MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: Duje Mitrović **BROJ INDEKSA:** 17-2-0205-2012

I1

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^4 - 4 + 4i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Riješi jednadžbu $\ln(x-4) = x - 4$ grafičkom metodom. *Provjeri uvrštavanjem!* 12+3
3. Ispitati domenu i sve asimptote funkcije $g(x) = (\sqrt{x^2+x} - x)$. 5+15 7
4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+4}$. 20(graf)
5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x-4))$. 15
6. Da li red $\sum_n \frac{5^n}{n^2}$ konvergira i zašto? 10
7. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 5

Ukupno: 82

① $z^4 - 4 + 4i = 0$

$z^4 = 4 - 4i$

$z = \sqrt[4]{4-4i}$

$w = 4 - 4i$

$|w| = \sqrt{16+16} = \sqrt{32}$

$\operatorname{tg}\varphi = \frac{y}{x} = \frac{-4}{4} = -1 \Rightarrow \varphi = \frac{7\pi}{4}$ / $k=3$

$k = 0, 1, 2, 3$

$k=0$

$z_1 = \sqrt[4]{\sqrt{32}}\left(\cos\frac{\varphi+2k\pi}{4} + i\sin\frac{\varphi+2k\pi}{4}\right)$

$z_1 = 1{,}5422(0{,}1950 + 0{,}9807i)$

$z_1 = 0{,}3007 + 1{,}5124i$ //

$k=1$

$z_2 = \sqrt[4]{\sqrt{32}}\left(\cos\frac{\varphi+2k\pi}{4} + i\sin\frac{\varphi+2k\pi}{4}\right)$

$z_2 = 1{,}5422(-0{,}9807 + 0{,}1950i)$

$z_2 = -1{,}5124 + 0{,}3007i$ //

$k=2$

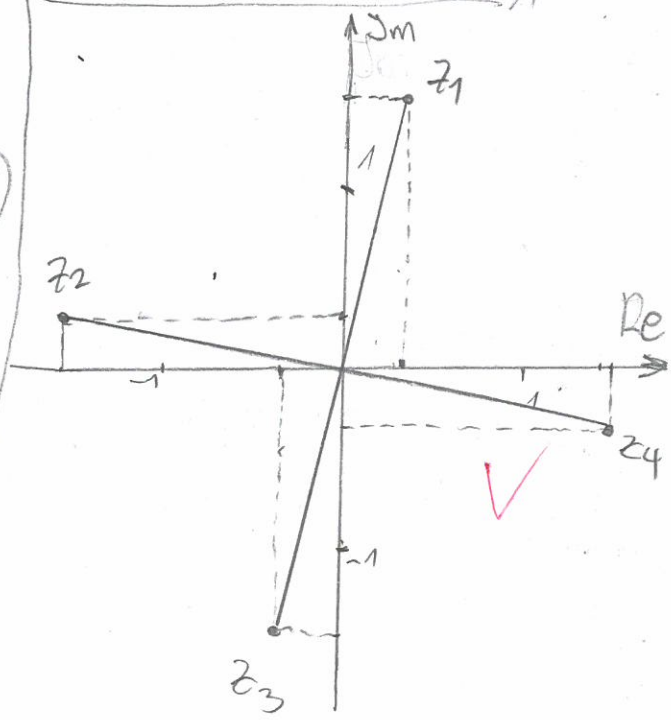
$z_3 = 1{,}5422(-0{,}1950 - 0{,}9807i)$

$z_3 = -0{,}3007 - 1{,}5124i$ //

$z_4 = \sqrt[4]{\sqrt{32}}\left(\cos\frac{\varphi+2k\pi}{4} + i\sin\frac{\varphi+2k\pi}{4}\right)$

$z_4 = 1{,}5422(0{,}9807 - 0{,}1950i)$

$z_4 = 1{,}5124 - 0{,}3007i$ //



V

② $\ln(x-4) = x-4$

U: $x-4>0$

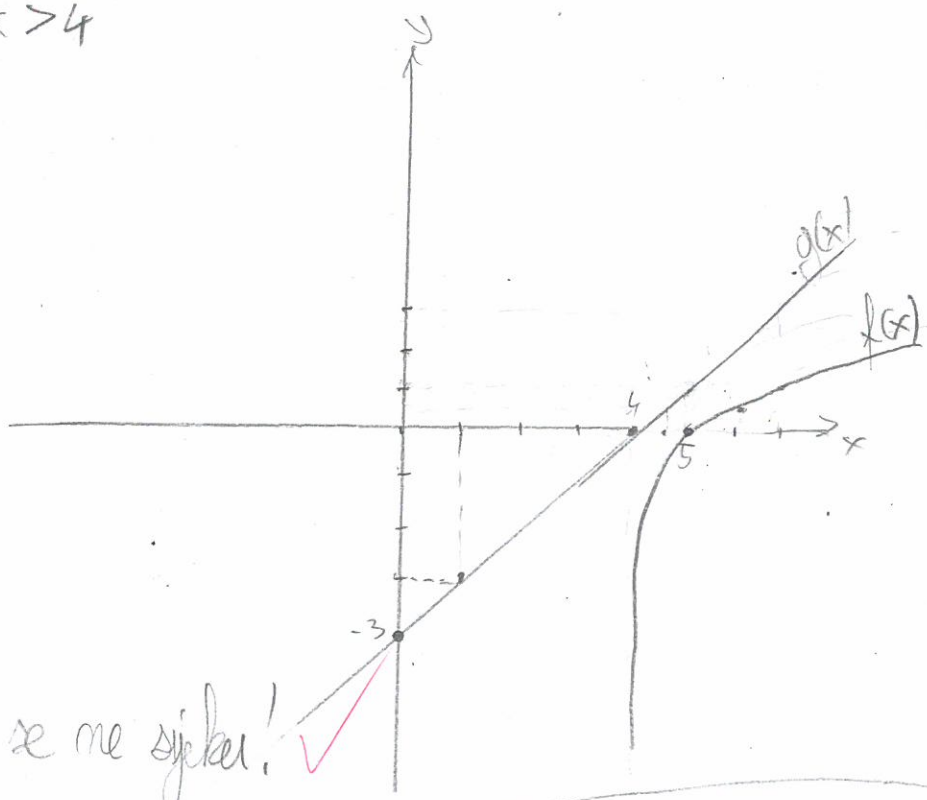
$x>4$

$f(x) = \ln(x-4)$

$g(x) = x-4$

x	f(x)
5	0
6	0,69
7	1,098
8	1,38

x	g(x)
1	-3
0	-4
4	0
5	1
7	3
8	4



Nema rješenja! Grafovi se ne sijeku! ✓

③ $g(x) = (\sqrt{x^2+x} - x)$

a) $D_f = \langle -\infty, -1] \cup [0, +\infty \rangle$ ✓

$x^2+x \geq 0$

$x_{1,2} = \frac{-1 \pm \sqrt{1-0}}{2}$

$x_{1,2} = \frac{-1 \pm 1}{2}$

$x_1 = -1$ //

$x_2 = 0$ //

b) Vertikalne asimptote:

$\lim_{x \to -1^-} \sqrt{x^2+x} - x = 0 + 1 = +1$ //

$\lim_{x \to -1^-} \sqrt{x^2+x} - x = 0 + 1 = 1$ //

$x = -1$ Nije V.A

$\lim_{x \to 0^-} \sqrt{x^2+x} - x = 0 - 0 = 0$ //

$\lim_{x \to 0^+} \sqrt{x^2+x} - x = 0 - 0 = 0$ //

$x = 0$ Nije V.A

-Horizontalne A.

