

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: SANDRO GRODVIĆ

BROJ INDEKSA: ~~17-2-0213-2012~~

H2

- Riješiti jednačbu:  $z^4 - (4 - i)^5 = 0$ . Prikaži rješenja u kompleksnoj ravnini! 12+3
- Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 5}$ . 5+15+5
- Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ . 5+5+10
- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 5x - (5 + 1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
- Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x + 2y - z + u &= 5 \\ 2x + 5y - z + 2u &= 5 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 5 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow 0} \frac{|x|}{x}$ .

5

Ukupno:

40

1.  $z^4 - (4 - i)^5 = 0$

$$4 - i = \sqrt{17} (\cos -0.24 + i \sin 0.24)$$

$$= \sqrt{4^2 + (-1)^2} = \sqrt{17}$$

$$\varphi = \arctg \frac{-1}{4} = -0.24 \checkmark$$

$$(4 - i)^5 = 1191 (\cos -1.2 + i \sin -1.2) \checkmark$$

$$z^4 = 1191 (\cos -1.2 + i \sin 1.2)$$

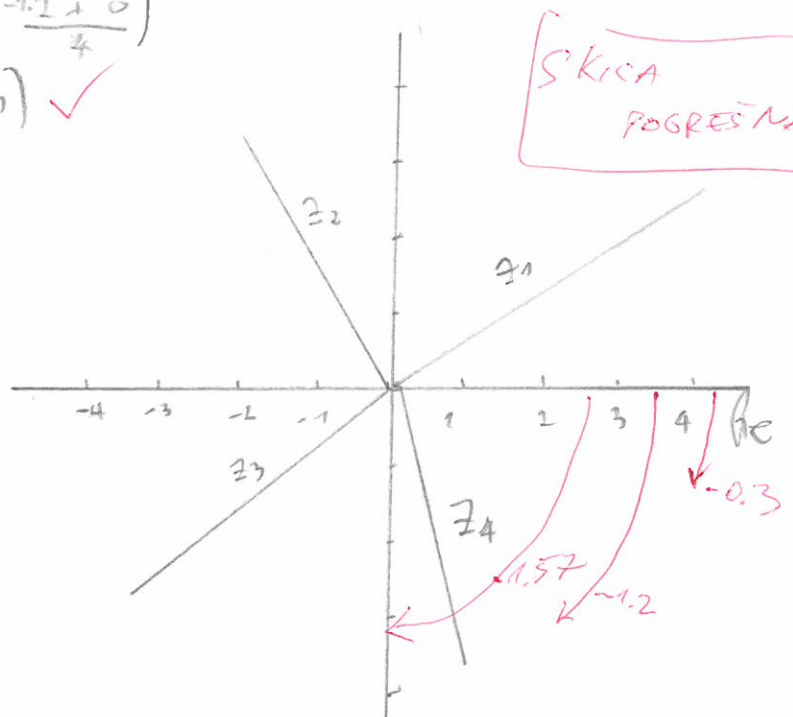
$$z_1 = \sqrt[4]{1191} (\cos \frac{-1.2 + 0}{4} + i \sin \frac{-1.2 + 0}{4})$$

$$z_1 = 5.87 (\cos -0.3 + i \sin -0.3) \checkmark$$

$$z_2 = 5.87 (\cos 1.27 + i \sin 1.27)$$

$$z_3 = 5.87 (\cos 2.84 + i \sin 2.84)$$

$$z_4 = 5.87 (\cos 4.41 + i \sin 4.41)$$



$$b. \quad x + 2y - z + u = 5$$

$$2x + 5y - z + 2u = 5$$

$$3x - y - 2z + u = 5$$

$$x - y + 5z - 5u = 5$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 2 & 5 & -1 & 2 & 5 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 5 & -5 & 5 \end{array} \right] \begin{array}{l} \\ \text{II} - 2\text{I} \\ \text{III} - 3\text{I} \\ \text{IV} - \text{I} \end{array}$$

$$\left[ \begin{array}{cccc|c} -2 & 5 & -1 & 2 & 5 \\ -2 & -4 & 2 & -2 & -10 \\ 0 & 1 & 1 & 0 & -5 \\ 2 & -1 & -2 & 1 & 5 \\ -3 & -6 & 3 & -3 & -15 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & -7 & 1 & -2 & -10 \\ 0 & -3 & 4 & -6 & 0 \end{array} \right] \begin{array}{l} \\ \\ \text{III} + 7\text{II} \\ \text{IV} + 3\text{II} \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 0 & -1 & 1 & 0 & -5 \\ 0 & -7 & 1 & -2 & -10 \\ 0 & -3 & 4 & -6 & 0 \end{array} \right] \begin{array}{l} \\ \\ \text{III} + 7\text{II} \\ \text{IV} + 3\text{II} \end{array}$$

