0, -1)

(1) (2) (2)



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\overline{AB} = y - 4 = \frac{5-4}{10+2}(x+1) = y - 4 = \frac{1}{12}(x+1)$$

×

$$y - 4 = \frac{1}{12}x + \frac{1}{12}$$

$$y = \frac{1}{12}x + \frac{49}{12}$$

$$\overline{AC} = y - 4 = \frac{-1-4}{0-2}(x+2)$$

$$y - 4 = -\frac{5}{2}(x+2) =$$

$$y - 4 = -\frac{5}{2}x + 5$$

$$y = -\frac{5}{2}x + 9$$

$$\overline{BC} = y - 5 = \frac{-1-5}{0-10}(x-10)$$

$$y - 5 = \frac{-6}{10}(x-10) = -\frac{3}{5}(x-10)$$

$$y - 5 = -\frac{3}{5}x + 6$$

$$y = -\frac{3}{5}x + 11$$

$$\int_{ABC} (x^2 - y)\,dx + \sin(y^3)\,dy$$

$x = r\cos\varphi$
$y = r\sin\varphi$
$dx\,dy = r\,dr\,d\varphi$

$$\int_{\frac{1}{12}x + \frac{49}{12}}^{-\frac{5}{2}x+9} (x^2 - y)\,dx + \int_{\frac{1}{12}x + \frac{49}{12}}^{-\frac{3}{5}x+11} \sin y^3\,dy$$

$$\int_{\frac{1}{12}x + \frac{49}{12}}^{-\frac{5}{2}x+9} (r\cos^2\varphi - r\sin\varphi)\,dx + \int_{\frac{1}{12}x + \frac{45}{12}}^{-\frac{3}{5}x+11} \sin(r\sin^3\varphi)\,dy$$

4. $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$

$\varphi \left[0, \dfrac{3\pi}{2}\right]$

$r \left[-2, 2\right]$

$x = \cos\varphi$
$y = \sin\varphi$
$dx\,dy = r\,dr\,dt$

$$\int_0^{3\frac{\pi}{2}} d\varphi \int_{-2}^{2} \frac{2}{\sqrt{\cos^2\varphi + \sin^2\varphi}}\, r\,dr$$

Luka Belošec

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ŠIME MATANOVIĆ **BROJ INDEKSA:** 57655

Grupa xxoxx POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20 ~~20~~
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20 ~~20~~
3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{A\widehat{B}C} (x^2 - y)\,\mathrm{d}x + \sin(y^3)\,\mathrm{d}y$. 20 ~~20~~
4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20 ~~20~~
5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$.

   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: 0

$f(0) = 1$

$f'(0) = 0$

$f''(0) = 1$

1. $f'''(t) + f''(t) = \sin(t)$

$s^3F(s) - s^2 \underbrace{f(0)}_{1} - s\underbrace{f'(0)}_{0} - \underbrace{f''(0)}_{1} + s^2F(s) - s\underbrace{f(0)}_{1} - \underbrace{f'(0)}_{0} = \frac{1}{s^2+1}$

$s^3F(s) - s^2 - 1 + s^2F(s) - s = \frac{1}{s^2+1}$

$F(s)(s^3+s^2) = \frac{1+}{s^2+1} + s^2 + s + 1$

$F(s)(s^3+s^2) = \frac{1 + s^2(s^2+1) + s(s^2+1) + 1(s^2+1)}{s^2+1}$

$F(s)(s^3+s^2) = \frac{1 + s^4 + s^2 + s^3 + s + s^2 + 1}{s^2+1}$

$F(s)(s^3+s^2) = \frac{s^4+s^3+2s^2+s+2}{s^2+1} \quad / : (s^3+s^2)$

$F(s) = \dfrac{\frac{s^4+s^3+2s^2+s+2}{s^2+1}}{\frac{s^3+s^2}{1}}$

$F(s) = \frac{s^4+s^3+2s^2+s+2}{s^2(s+1)(s^2+1)}$

$F(s) = \frac{s^4+s^3+2s^2+s+2}{(s^3+s^2)(s^2+1)}$

$$F(s)=\frac{s^4+s^3+2s^2+s+2}{s^2(s+1)(s^2+1)} \quad / \cdot s^2$$

$$F(s) = \frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+1}+\frac{Ds+E}{s^2+1} \quad / \cdot s^4+s^4$$

$$s^4+s^3+2s^2+s+2=A(s+1)(s^2+1)+Bs(s+1)(s^2+1)+(Ds+E)(s^2)(s+1)$$

$$s^4+s^3+2s^2+s+2=A(s^3+s+s^2+1)+Bs(s^3+s+s^2+1)+C(s^2)(s^2+1)+(Ds+E)(s^2)(s+1)$$

$$s^4+s^3+2s^2+s+2=A(s^3+s+s^2+1)+Bs(s^3+s+s^2+1)+C(s^4+s^2)+(Ds+E)(s^3+s^2)$$

$$s^4+s^3+2s^2+s+2=As^3+As+As^2+A+Bs+Bs^4+Bs^2+Bs^3+Bs+Cs^4+Cs^2+Ds^4+Ds^3+Es^3+Es^2$$