$\lim_{x \to \infty} \sqrt{x^2+x} - x = \sqrt{x^2+x} - x \cdot \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x} = \frac{x^2+x-x^2}{\sqrt{x^2+x}+x} \,/:x = \frac{1}{2}$ //

$y = \frac{1}{2}$ ✓

$\lim_{x \to -\infty} \sqrt{x^2+x} - x = \left| \begin{matrix} -\infty \to \infty \\ x \to -x \end{matrix} \right| = \sqrt{x^2-x} + x \cdot \frac{\sqrt{x^2-x}-x}{\sqrt{x^2-x}-x} = \frac{x^2-x-x^2}{\sqrt{x^2-x}-x} \,/:x = \frac{-1}{0} = -\infty$

-Nema kose asimptote!

MA L.K.A!

Dure Mitrović

⑤ $f(x) = \ln(\sin(4x-4))$

$f'(x) = ?$

$f'(x) = \frac{1}{\sin(4x-4)} \cdot \cos(4x-4) \cdot 4$ ✓

$f'(x) = \frac{4\cos(4x-4)}{\sin(4x-4)}$

---

④ $h(x) = \frac{x^2-4}{x^2+4}$ $D_f = \mathbb{R}$

u. $x^2+4 \neq 0$

$x^2 = -4$

$x = \sqrt{-4}$

Nultočke:

$x^2-4=0$

$x^2=4$

$x = \pm 2$ //

$f(0) = \frac{0-4}{0+4} = -1$ //

$T(0,-1)$ sjecište s osi y

Asimptote

- Nema vertikalnih asimptota

$\lim_{x\to\infty} \frac{x^2-4 \,/\, x^2}{x^2+4 \,/\, x^2} = \frac{1}{1} = 1$ //

$y = 1$ desnostrana horizontalna asimptota!

$\lim_{x\to\infty} \frac{x^2-4}{x^2+4} = \left| \begin{matrix} -\infty \Rightarrow \infty \\ x \Rightarrow -x \end{matrix} \right| = \frac{x^2-4 \,/\, x^2}{x^2+4 \,/\, x^2} = \frac{1}{1} = 1$ //

- Nema kose asimptote.

Derivacija!

$f(x)' = \frac{x^2-4}{x^2+4} = \frac{2x(x^2+4) - (x^2-4)2x}{(x^2+4)^2} = \frac{2x^3+8x-2x^3+8x}{(x^2+4)^2} =$

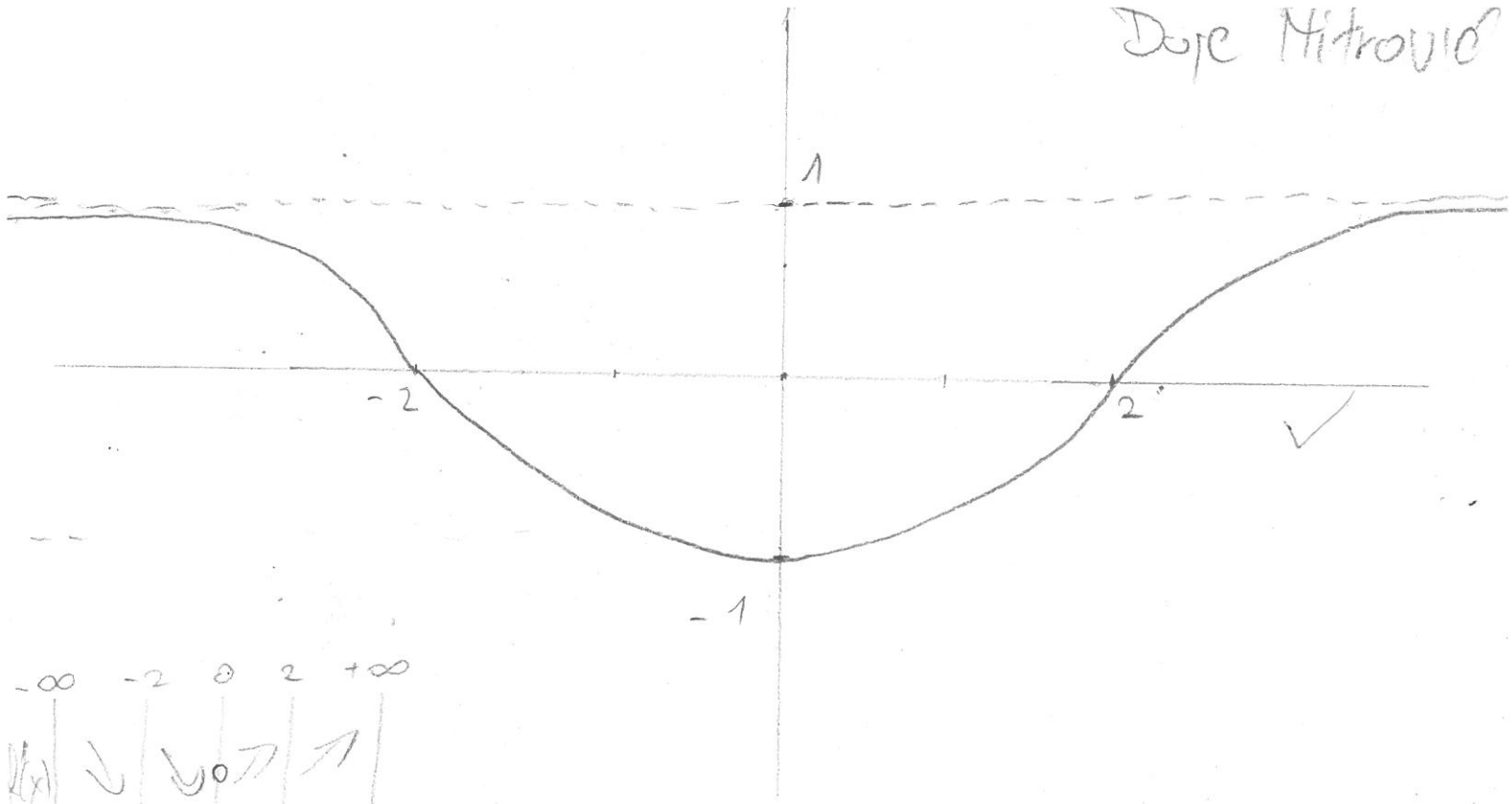
$= \frac{16x}{(x^2+4)^2} \Rightarrow 16x = 0$

$x = 0$

$f'(0) = \frac{0}{16} = 0$ //

$\Rightarrow$ 12A

Doje Mitrovic



$-\infty$	-2	0	2	$+\infty$
$f(x)$	↘	↘ 0	↗	↗ 1

T(0,1)

(7.)

$$\begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \begin{matrix} \\ \\ \text{III}-2\text{I} \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 1 & 0 & -13 & 4 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \begin{matrix} \text{I}-\text{II} \\ \text{III}-\text{II} \\ \text{IV}-\text{II} \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -4 & 6 & -3 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 4 & -11 & 3 \\ 0 & 0 & -4 & 4 & 3 \end{bmatrix} \begin{matrix} \text{I}+\text{III} \\ \text{II}-\text{III} \\ \text{IV}+\text{III} \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 & -5 & 0 \\ 0 & 1 & 0 & 9 & -2 \\ 0 & 0 & 4 & -11 & 3 \\ 0 & 0 & 0 & -7 & 4 \end{bmatrix} \sim \quad \cdot(-7)$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -5 & 0 \\ 0 & 1 & 0 & 9 & -2 \\ 0 & 0 & 4 & -11 & 3 \\ 0 & 0 & 0 & 1 & \frac{4}{7} \end{bmatrix} \begin{matrix} \text{I}+5\text{IV} \\ \text{II}-9\text{IV} \\ \text{III}+11\text{IV} \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{34}{7} \\ 0 & 1 & 0 & 0 & \frac{81}{7} \\ 0 & 0 & 1 & 0 & \frac{51}{14} \\ 0 & 0 & 0 & 1 & \frac{4}{7} \end{bmatrix} \quad r(A) = 4 //$$