$$\left[ \begin{array}{cccc|c} 0 & -7 & 1 & -2 & -10 \\ 1 & -1 & 3 & -3 & 3 \\ -1 & -2 & 1 & -1 & -3 \\ 0 & -3 & 4 & -6 & 0 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 0 & -3 & 4 & -6 & 0 \\ 0 & 3 & 3 & 0 & -15 \\ 0 & 0 & 7 & -6 & -15 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 0 & 8 & -2 & -45 \\ 0 & 0 & 7 & -6 & -15 \end{array} \right]$$

6.  $\lim_{x \rightarrow 0} \frac{|x|}{x} = \left[ \frac{0}{0} \right] \text{N/A}$

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$\lim_{x \rightarrow 0^-} \frac{|x|}{x} = \frac{0^+}{0^-} = -1 \checkmark$

$f(x) = \begin{cases} x & x > 0 \\ -1 & x < 0 \end{cases} \checkmark$

$\lim_{x \rightarrow 0^+} \frac{|x|}{x} = \frac{0^+}{0^+} = 1 \checkmark$

PROVERA  $\checkmark$

$\exists A \ 0^- \quad x = -0.1$   
 $\frac{|-0.1|}{-0.1} = \frac{0.1}{-0.1} = -1 \parallel \checkmark$

$\exists A \ 0^+ \quad x = 0.1$   
 $\frac{|0.1|}{0.1} = \frac{0.1}{0.1} = 1 \parallel \checkmark$

2.  $f(x) = x - \sqrt{x^2 - 5}$

DOMENA

$x^2 - 5 \geq 0$

$x^2 \geq 5 \quad | \sqrt{\phantom{x}}$

$\sqrt{5} \leq x \leq \sqrt{5}$

$D_f = \langle -\infty, -\sqrt{5} \rangle \cup [\sqrt{5}, +\infty) \checkmark$

5

NEMA VERTIKALNE ASIMPTOTE

$\lim_{x \rightarrow -\infty} x - \sqrt{x^2 - 5} = -\infty - \infty = -\infty$

NEMA L.H.A

H.A

$\lim_{x \rightarrow +\infty} x - \sqrt{x^2 - 5} = \lim_{x \rightarrow +\infty} \frac{x + \sqrt{x^2 - 5}}{x + \sqrt{x^2 - 5}} = \lim_{x \rightarrow +\infty} \frac{x^2 - x^2 + 5}{x^2 + \sqrt{x^2 + 5}} \cdot \frac{1}{x} = \frac{0}{2} = 0 \checkmark$

$\lim_{x \rightarrow -\infty} x - \sqrt{x^2 - 5} = \begin{cases} x \rightarrow -x \\ -\infty \rightarrow +\infty \end{cases} = \lim_{x \rightarrow -\infty} \frac{-x - \sqrt{x^2 - 5}}{-x} \cdot \frac{1}{x} = \frac{-1 - 1}{-1} = \frac{-2}{-1} = 2$

L.C.A

$\lim_{x \rightarrow -\infty} x - \sqrt{x^2 - 5} - 2x = \lim_{x \rightarrow -\infty} -3x - \sqrt{x^2 - 5}$

L.K.A.?

$= \lim_{x \rightarrow +\infty} 3x - \sqrt{x^2 - 5} = \lim_{x \rightarrow +\infty} \frac{3x + \sqrt{x^2 - 5}}{3x + \sqrt{x^2 - 5}} = \lim_{x \rightarrow +\infty} \frac{3x^3 - x^2 + 5}{3x + \sqrt{x^2 - 5}}$