$1=B+C+D$

$1=A+B+D+E$

$2=A+B+C+E$

$1=A+B \longrightarrow 1=A+B$

$A+B=1$

$B=1-A$

$B=1-2$

$\boxed{B=-1}$

$2=A$

$\boxed{A=2}$

$1=A+B$

$1=B+C+D$

$B+C+D=1$

$-1+C+D=0$

$C+D=0+1$

$C+D=1$

$\boxed{C=-D+1}$

$C=0+1$

$\boxed{C=1}$

$$\left.\begin{array}{l}1=A+B+\cancel{D}+E\\2=A+B\cancel{-D}+1+E\end{array}\right] +$$

$3=2A+2E+1$

$3=2\cdot 2+2E+1$

$3=4+2E+1$

$4+2E+1=3$

$2E=3-4-1$

$2E=-2 \;/ : (-2)$

$E=\frac{-2}{-2}$

$\boxed{E=1}$

$1=A+B+D+E$

$A+B+D+E=1$

$E=1-A-B-D$

$E=1-2+1$

$1=B+C+D$

$1=-1-D+1+D$

$1=0$

$\boxed{D=0}$

$$F(t) = \frac{2}{s^2} - \frac{1}{s} + \frac{1}{s+1} + \frac{1}{s^2+1}$$

$$f(t) = 2t - 1 + e^{-t} + \sin t$$

X

PROVJERA ?

12. $x^2+y^2 = a\cdot z$ PARABOLOID

$\varphi \in [0, 2\pi]$

$r \in [0, 1]$

$z \in [5z, -5z]$

$x^2+y^2 = 5z$

$x^2+y^2 = r^2$

$r^2 = 5z$

$5z = r^2 /:5$

$r^2 = 5z$

$r = \pm\sqrt{5z}$

$$\int_{5z}^{-5z}\int_0^1\int_0^{2\pi} x^2+y^2\,dx\,dy = \int_{5z}^{-5z}\int_0^1 \left.\frac{x^3}{3}+\frac{y^3}{3}\right|_0^{2\pi} dx\,dy =$$