/14

+TREBA DONIJETI IDENTIFIKACIJU. DONIO ~~PROMETNU~~ VOZAČKU. Koso

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{I}1$

IME I PREZIME: PETAR JELAVIĆ MITROVIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK — Broj ↓ bodova

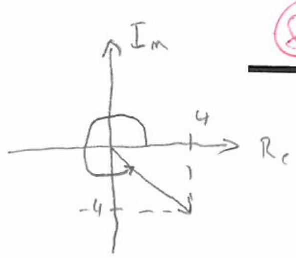
Zadatak	Broj bodova
1. Riješi jednadžbu među kompleksnim brojevima: $z^4 - 4 + 4i = 0$. *Prikaži rješenja u kompleksnoj ravnini!*	12+3
2. Riješi jednadžbu $\ln(x-4) = x - 4$ grafičkom metodom. *Provjeri uvrštavanjem!*	12+3
3. Ispitati domenu i sve asimptote funkcije $g(x) = \left(\sqrt{x^2+x} - x\right)$.	5+15 7
4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+4}$.	20(graf)
5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x-4))$.	15
6. Da li red $\sum_n \frac{5^n}{n^2}$ konvergira i zašto?	10
7. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$.	5

Ukupno: 82

(1.) $z^4 = 4 - 4i \Rightarrow z = \sqrt[4]{4-4i}$

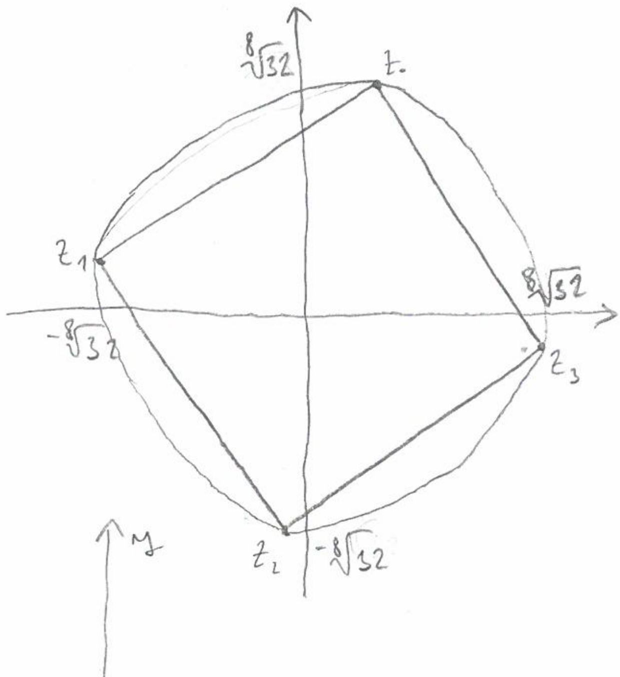
$r = \sqrt{4^2 + (-4)^2} = \sqrt{16+16} = \sqrt{2 \cdot 16} = 4\sqrt{2}$

$\operatorname{tg}\varphi = \frac{-4}{4} = -1 \qquad \varphi = \frac{7\pi}{4}$



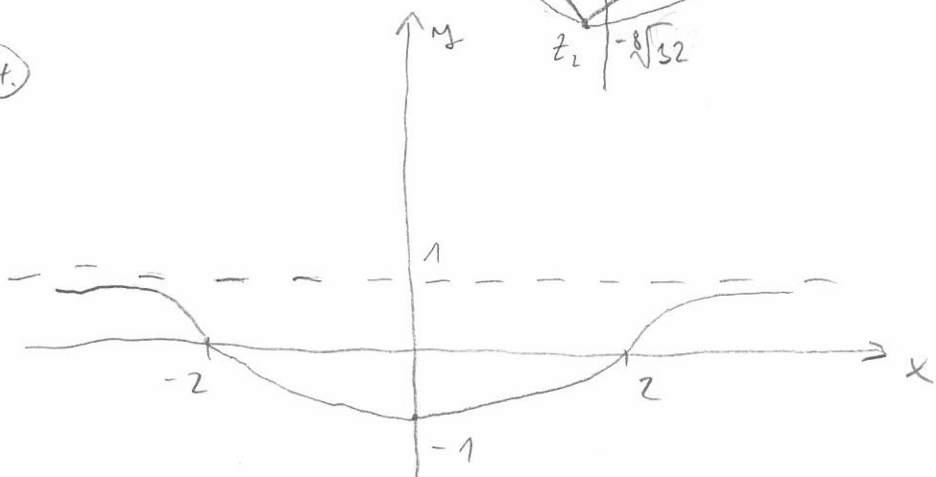
$z = \sqrt[4]{4\sqrt{2}} \cdot \left(\cos \frac{\frac{7\pi}{4} + 2k\pi}{4} + i \sin \frac{\frac{7\pi}{4} + 2k\pi}{4}\right)$

$\sqrt[4]{4\sqrt{2}} = \sqrt[8]{32}$



✓

(4.)



3. $g(x) = \sqrt{x^2+x} - x$

$x^2 + x \geq 0$

$x^2 + x = 0$

$x(x+1) = 0$

$x_1 = 0 \qquad x+1 = 0$

$x_2 = -1$

$D = x \in \langle -\infty, -1] \cup [0, +\infty \rangle$ ✓

HA

$$\lim_{x \to \pm\infty} \left(\sqrt{x^2+x} - x\right) \cdot \frac{\sqrt{x^2+x} + x}{\sqrt{x^2+x} + x} =$$

$$= \lim_{x \to \pm\infty} \frac{x^2 + x - x^2}{\sqrt{x^2+x} + x} = \lim_{x \to \pm\infty} \frac{x}{\sqrt{x^2+x} + x} \quad /:1$$

$$= \lim_{x \to \pm\infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1} = \frac{1}{2}$$ ✓ SAMO L. +! A.