~~8~~

$$f'(x) = (x - \sqrt{x^2 - 5})' = 1 - \frac{1}{2} (x^2 - 5)^{-\frac{1}{2}} \cdot 2x = 1 - \frac{x}{\sqrt{x^2 - 5}} \quad \checkmark$$

$$f''(x) = \left( 1 - \frac{x}{\sqrt{x^2 - 5}} \right)' = 0 - \frac{1 \cdot \sqrt{x^2 - 5} - x \cdot \frac{1}{2} (x^2 - 5)^{-\frac{1}{2}} \cdot 2x}{x^2 - 5} = \checkmark$$

$$f''(x) = \frac{\sqrt{x^2 - 5} - \frac{x^2}{\sqrt{x^2 - 5}}}{x^2 - 5}$$

5

3.  $f(x) = \ln(4 - x^2)$

DOMENA

$$4 - x^2 > 0$$

$$x^2 > 4$$

$$-2 < x < 2$$

$$D_f = (-2, 2) \quad \checkmark$$

$$f(-x) = \ln(4 - (-x)^2) = \ln(4 - x^2) = f(x) \quad ? \rightarrow$$

funkcija nije parna ~~X~~

5.  $x + 2y - z + u = 5$   
 $2x + 5y - z + 2u = 5$   
 $3x - y - 2z + u = 5$   
 $x - y + 3z - 5u = 5$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 2 & 5 & -1 & 2 & 5 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 5 \end{array} \right] \begin{array}{l} -2R_1 \\ -2R_1 \\ -R_1 \end{array} = \left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & -7 & 1 & -2 & -10 \\ 0 & -3 & 4 & -6 & 0 \end{array} \right] \begin{array}{l} -2R_2 \\ +7R_2 \\ +3R_1 \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & -3 & 1 & 15 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 8 & -2 & -45 \\ 0 & 0 & 2 & 0 & -15 \end{array} \right] \begin{array}{l} +R_4 \\ -R_3 \end{array} = \left[ \begin{array}{cccc|c} 1 & 0 & -3 & 1 & 15 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & -2 & -30 \\ 0 & 0 & 0 & 0 & -15 \end{array} \right] \begin{array}{l} +3R_3 \\ -R_3 \\ -7R_3 \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 7 & -75 \\ 0 & 1 & 0 & 2 & 25 \\ 0 & 0 & 1 & -2 & -30 \\ 0 & 0 & 0 & 14 & 175 \end{array} \right] \begin{array}{l} -14 \\ -7R_4 \\ -2R_1 \\ +2R_4 \end{array} = \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 7 & -75 \\ 0 & 1 & 0 & 2 & 25 \\ 0 & 0 & 1 & -2 & -30 \\ 0 & 0 & 0 & 1 & 13.93 \end{array} \right] \begin{array}{l} -7R_4 \\ -2R_1 \\ +2R_4 \end{array}$$

$$= \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & -171.5 \\ 0 & 1 & 0 & 0 & -2.86 \\ 0 & 0 & 1 & 0 & -2.94 \\ 0 & 0 & 0 & 1 & 13.93 \end{array} \right]$$

$$x = -171.5$$

$$y = -2.86$$

$$z = -2.94$$

$$u = 13.93$$

PROVJERA ?

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

IME I PREZIME: JURAJ POVIJAK

BROJ INDEKSA: 17-2-0201-2012

POPUNJAVA  
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Broj ↓  
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H2

- Riješiti jednadžbu:  $z^4 - (4-i)^5 = 0$ . Prikaži rješenja u kompleksnoj ravni! 12+3
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- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 5x - (5+1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira. ~~20(graf)~~
- Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje: ~~15~~

$$\begin{aligned} x + 2y - z + u &= 5 \\ 2x + 5y - z + 2u &= 5 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 5 \end{aligned}$$

6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow 0} \frac{|x|}{x}$ . *lijevo, desno  
da li su isti*

~~5~~

Ukupno:

10

2.  $f(x) = x - \sqrt{x^2 - 5}$

a) Domena

$$\begin{aligned} x^2 - 5 &\geq 0 \\ x_{1,2} &= \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot 0}}{2} \\ &= \frac{5 \pm \sqrt{25}}{2} \end{aligned}$$

$$\begin{aligned} x_1 &= \frac{5 + \sqrt{25}}{2} \\ x_2 &= \frac{5 - \sqrt{25}}{2} \end{aligned}$$

$$D_f = \left(-\infty, \frac{5 - \sqrt{25}}{2}\right] \cup \left[\frac{5 + \sqrt{25}}{2}, +\infty\right)$$

b) Prva derivacija

$$\begin{aligned} (x - \sqrt{x^2 - 5})' &= (x)' - \frac{1}{2\sqrt{x^2 - 5}} \cdot (x^2 - 5)' \\ &= 1 - \frac{1}{2\sqrt{x^2 - 5}} \cdot 2x \\ &= 1 - \frac{x}{\sqrt{x^2 - 5}} \end{aligned}$$