$$= \int_{5z}^{-5z}\int_0^1 \frac{2\pi^3}{3}+\frac{2\pi^3}{3} =$$

1.) $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$

$\varphi \in [0, 3\frac{\pi}{2})$

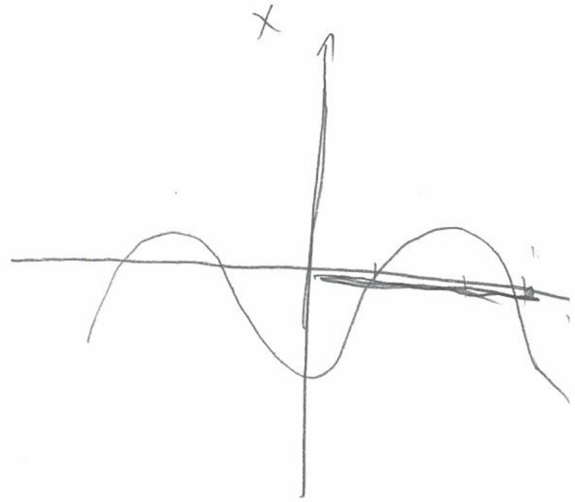
$r \in [0, 2]$

$$\int_0^2\int_0^{3\frac{\pi}{2}} \frac{2}{\sqrt{x^2+y^2}} = 2$$

$f(x,y) = -y$

$X \ldots \begin{cases} x = \sin y \\ y = -x \\ y = \pi \end{cases}$

$X' \colon \begin{cases} x = \cos x \\ y = x \\ y = \pi \end{cases}$



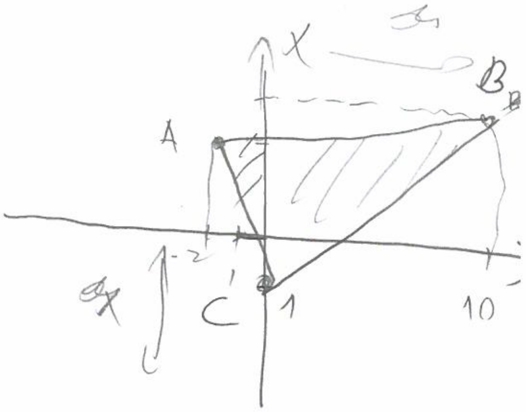
$x = \sqrt{x^2 + y^2 + z^2}$

$x = \sqrt{\cos x^2 + x^2 + \pi^2}$

$x =$

3. $(y_2 - y_1)(x - x_1) = (x_2 - x_1)(y - y_1)$

$A(-2,4) \quad B(10,5) \quad C(0,-1)$



$AB \ldots (5-4)(x+2) = (10+2)(y-4)$

$(1)(x+2) = (12)(y-4)$

$x + 2 = 12y - 48$

$x = 12y - 48 - 2$

$\boxed{x = 12y - 50}$ ×

$AC \ldots (-1-4)(x+2) = (0+2)(y-4)$

$-5x - 10 = 2y - 8$

$-5x = 2y - 8 + 10$

$-5x = 2y - 2 \;/ : 5$

$\boxed{x = -\frac{2}{5}y + \frac{2}{5}}$

$BC \ldots (-1-5)(x-10) = (0-10)(y-5)$

$-6x + 60 = -10y + 50$

$-6x = -10y + 50 - 60$

$-6x = -10y - 10 \;/ : -6$

$x = \frac{10}{6}y + \frac{10}{6}$

$\boxed{x = \frac{5}{3}y + \frac{5}{3}}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** DAMIR DVORNIK **BROJ INDEKSA:**	Grupa xxoxx POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. ~~20~~

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4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20

5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$

   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. 20

Ukupno: $\cancel{0}$

1. $f'''(t) + f''(t) = \sin t$ $\qquad f'(0) = 0,\ f(0) = 1,\ f'' = 1$

$s^3F(s) - s^2f(0) - \cancel{sf'(0)} - f''(0) + s^2F(s) - sf(0) - \cancel{f'(0)} = \frac{1}{s^2-1}$

$s^3F(s) - s^2 - 1 + s^2F(s) - 1 = \frac{1}{s^2-1}$

$s^3F(s) + s^2F(s) - s^2 - 2 = \frac{1}{s^2-1}$

$F(s)(s^3+s^2) = \frac{1+s^2+2}{s^2-1} \quad / : s^3+s^2$

$F(s) = \frac{1+s^2+2}{(s^2-1)(s^3+s^2)} = \frac{s^2+3}{(s^2-1)s^2(s-1)} = \frac{As}{(s-1)} + \frac{B}{s} + \frac{C}{s^2} + \frac{D}{(S-1)} \quad / \cdot (s^2-1)s^2(s-1)$

$F(s) = As \cdot s^2(s-1) + Bs(s^2-1)(s-1) + C(s^2-1)(s-1) + D(s^2-1)s^2$

$F(s) = As(s^3-s^2) + Bs(s^3-s^2-s+1) + C(s^3-s^2-s+1) + Ds^2(s^2-1)$

$F(s) = As^4 - As^3 + Bs^4 - Bs^3 - Bs^2 + Bs + Cs^3 - Cs^2 - Cs + \underline{C} + Ds^4 - Ds^2$

$s^4 \ldots\ 0 = (A+B+D)$ $\qquad$ sl. člon ... $\boxed{1=C}$ $\qquad 0 = -1+1-2$

$s^3 \ldots\ 1 = (-A-B+C) \Rightarrow 1 = -A-1+1 \Rightarrow A = -1$

$s^2 \ldots\ 0 = (-B-C-D) \Rightarrow 0 = -1-1-D \Rightarrow \boxed{D=-2}$

$s \ldots\ 0 = (B-C) \Rightarrow 0 = B-1 \Rightarrow \boxed{B=1}$

$$F(s) = \frac{-1}{(s-1)} + \frac{1}{s} + \frac{1}{s^2} + \frac{-2}{(s-1)}$$

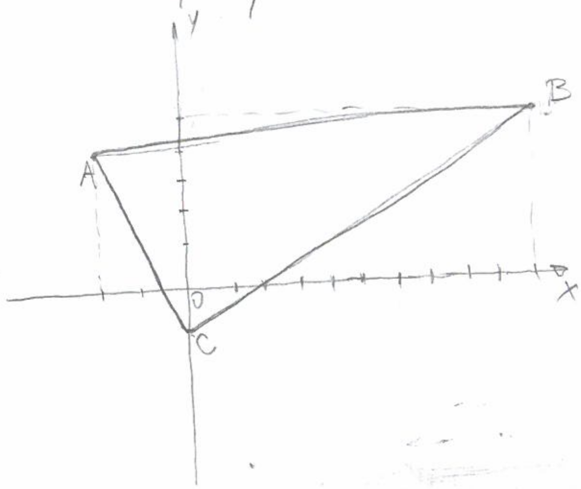
$$F(s) = e^t + 1 + t - e^t$$

PROVJERA:

$f(0) = e^0 + 1 - 0 - e^0 \ldots$

3. $A(\overset{x}{-2},\overset{y}{4})$
$B(10,5)$
$C(0,-1)$

$$\int_{ABC} (x^2 - y)\,dx + \sin(y^3)\,dy$$



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: TONY CAR **BROJ INDEKSA:** 17-2-0045-2010	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. — 20

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. — 20

3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{\widehat{ABC}} (x^2 - y)\,dx + \sin(y^3)\,dy$. — 20

4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. — 20

5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = -x, \\ y = \pi. \end{cases}$.

   Pomoć: $\int y \sin y dy$ može se riješiti parcijalnom integracijom. — 20

Ukupno: $\emptyset$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** JURE SUILIČIĆ **BROJ INDEKSA:** 17-2-0043-2010	Grupa XXOXX POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20

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