4. $h(x) = \frac{x^2-4}{x^2+4}$

DOMINA:

$x^2 + 4 \neq 0$

$x^2 \neq -4 \quad /^{\sqrt{}}$

$x \neq \pm\sqrt{-4}$

$D = x \in \mathbb{R}$

NUL TOČKA

$y = 0 \Rightarrow x^2 - 4 = 0$

$x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

$T_1(-2, 0)$

$T_2(2, 0)$

ASIMPTOTE

VA - nema je

HA $\lim_{x \to \pm\infty} \frac{x^2-4}{x^2+4} \quad /:x^2 = \lim_{x \to \pm\infty} \frac{1 - \frac{4}{x^2}}{1 + \frac{4}{x^2}} = \frac{1}{1} = 1$

EKSTREM

$$y' = \frac{2x(x^2+4) - (x^2-4) \cdot 2x}{(x^2+4)^2} = \frac{2x^3 + 8x - 2x^3 + 8x}{(x^2+4)^2}$$

$$y' = \frac{16x}{(x^2+4)^2} \qquad y' = 0 \Rightarrow 16x = 0$$

$$y'_{54} = \frac{16}{(x^2+4)^2} \qquad x = 0$$

$$y''_{(x=0)} = \frac{+}{+} > 0 \Rightarrow \text{min} \cup$$

$$y_{\min(x=0)} = \frac{0^2-4}{0^2+4} = -\frac{4}{4} = -1$$

PETAR JELAVIĆ MITROVIĆ

(5.) $f(x) = \ln(\sin(4x-4))$

$$f'(x) = \frac{1}{\sin(4x-4)} \cdot (\sin(4x-4))'$$

$$= \frac{1}{\sin(4x-4)} \cdot \cos(4x-4) \cdot (4x-4)'$$

$$= \frac{\cos(4x-4)}{\sin(4x-4)} \cdot 4 = 4 \cdot \operatorname{ctg}(4x-4)$$

(2.) $\ln(x-4) = x-4$

$f_1(x) = \ln(x-4)$

$x-4 > 0$

$x > 4$

$f_2(x) = x-4$

$x-4 = 0$

$x = 4$

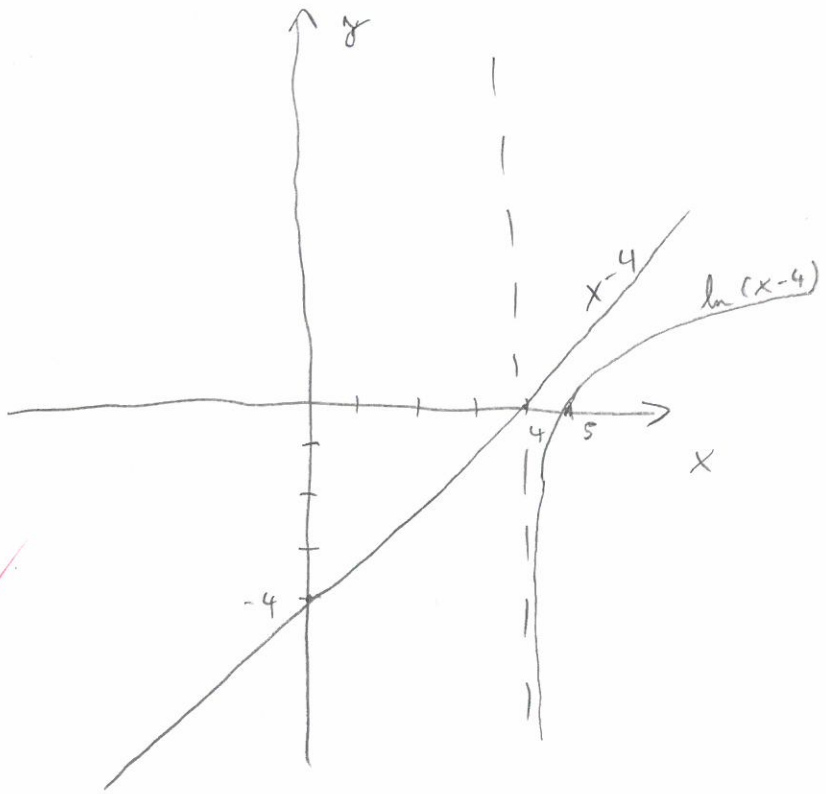
x	y
0	-4
4	0

$\ln(x-4) = 0$

$x-4 = e^0$

$x-4 = 1$

$x = 5$



NEMA REŠENJA

7.

$$\begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \leftarrow \sim$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \begin{matrix} (-2) \\ + \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 1 & 0 & -13 & 4 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \begin{matrix} (-1) \\ + \\ + \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & -4 & -11 & 3 \\ 0 & 0 & -4 & 4 & 3 \end{bmatrix} \begin{matrix} (-1) \\ + \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & -4 & -11 & 3 \\ 0 & 0 & 0 & -15 & 0 \end{bmatrix}$$

$r(A) = 4$ ✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ I1

IME I PREZIME: KRISTIAN MARTINOVIĆ **BROJ INDEKSA:** 17-2-0110-2011

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^4 - 4 + 4i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Riješi jednadžbu $\ln(x-4) = x - 4$ grafičkom metodom. *Provjeri uvrštavanjem!* 12+3
3. Ispitati domenu i sve asimptote funkcije $g(x) = (\sqrt{x^2+x} - x)$. 5+15
4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+4}$. 20(graf)
5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x-4))$. 15
6. Da li red $\sum_n \frac{5^n}{n^2}$ konvergira i zašto? 10
7. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 5

Ukupno: 77

1. $z^4 - 4 + 4i = 0$

$z^4 = 4 - 4i$

$z = \sqrt[4]{4-4i}$

$r = \sqrt{x^2+y^2}$

$r = \sqrt{4^2+4^2}$

$r = 4\sqrt{2} \approx 5.65$

$\operatorname{tg}\varphi = \frac{y}{x} = \frac{-4}{4} = -1$

$\varphi = -45°$

$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right)$

$k = 0$

$\sqrt[4]{z} = \sqrt[4]{5.65}\left(\cos\frac{-45°+2\cdot 0\cdot\pi}{4} + i\sin\frac{-45°+2\cdot 0\cdot\pi}{4}\right)$

$\sqrt[4]{z} = \sqrt[4]{5.65}\left(0.9807 + (-0.1950i)\right)$

$\sqrt[4]{z} = 1.54\left(0.9807 - 0.1950i\right)$

$\sqrt[4]{z} = 1.5102 - 0.3003i$

$k = 1$

$\sqrt[4]{z} = \sqrt[4]{5.65}\left(\cos\frac{-45°+2\pi}{4} + i\sin\frac{-45°+2\pi}{4}\right)$ ($2\pi = 360°$)

$\sqrt[4]{z} = 1.54\left(0.1950 + 0.9807i\right)$

$\sqrt[4]{z} = 0.3003 + 1.5102i$

$k=2$

$$\sqrt[4]{z} = \sqrt[4]{r}\left(\cos\frac{-45^\circ + (4\pi)}{4} + i\sin\frac{-45^\circ + (4\pi)}{4}\right)$$

($4\pi = 720^\circ$; $-45^\circ + 720^\circ = 675^\circ$)