$f'(x) = 1 - \frac{x}{\sqrt{x^2 - 5}}$

c) Druga derivacija

$$\begin{aligned} \left(1 - \frac{x}{\sqrt{x^2 - 5}}\right)' &= (1)' - \frac{(x)' \cdot \sqrt{x^2 - 5} - (x) \cdot (\sqrt{x^2 - 5})'}{(\sqrt{x^2 - 5})^2} \\ &= - \frac{\sqrt{x^2 - 5} - (x) \cdot \left(\frac{1}{2\sqrt{x^2 - 5}}\right) \cdot (x^2 - 5)'}{x^2 - 5} \\ &= - \frac{\sqrt{x^2 - 5} - (x) \cdot \left(\frac{1}{2\sqrt{x^2 - 5}}\right) \cdot 2x}{x^2 - 5} \\ &= - \frac{\sqrt{x^2 - 5} - x \cdot \frac{2x}{2\sqrt{x^2 - 5}}}{x^2 - 5} \\ &= - \frac{\sqrt{x^2 - 5} - \frac{x^2}{\sqrt{x^2 - 5}}}{x^2 - 5} \\ &= - \frac{\frac{x^2 - 5 - x^2}{\sqrt{x^2 - 5}}}{x^2 - 5} = - \frac{-5}{\sqrt{x^2 - 5} \cdot (x^2 - 5)} \end{aligned}$$

$f''(x) = \frac{5}{(x^2 - 5)^{\frac{3}{2}}}$

3.  $g(x) = \ln(4-x^2)$

$\ln > 0$

a) Domena

$4-x^2 > 0$

$-x^2 > -4 \quad | :(-1)$

$x^2 > 4 \quad | \sqrt{\quad}$

$x < 2$

b) Welpamast

$g(-x) = \ln(4-(-x)^2)$   
 $= \ln(4-x^2)$

parna funkcija



$D_g = \langle -\infty, 2 \rangle$

5.

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 5 \\ 2 & 5 & -1 & 2 & 5 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 5 \end{bmatrix} \begin{array}{l} I \cdot (-2) + II \\ I \cdot (-3) + III \\ I \cdot (-1) + IV \\ \sim \end{array} \begin{bmatrix} 1 & 2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & -7 & 1 & -2 & -10 \\ 0 & -3 & 4 & -6 & 0 \end{bmatrix} \begin{array}{l} II \cdot (-2) + I \\ \sim \end{array} \begin{bmatrix} 1 & 0 & -3 & 1 & 15 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & -7 & 1 & -2 & -10 \\ 0 & -3 & 4 & -6 & 0 \end{bmatrix} \begin{array}{l} II \cdot 7 + III \\ II \cdot 3 + IV \\ \sim \end{array}$$

$$\begin{bmatrix} 1 & 0 & -3 & 1 & 15 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 0 & 8 & -2 & -45 \\ 0 & 0 & 7 & -6 & -15 \end{bmatrix} \begin{array}{l} II \cdot 8 - III \\ \sim \end{array} \begin{bmatrix} 1 & 0 & -3 & 1 & 15 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 8 & 0 & 2 & 5 \\ 0 & 0 & 7 & -6 & -15 \end{bmatrix} \begin{array}{l} II \cdot 7 - IV \\ \sim \end{array} \begin{bmatrix} 1 & 0 & -3 & 1 & 15 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 8 & 0 & 2 & 5 \\ 0 & 7 & 0 & 6 & -20 \end{bmatrix} \begin{array}{l} II \cdot 3 + I \\ \sim \end{array}$$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 8 & 0 & 2 & 5 \\ 0 & 7 & 0 & 6 & -20 \end{bmatrix}$$

duparametersko rjesenje

PROVJERA

6.  $\lim_{x \rightarrow 0} \frac{|x|}{x} = \frac{0}{0}$

Polyak

$\lim_{x \rightarrow 0^+} \frac{|x|}{x} = \frac{0^+}{0^+} = +\infty = \lim_{x \rightarrow 0^-} \frac{|x|}{x} = \frac{0^-}{0^-} = +\infty$