$$\sqrt[4]{z} = 1.54\,(-0.9807 + 0.1950\,i)$$

$$\sqrt[4]{z} = -1.5102 + 0.3003\,i$$

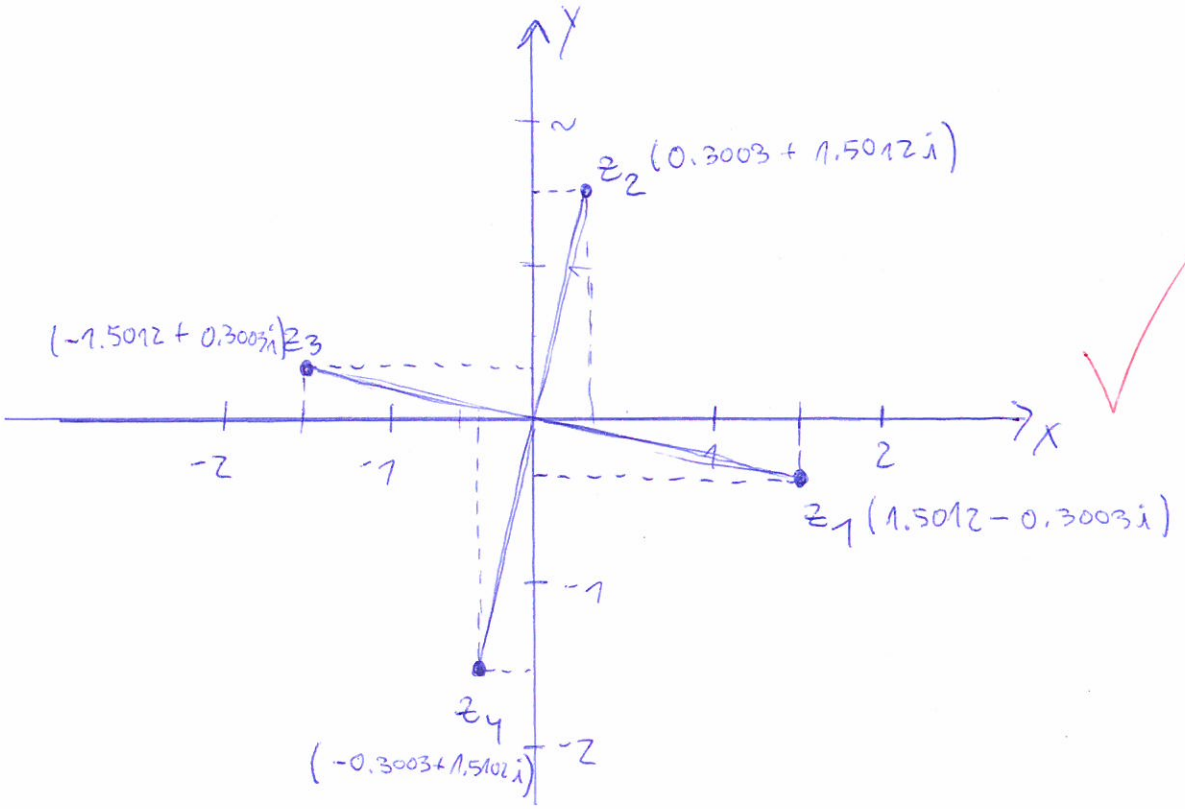
$k=3$

$$\sqrt[4]{z} = \sqrt[4]{r}\left(\cos\frac{-45^\circ + (6\pi)}{4} + i\sin\frac{-45^\circ + (6\pi)}{4}\right)$$

($6\pi = 1080^\circ$; $-45^\circ + 1080^\circ = 1035^\circ$)

$$\sqrt[4]{z} = 1.54\,(-0.1950 + (-0.9807\,i))$$

$$\sqrt[4]{z} = -0.3003 - 1.5102\,i$$



KRISTIAN MARTINOVIĆ

2. $\ln(x-4) = x-4$

UVJET ZA $\ln$

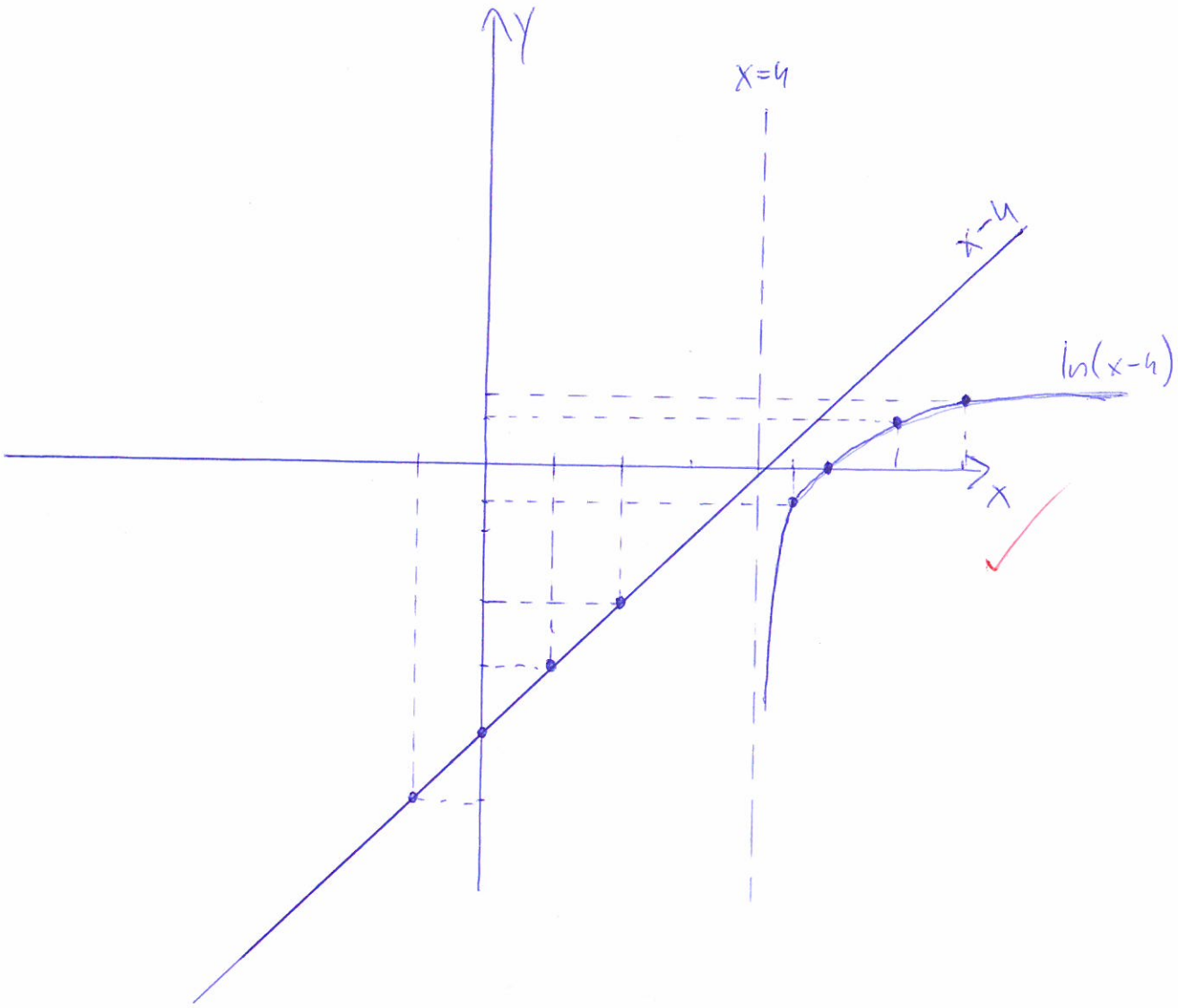
$x-4 > 0$

$x > 4$

$x = 4$, V.A.

x	4.5	5	6	7
$\ln(x-4)$	-0.69	0	0.69	1.09

x	-1	0	1	2
$x-4$	-5	-4	-3	-2



3. $g(x) = (\sqrt{x^2+x} - x)$

DOMENA

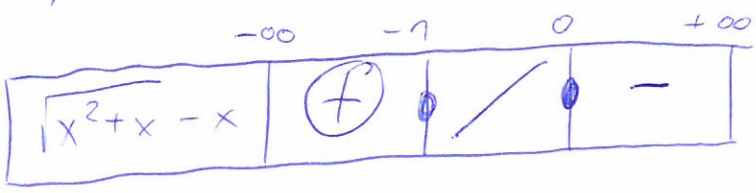
$x^2 + x \geq 0$

$x(x+1) \geq 0$

$x = 0$

$x + 1 = 0$

$x = -1$



$D_f < -\infty, -1]$ ✗

ASIMPTOTE

V.A.

$\lim_{x \to 0^-} (\sqrt{0+0} - 0) = 0^-$ ⟩ nije V.A.