$\lim_{x \rightarrow 0} \frac{|x|}{x} = +\infty$  X

4.  $h(x) = \frac{x^2 - 5x - (5+1)}{x^2 + 1}$

a) Domena

$x^2 - 5x - (5+1) > 0$   
 $x^2 - 5x - 5 - 1 > 0$   
 $x^2 - 5x - 6 > 0$



$D_h = \mathbb{R} \setminus \{-1, 6\}$

$x_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot (-6)}}{2}$   
 $x_{1,2} = \frac{5 \pm \sqrt{49}}{2}$

h) (Ne)parnost

$h(-x) = \frac{(-x)^2 - 5 \cdot (-x) - (5+1)}{(-x)^2 + 1}$   
 $= \frac{x^2 + 5x - 6}{x^2 + 1}$

miti parna

miti neparna

$x_1 = \frac{5+7}{2} = \frac{12}{2} = 6$   
 $x_2 = \frac{5-7}{2} = \frac{-2}{2} = -1$

} multičke

c) lok. ekstremi; prva derivacija

$\left( \frac{x^2 - 5x - (5+1)}{x^2 + 1} \right)' = \frac{(x^2 - 5x - 6)' \cdot (x^2 + 1) - (x^2 - 5x - 6) \cdot (x^2 + 1)'}{(x^2 + 1)^2}$   
 $= \frac{(2x - 5) \cdot (x^2 + 1) - (x^2 - 5x - 6) \cdot 2x}{(x^2 + 1)^2}$   
 $= \frac{2x^3 + 2x - 5x^2 - 5 - 2x^3 - 10x^2 - 12x}{(x^2 + 1)^2}$   
 $= \frac{-10x - 15x^2 - 5}{(x^2 + 1)^2}$

$h'(x) = \frac{-15x^2 - 10x - 5}{(x^2 + 1)^2}$

ispitivanje lok. ekstremi

$\frac{-15x^2 - 10x - 5}{x^2 + 2x^2 + 1} = 0$

$-15x^2 - 10x - 5 \neq 0 \quad | :5$

$-3x^2 - 2x - 1 \neq 0$

$x_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot (-3) \cdot (-1)}}{-6}$

$x_{1,2} = \frac{2 \pm \sqrt{16}}{-6}$

d) toche inflexiõe

$$\begin{aligned} \left( \frac{-15x^2 - 10x - 5}{x^2 + 1} \right)' &= \frac{(-15x^2 - 10x - 5)' \cdot (x^2 + 1) - (-15x^2 - 10x - 5) \cdot (x^2 + 1)'}{(x^2 + 1)^2} \\ &= \frac{(-30x - 10) \cdot (x^2 + 1) - (-15x^2 - 10x - 5) \cdot (2x)}{x^4 + 2x^2 + 1} \\ &= \frac{-30x^3 - 30x - 10x^2 - 10 - (-30x^3 - 20x^2 - 10x - 10x)}{x^4 + 2x^2 + 1} \\ &= \frac{-30x^3 - 30x - 10x^2 - 10 + 30x^3 + 20x^2 + 10x + 10x}{x^4 + 2x^2 + 1} \\ &= \frac{-90x^3 + 20x^2 - 10x + 10x^2 + 20x^2 - 10}{x^4 + 2x^2 + 1} \end{aligned}$$

e) asimptote

V. A.  $\lim_{x \rightarrow -1}$

GRAF ?



$$1. z^4 - (4-i)^5 = 0$$

$$z^4 - (1024 - i^5) = 0$$

$$z^4 - 1024 + i = 0$$

$$z^4 = 1024 - i$$

$$w = 1024 - i; \quad x = 1024 \quad y = -1$$

$$r = \sqrt{1024^2 + 1}$$

$$r = 5\sqrt{99}$$

$$\tan \theta = \frac{y}{x} \\ = -\frac{1}{1024}$$

Poljil

2. asimptote

V.A. nema jer funkcija nema produkt

$$H.A. \lim_{x \rightarrow \infty} x - \sqrt{x^2 - 5} = \infty$$

$$K.A. k = \lim_{x \rightarrow \infty} \frac{x - \sqrt{x^2 - 5}}{x} \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{1 - \sqrt{1 - \frac{5}{x^2}}}{1} = 0$$

$$L = \lim_{x \rightarrow \infty} x - \sqrt{x^2 - 5} + 0 \cdot \frac{x - \sqrt{x^2 - 5} - 0}{x - \sqrt{x^2 - 5} - 0} = \lim_{x \rightarrow \infty} \frac{(x - \sqrt{x^2 - 5})^2}{x - \sqrt{x^2 - 5}} = \lim_{x \rightarrow \infty} \frac{x - x^2 - 5}{x - \sqrt{x^2 - 5}} \stackrel{L'H}{=} \\ = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 1 - \frac{5}{x^2}}{\frac{1}{x}}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 5 \\ 2 & 5 & -1 & 2 & 5 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 5 \end{bmatrix} \xrightarrow{IV \cdot (-3) + III} \begin{bmatrix} 1 & 2 & -1 & 1 & 5 \\ 2 & 5 & -1 & 2 & 5 \\ 0 & 2 & -11 & 16 & -10 \\ 1 & -1 & 3 & -5 & 5 \end{bmatrix} \xrightarrow{I \cdot (-2) + II} \begin{bmatrix} 1 & 2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 2 & -11 & 16 & -10 \\ 1 & -1 & 3 & -5 & 5 \end{bmatrix} \xrightarrow{I \cdot (-2) + II}$$