$\lim_{x \to 0^+} (\sqrt{0+0} - 0) = 0^+$

NEMA V.A.

$\lim_{x \to -1^-} (\sqrt{(-1)^2 - 1} + 1) = 1^-$ ⟩ nije V.A.

$\lim_{x \to -1^+} (\sqrt{(-1)^2 - 1} + 1) = 1^+$

H.A.

$$\lim_{x \to \infty} \sqrt{x^2+x} - x \cdot \frac{\sqrt{x^2+x} + x}{\sqrt{x^2+x} + x} = \lim_{x \to \infty} \frac{x^2 + x - x^2}{\sqrt{x^2+x} + x} \quad /:x \quad /:x$$

$$\lim_{x \to \infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1} = \frac{1}{2}, \quad \boxed{y = \frac{1}{2}, \text{ H.A.}}$$

$\frac{1}{x} \to 0$

D. V. A

NEMA K.A.

$$\lim_{x \to -\infty} \sqrt{x^2+x} - x = \begin{bmatrix} x \to -x \\ -\infty \to \infty \end{bmatrix} = \lim_{x \to \infty} \sqrt{x^2-x} + x \cdot \frac{\sqrt{x^2-x} - x}{\sqrt{x^2-x} - x}$$

$$= \lim_{x \to \infty} \frac{x^2 - x - x^2}{\sqrt{x^2-x} - x} \quad /:x \quad /:x = \lim_{x \to \infty} \frac{-1}{\sqrt{1 - \frac{1}{x}} - 1} = \frac{-1}{0} = -\infty, \text{ NIJE H.A.}$$

$\frac{1}{x} \to 0$

L.K.A ?

(4.) $h(x) = \frac{x^2-4}{x^2+4}$

PARNOST / NEPARNOST

$h(-x) = \frac{(-x)^2-4}{(-x)^2+4} = \frac{x^2-4}{x^2+4} = f(x)$

PARNA, NIJE PERIODIČNA

DOMENA

$x^2+4 \neq 0$ $\quad D_f = \mathbb{R}$

$x^2 \neq -4$

NULTOČKE

$h(x) = 0$

$x^2-4 = 0$

$x^2 = 4$

$x_1 = 2$

$x_2 = -2$

$h(2) = \frac{4-4}{4+4} = 0$

$h(-2) = 0$

$NT(2,0)$

$NT(-2,0)$

$h(0) = \frac{0-4}{0+4} = -1$

$NT(0,-1)$

ASIMPTOTE

V.A. NEMA

H.A.

$\lim_{x\to\infty} \frac{x^2-4}{x^2+4} \quad /:x^2 \quad /:x^2 = \lim_{x\to\infty} \frac{1-\frac{4}{x^2}}{1+\frac{4}{x^2}} = \frac{1}{1} = 1$, $y = 1$, H.A.

NEMA K.A.

DERIVACIJA

$f'(x) = \frac{(x^2-4)'(x^2+4) - (x^2-4)(x^2+4)'}{(x^2+4)^2}$

$f'(x) = \frac{2x(x^2+4) - (x^2-4)2x}{(x^2+4)^2}$

$f'(x) = \frac{2x^3+8x-(2x^3-8x)}{(x^2+4)^2}$

$f'(x) = \frac{2x^3+8x-2x^3+8x}{(x^2+4)^2}$

$f'(x) = \frac{16x}{(x^2+4)^2}$

EKSTREMI

$f'(x)=0$

ST $(0,-1)$, JEDINO I NT

$16x=0$

$x=0$

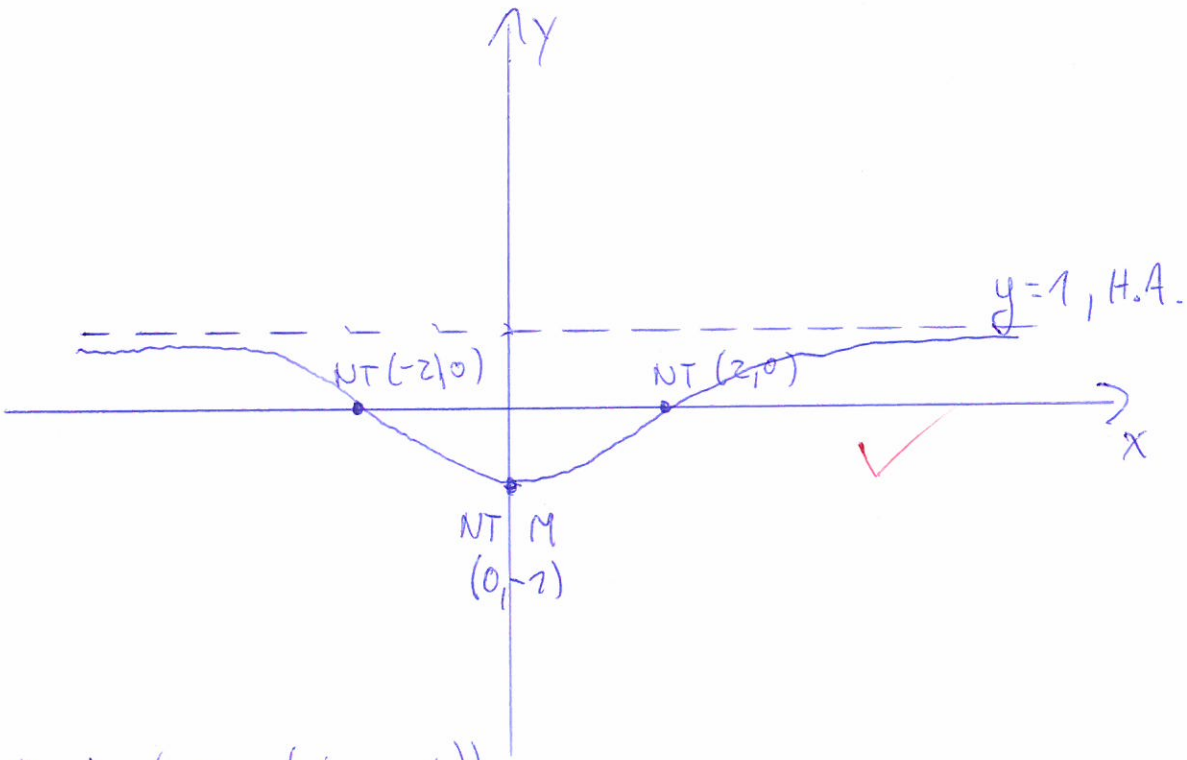
	$-\infty$		0		$+\infty$
$f'(x)$		−	0	+	
$f(x)$		↘	M	↗	

NT, M $(0,-1)$

NT $(2,0)$

NT $(-2,0)$

$y=1$, H.A.



5. $f(x)=\ln(\sin(4x-4))$

$$f'(x)=\frac{1}{\sin(4x-4)}\cdot\cos(4x-4)\cdot 4$$

$$f'(x)=\frac{4\cos(4x-4)}{\sin(4x-4)}$$

KRISTIAN MARTINOVIĆ

KRISTIAN MARTINOVIĆ

7)

$$\begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \begin{matrix} \\ \\ -2R1+R3 \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 1 & 0 & 3 & -4 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \begin{matrix} -R2+R1 \\ \\ -R2+R3 \\ -R2+R4 \end{matrix} \sim \begin{bmatrix} 1 & 0 & -4 & 6 & -3 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & -4 & 5 & -5 \\ 0 & 0 & -4 & 4 & 3 \end{bmatrix} \begin{matrix} -R3+R1 \\ +R3+R2 \\ \\ -R3+R4 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 & -4 \\ 0 & 0 & -4 & 5 & -5 \\ 0 & 0 & 0 & 1 & -8 \end{bmatrix} \begin{matrix} \\ \\ /(-4) \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 & -4 \\ 0 & 0 & 1 & -5/4 & 5/4 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & -6 \\ 0 & 1 & 0 & 0 & -28 \\ 0 & 0 & 1 & 0 & 45/4 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$$

RANG MATRICE JE 4 ✓

$$\begin{array}{ccccc} -1 & -1 & 0 & -4 & 2 \\ 0 & 1 & 4 & -2 & 1 \\ \hline -1 & 0 & & & \end{array}$$

$$\begin{array}{ccccc} -2 & -2 & 0 & 8 & -4 \\ 2 & 3 & 0 & -5 & 0 \\ \hline 0 & 1 & 0 & 3 & -4 \end{array}$$

$$\begin{array}{ccccc} 0 & 0 & +4 & -5 & +5 \\ 1 & 0 & -4 & 6 & -3 \\ \hline 1 & 0 & 0 & 1 & 2 \end{array}$$

$$\begin{array}{ccccc} 0 & 0 & 4 & -5 & 5 \\ 0 & 0 & -4 & 4 & 3 \\ \hline 0 & 0 & 0 & -1 & 8 \\ 0 & 0 & 0 & 1 & -8 \end{array}$$

$$\begin{array}{ccccc} 0 & 0 & 0 & 5/4 & 10 \\ 0 & 0 & 4 & -5/4 & 5/4 \\ \hline 0 & 0 & 4 & 0 & 45/4 \end{array}$$

$$\begin{array}{ccccc} 0 & -1 & -4 & 2 & -1 \\ 1 & 1 & 0 & 4 & -2 \\ \hline 1 & 0 & -4 & 6 & -3 \end{array}$$

$$\begin{array}{ccccc} 0 & -1 & -4 & 2 & -1 \\ 0 & 1 & 0 & 3 & -4 \\ \hline 0 & 0 & -4 & 5 & -5 \end{array}$$

$$\begin{array}{ccccc} 3 & -1 & -4 & 2 & -1 \\ 0 & 1 & 0 & 2 & 4 \\ \hline 0 & 0 & -4 & 4 & 3 \end{array}$$

$$\begin{array}{ccccc} 0 & 0 & -4 & 5 & -5 \\ 0 & 1 & 4 & -2 & 1 \\ \hline 0 & 1 & 0 & 3 & -4 \end{array}$$

$$\begin{array}{ccccc} 0 & 0 & 0 & -3 & -24 \\ 0 & 4 & 0 & 3 & -4 \\ \hline 0 & 1 & 0 & 0 & -24 \end{array}$$

$$\begin{array}{ccccc} 0 & 0 & 0 & -1 & -8 \\ 4 & 0 & 0 & 1 & 2 \\ \hline 1 & 0 & 0 & 0 & -6 \end{array}$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ | $\mathcal{I}1$

IME I PREZIME: VESNA ŠARIĆ **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješi jednadžbu među kompleksnim brojevima: $z^4 - 4 + 4i = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Riješi jednadžbu $\ln(x-4) = x-4$ grafičkom metodom. *Provjeri uvrštavanjem!* 12+3
3. Ispitati domenu i sve asimptote funkcije $g(x) = (\sqrt{x^2+x} - x)$. 5+15
4. Ispitati tok i nacrtati graf funkcije: $h(x) = \frac{x^2-4}{x^2+4}$. 20(graf)
5. Odrediti prvu derivaciju funkcije: $f(x) = \ln(\sin(4x-4))$. 15
6. Da li red $\sum_n \frac{5^n}{n^2}$ konvergira i zašto? 10
7. Izračunati rang matrice: $\begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix}$. 5

Ukupno: 30

7. $\begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 1 & -2 & -13 & 4 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim$

$1r \cdot (-2) + 3r$ $\quad 2r \cdot (-1) + 3r$

$\sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & -6 & -11 & 3 \\ 0 & 0 & -4 & 4 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 1 & -11/4 & -3/4 \\ 0 & 0 & -4 & 4 & 3 \end{bmatrix} \sim$ $\quad 2r \cdot (-1) + 4r$

$3r \cdot 4 + 4r$

$\sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 1 & -11/4 & -3/4 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 1 & -11/4 & -3/4 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} /4$

$4r \cdot \frac{11}{4} + 3r$

⑤ $f(x) = \ln(\sin(4x-4))$

$$f'(x) = \frac{1}{\sin(4x-4)} \cdot (\sin(4x-4))'$$

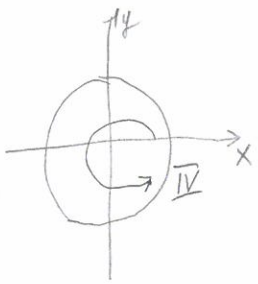
$$f'(x) = \frac{1}{\sin(4x-4)} \cdot \cos(4x-4)$$ ✗

$$f'(x) = \frac{\cos(4x-4)}{\sin(4x-4)}$$

$$f'(x) = \tan g(4x-4)$$

① $z^4 - 4 + 4i = 0$

$z^4 = 4 - 4i$ $\quad x = 4$ $\quad y = -4$



$\operatorname{tg} f = \frac{7}{4}\pi$

$|z| = \sqrt{x^2+y^2} = \sqrt{(4)^2+(-4)^2} = \sqrt{36} = 6$

$k = 0, 1, 2, 3$

$$z_1 = \sqrt[n]{z}\left(\cos\frac{f+2k\pi}{n} + i\sin\frac{f+2k\pi}{n}\right)$$

$$z_1 = \sqrt[4]{6}\left(\cos\frac{\frac{7}{4}\pi}{4} + i\sin\frac{\frac{7}{4}\pi}{4}\right)$$

$$z_1 = 1.56\left(\cos\frac{7}{16}\pi + i\sin\frac{7}{16}\pi\right)$$

$z_1 = 1.56(0.19 + i0.98)$

$\boxed{z_1 = 0.29 + 1.52i}$

(1) $z_2 = 4\sqrt{6}\left(\cos\frac{\frac{7}{4}\pi + 2\cdot 1\cdot\pi}{4} + i\sin\frac{\frac{7}{4}\pi + 2\pi}{4}\right)$

$z_2 = 4\sqrt{6}\left(\cos\frac{15}{16}\pi + i\sin\frac{15}{16}\pi\right)$

$z_2 = 4\sqrt{6}(-0.98 + i\,0.19)$

$z_2 = 1.56(-0.98 + i\,0.19)$

$\underset{x}{} \quad \underset{y}{}$

$\boxed{z_2 = -1.52 + i\,0.29}$

$z_3 = 4\sqrt{6}\left(\cos\frac{\frac{7}{4}\pi + 2\cdot 2\cdot\pi}{4} + i\sin\frac{\frac{7}{4}\pi + 2\cdot 2\cdot\pi}{4}\right)$

$z_3 = 4\sqrt{6}\left(\cos\frac{\frac{7}{4}\pi + 4\pi}{4} + i\sin\frac{\frac{7}{4}\pi + 4\pi}{4}\right)$