$$\xrightarrow{II \cdot (-2) + III} \begin{bmatrix} 1 & 2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 2 & -11 & 16 & -10 \\ 0 & -3 & 4 & -6 & 0 \end{bmatrix} \xrightarrow{II \cdot 3 + IV} \begin{bmatrix} 1 & 2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 0 & -13 & 16 & 0 \\ 0 & -3 & 4 & -6 & 0 \end{bmatrix} \xrightarrow{II \cdot (-2) + I} \begin{bmatrix} 1 & 2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 0 & -13 & 16 & 0 \\ 0 & 0 & 7 & -6 & -15 \end{bmatrix}$$

$$\xrightarrow{II \cdot (-2) + I} \begin{bmatrix} 1 & 0 & -3 & 1 & 5 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 0 & -13 & 16 & 0 \\ 0 & 0 & 7 & -6 & -15 \end{bmatrix}$$

$$x - 3z + u = 5$$

$$y + z = -5 \implies z = -5 - y$$

$$-13z + 16u = 0$$

$$7z - 6u = -15$$

$$7 \cdot (-5 - y) - 6u = -15$$

$$-35 - 7y - 6u = -15$$

$$-7y = -15 + 35 + 6u$$

$$-7y = 20 + 6u \quad | : (-7)$$

$$y = -\frac{20}{7} - \frac{6}{7}u$$

PROVJERA?

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H2

IME I PREZIME: Matea Čulina

BROJ INDEKSA: 17-2-0206-2012

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6. Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow 0} \frac{|x|}{x}$ . 5

Ukupno:

~~0~~

1.)  $z^4 - (4-i)^5 = 0$

$z^4 = (4-i)^5 / 4$

$z = \sqrt[4]{(4-i)^5}$

$z = \sqrt[4]{(4-i)^3 \cdot (4-i)^2}$

$z = \sqrt[4]{(64 - 48i + 12i^2 + i) \cdot (16 - 8i + i^2)}$

$z = \sqrt[4]{(52 - 47i) \cdot (15 - 8i)}$

$z = \sqrt[4]{780 - 416i - 705i - 316}$

$z = \sqrt[4]{404 - 1121i}$

$n = 4$

$x = 404$

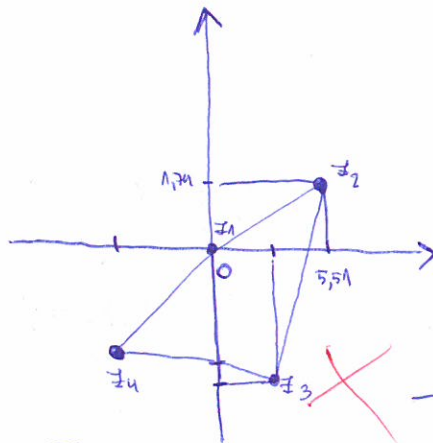
$y = -1121$

~~z =~~

$r = \sqrt{x^2 + y^2}$

$= \sqrt{163216 + 1256641}$

$= 1191,57$



$\text{tg } \varphi = \frac{y}{x} = \frac{-1121}{404} = -2,77$

$\varphi = -70^\circ 12' 57''$

$360^\circ - \varphi = 289^\circ 47' 3''$

$\sqrt[n]{r} (\cos \frac{360^\circ - k \cdot \varphi}{n} + i \sin \frac{360^\circ - k \cdot \varphi}{n})$

$k=0$

$z_1 = \sqrt[4]{1191,57} (\cos \frac{360^\circ - 0 \cdot 289^\circ 47' 3''}{4} + i \sin \frac{360^\circ - 0 \cdot 289^\circ 47' 3''}{4})$

$z_1 = 5,8 (\cos 90 + i \sin 90)$

$z_1 = 5,8 (0 + 0i)$

$z_1 = 0 + 0i$   
x y

$k=2$

$z_3 = 5,8 (\cos \frac{360^\circ - 2 \cdot 289^\circ 47' 3''}{4} + i \sin \frac{360^\circ - 2 \cdot 289^\circ 47' 3''}{4})$

$= 5,8 (0,57 + 0,81i)$

$= 3,3 + 4,68i$   
x y

$k=1$

$z_2 = 5,8 (\cos \frac{360^\circ - 1 \cdot 289^\circ 47' 3''}{4} + i \sin \frac{360^\circ - 1 \cdot 289^\circ 47' 3''}{4})$

$= 5,8 (0,95 + 0,30i)$

$= 5,51 + 1,74i$   
x y

$k=3$

$z_4 = 5,8 (\cos \frac{360^\circ - 3 \cdot 289^\circ 47' 3''}{4} + i \sin \frac{360^\circ - 3 \cdot 289^\circ 47' 3''}{4})$

$= 5,8 (-0,60 - 0,8i)$

$= -3,48 - 4,64i$   
x y

2.)  $f(x) = x - \sqrt{x^2 - 5}$

~~Domena~~ / SVE ASIMPTOTE / DRUGA DERIVACIJA /

Domena

$\sqrt{x^2 - 5} \geq 0 \quad | \quad ^2$

$x^2 - 5 \geq 0$

$x^2 \geq 5 \quad | \quad \sqrt{\quad}$

$x \geq \pm \sqrt{5}$  ~~X~~

$DE [\sqrt{5}, +\infty)$  ~~X~~

ASIMPTOTE

V.A.  
 $f(x) = x - \sqrt{x^2 - 5}$

$\lim_{x \rightarrow \sqrt{5}} (\sqrt{5} - \sqrt{(\sqrt{5})^2})$

$= (\sqrt{5} - 0) = \sqrt{5}$   $x = \sqrt{5}$  ~~X~~

HORIZONTALNA ASIMPTOTA

$\lim_{x \rightarrow \infty} (x - \sqrt{x^2 - 5}) / : x^2$  ~~X~~

$\lim_{x \rightarrow \infty} \frac{x}{x} - \sqrt{\frac{x^2}{x^2} - \frac{5}{x^2}}$

$\lim_{x \rightarrow \infty} 1 - \sqrt{1 - 0}$

$\lim_{x \rightarrow \infty} 1 - 1$

$\lim_{x \rightarrow \infty} 0$

$y = 0$

KOSE ASIMPTOTE  
NEMA.

DRUGA DERIV

$f'(x) = x^{-1} \cdot (-\sqrt{x^2 - 5})'$

$f'(x) = 1 - ((x^2 - 5)^{\frac{1}{2}})'$

$f'(x) = 1 - (x^2 - 5)^{-\frac{1}{2}} \cdot (x^2 - 5)'$

$= 1 - \frac{1}{2} \cdot (x^2 - 5)^{-1} \cdot 2x$

$= 1 - x \cdot (x^2 - 5)^{-\frac{1}{2}}$

$f''(x) = (1 - x \cdot (x^2 - 5)^{-\frac{1}{2}})'$

$= -x \cdot (x^2 - 5)^{-\frac{1}{2}} \cdot x \cdot [(x^2 - 5)^{-\frac{1}{2}}]'$

$= -x \cdot (x^2 - 5)^{-\frac{1}{2}} \cdot x \cdot (-\frac{1}{2} (x^2 - 5)^{-\frac{3}{2}})$

~~$= -x \cdot (x^2 - 5)^{-\frac{1}{2}} \cdot x \cdot (-\frac{1}{2} (x^2 - 5)^{-\frac{3}{2}})$~~

$= -x \cdot (x^2 - 5)^{-\frac{1}{2}} - x^2 \cdot (x^2 - 5)^{-\frac{3}{2}}$

~~$x + 2y - z + u = 5$   
 $2x + 5y - z + 2u = 5$   
 $3x - y - 2z + u = 5$   
 $x - y + 3z - 5u = 5$~~

~~$\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & & 5 \\ 2 & 5 & -1 & 2 & & 5 \\ 3 & -1 & -2 & 1 & & 5 \\ 1 & -1 & 3 & -5 & & 5 \end{array}$  (III:II)~~

~~$\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & & 5 \\ 1 & -6 & -3 & -1 & & 0 \\ 3 & -1 & -2 & 1 & & 5 \\ 1 & -1 & 3 & -5 & & 5 \end{array}$  II-III~~

~~$\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & & 5 \\ 1 & -6 & -3 & -1 & & 0 \\ 3 & -1 & -2 & 1 & & 5 \\ 0 & 5 & 0 & -6 & & -5 \end{array}$  II-III~~

~~$\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & & 5 \\ 1 & -6 & -3 & -1 & & 0 \\ -2 & -5 & -1 & -2 & & -5 \\ 0 & 5 & 0 & -6 & & -5 \end{array}$  (-1)~~

~~$\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & & 5 \\ 1 & -6 & -3 & -1 & & 0 \\ 2 & 5 & 1 & 2 & & 5 \\ 0 & 5 & 0 & -6 & & -5 \end{array}$  III-IV~~

~~$\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & & 5 \\ 1 & -6 & -3 & -1 & & 0 \\ 2 & 5 & 1 & 2 & & 5 \\ 2 & 0 & 1 & 8 & & 10 \end{array}$~~

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: *Filip Štnlek*

BROJ INDEKSA: *0269072072*

H2

- Riješiti jednačbu:  $z^4 - (4-i)^5 = 0$ . Prikaži rješenja u kompleksnoj ravnini! ~~12+3~~
- Odrediti domenu, sve asimptote i drugu derivaciju funkcije  $f(x) = x - \sqrt{x^2 - 5}$ . ~~5+15+5~~
- Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije  $g(x) = \ln(4 - x^2)$ . 5+5+10
- Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije  $h(x) = \frac{x^2 - 5x - (5+1)}{x^2 + 1}$ . Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
- Gaussovom metodom riješiti matricni sustav i obavezno provjeri rješenje: ~~15~~

$$\begin{aligned} x + 2y - z + u &= 5 \\ 2x + 5y - z + 2u &= 5 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 5 \end{aligned}$$

- Izračunati i provjeriti uvrštavanjem:  $\lim_{x \rightarrow 0} \frac{|x|}{x}$ . 5

Ukupno:

~~0~~

①  ~~$z^4 - (4-i)^5 = 0$~~

~~$z^4 = (4-i)^5$~~

~~$z^4 = (4-i)(4-i)(4-i)(4-i)(4-i)$~~

~~$z^4 = (16 - 8i + i^2)(16 - 8i + i^2)(4-i)$~~

~~$z^4 = 225 - 240i - 64)(4-i)$~~

~~$z^4 = (161 - 240i)(4-i)$~~

~~$z^4 = 644 - 161i - 960i^2 + 240i^2$~~

~~$z^4 = 644 - 1121i - 240$~~

~~$z^4 = 404 - 1121i$~~

②  ~~$f(x) = x - \sqrt{x^2 - 5}$~~

~~1 DOMENA~~

~~$x^2 \geq 5$~~

~~NEMA REALNOG RJEŠENJA~~

$$\begin{aligned}
 (5) \quad & x + 2y - z + u = 5 \\
 & 2x + 5y - z + 2u = 5 \\
 & 3x - y - 2z + u = 5 \\
 & x - y + 3z - 5u = 5
 \end{aligned}$$

$$\left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 2 & 5 & -1 & 2 & 5 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 5 \end{array} \right] \approx \left[ \begin{array}{cccc|c} 1 & 2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 0 & -5 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 5 \end{array} \right] \approx \left[ \begin{array}{cccc|c} 1 & 0 & -3 & 1 & 15 \\ 0 & 1 & 1 & 0 & -5 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 5 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & -3 & 1 & 15 \\ 0 & 1 & 1 & 0 & -5 \\ 3 & -1 & -2 & 1 & 5 \\ 0 & -1 & 6 & -6 & -10 \end{array} \right] \approx \left[ \begin{array}{cccc|c} 1 & 0 & -3 & 1 & 15 \\ 0 & 1 & 1 & 0 & -5 \\ 3 & 0 & -1 & 1 & 0 \\ 0 & 0 & 7 & -6 & -15 \end{array} \right] \approx \left[ \begin{array}{cccc|c} 1 & 0 & -3 & 1 & 15 \\ 0 & 1 & 1 & 0 & -5 \\ 3 & 0 & -1 & 0 & -15 \\ 0 & 0 & 7 & -6 & -15 \end{array} \right]$$

$$\left[ \begin{array}{cccc|c} -3 & 0 & 0 & 1 & 60 \\ 0 & 1 & 1 & 0 & -5 \\ 3 & 0 & -1 & 0 & -15 \\ 0 & 0 & 7 & -6 & -15 \end{array} \right]$$