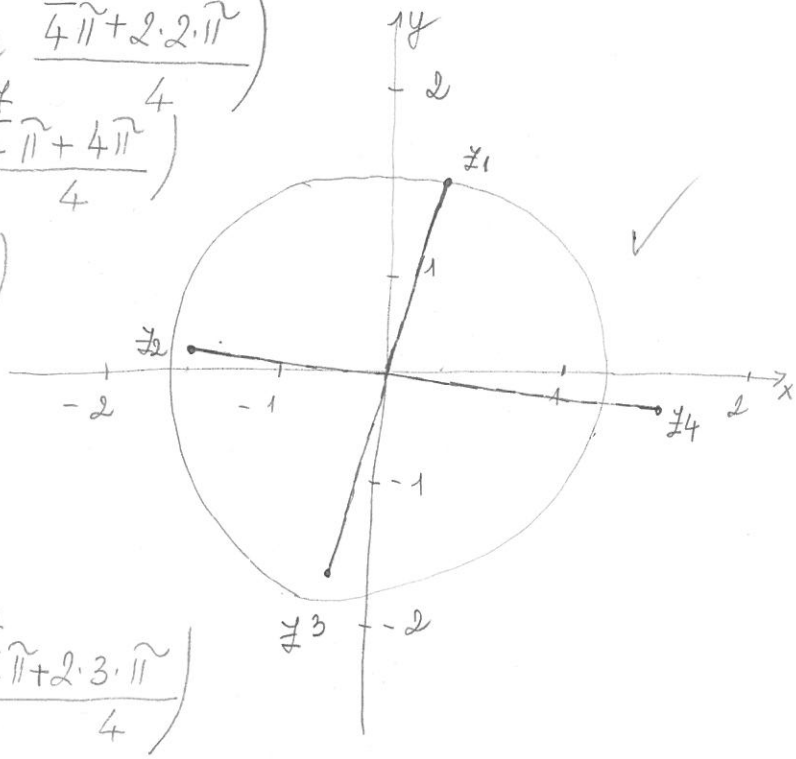
$z_3 = 4\sqrt{6}\left(\cos\frac{23}{16}\pi + i\sin\frac{23}{16}\pi\right)$

$z_3 = 4\sqrt{6}(-0.19 + i\,-0.98)$

$z_3 = 1.56(-0.19 + i\,-0.98)$

$\underset{x}{} \quad \underset{y}{}$

$\boxed{z_3 = -0.29 + i\,-1.52}$



$z_4 = 4\sqrt{6}\left(\cos\frac{\frac{7}{4}\pi + 2\cdot 3\cdot\pi}{4} + i\sin\frac{\frac{7}{4}\pi + 2\cdot 3\cdot\pi}{4}\right)$

$z_4 = 4\sqrt{6}\left(\cos\frac{\frac{7}{4}\pi + 6\pi}{4} + i\sin\frac{\frac{7}{4}\pi + 6\pi}{4}\right)$

$z_4 = 4\sqrt{6}\left(\cos\frac{31}{16}\pi + i\sin\frac{31}{16}\pi\right)$

$z_4 = 1.56(0.98 + i\,-0.19)$

$\underset{x}{} \quad \underset{y}{}$

$\boxed{z_4 = 1.52 + i\,-0.29}$

VESNA ŠAKIĆ

③ $g(x) = (\sqrt{x^2+x} - x)$

$x^2+x \geq 0$

$x^2+x=0$

$x(x+1)=0$

$\boxed{x=0}$

$x+1=0$

$\boxed{x=-1}$

$D(f) = \mathbb{R} \setminus \{0, -1\}$ ✗

① V.A $\lim\limits_{x \to 0} (\sqrt{x^2+x} - x = 0$

nema vertikalne asimptote

$\lim\limits_{x \to -1} (\sqrt{x^2+x} - x = -1$

② H.A 1) D. H. A

desna horizontalna asimptota je $\frac{1}{2}$ ✓

$$\lim_{x \to \infty} \sqrt{x^2+x} - x \cdot \frac{\sqrt{x^2+x} + x}{\sqrt{x^2+x} + x} = \lim_{x \to \infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}+x} =$$

$$\lim_{x \to \infty} \frac{x \,/\!: x}{\sqrt{x^2+x}+x \,/\!: x} = \lim_{x \to \infty} \frac{\frac{x}{x} \to 1}{\sqrt{\frac{x^2}{x^2} + \frac{x}{x^2}} + \frac{x}{x}} = \lim_{x \to \infty} \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

($\frac{x^2}{x^2} \to 1$, $\frac{x}{x^2} \to 0$, $\frac{x}{x} \to 1$)

2) L. H. A

$$\lim_{x \to -\infty} \sqrt{x^2+x} - x = \left[\begin{matrix} x \mapsto (-x) \\ -\infty \mapsto +\infty \end{matrix} \right] = \lim_{x \to \infty} \sqrt{(-x)^2 - x} + x$$

$$= \lim_{x \to \infty} \sqrt{x^2-x} + x \cdot \frac{\sqrt{x^2-x} - x}{\sqrt{x^2-x} - x} = \lim_{x \to \infty} \frac{x^2-x-x^2}{\sqrt{x^2-x}-x}$$

$$= \lim_{x \to \infty} \frac{-x \,/\!: x}{\sqrt{x^2-x} - x \,/\!: x} = \lim_{x \to \infty} \frac{\frac{-x}{x} \to -1}{\sqrt{\frac{x^2}{x^2} - \frac{x}{x^2}} - \frac{x}{x}} = \frac{-1}{1-1} = \frac{-1}{0}$$

($\frac{x^2}{x^2} \to 1$, $\frac{x}{x^2} \to 0$, $\frac{x}{x} \to 1$)

Lijeve horizontalne asimptote nema.

(3) K. L. A $\quad g(x) = kx + l$

(k) $\quad k = \lim\limits_{x \to (-\infty)} \frac{f(x)}{x} = \lim\limits_{x \to (-\infty)} \frac{\sqrt{x^2+x} - x}{x} = \left[\begin{matrix} x \mapsto (-x) \\ -\infty \mapsto (+\infty) \end{matrix}\right] = \lim\limits_{x \to \infty} \frac{\sqrt{(-x)^2 - x} + x}{-x}$

$= \lim\limits_{x \to \infty} \frac{\sqrt{x^2 - x} + x \,/:x}{x \,/:x} = \frac{\sqrt{\frac{x^2}{x^2} - \frac{x}{x^2}} + \frac{x}{x}}{\frac{x}{x}} = \boxed{2 \;//}$

(l) $\quad l = f(x) - kx$

$\lim\limits_{x \to (-x)} \sqrt{x^2+x} - x - 2x = \left[\begin{matrix} x \mapsto (-x) \\ -\infty \mapsto +\infty \end{matrix}\right] = \lim\limits_{x \to \infty} \sqrt{(-x)^2 - x} + x + 2x$

$\lim\limits_{x \to \infty} \sqrt{x^2 - x} + 3x \cdot \frac{\sqrt{x^2 - x} - 3x}{\sqrt{x^2 - x} - 3x} = \lim\limits_{x \to \infty} \frac{x^2 - x - 9x^2}{\sqrt{x^2 - x} - 3x}$

$= \lim\limits_{x \to \infty} \frac{-8x^2 - x \,/:x}{\sqrt{x^2 - x} - 3x \,/:x} = \lim\limits_{x \to \infty} \frac{\frac{-8x^2}{x} - \frac{x}{x}}{\sqrt{\frac{x^2}{x^2} - \frac{x}{x^2}} - \frac{3x}{x}} = \frac{-1}{1-3} = \frac{-1}{-2} = \frac{1}{2}$

$\boxed{f(x) = 2x + \frac{1}{2}}$ lijeva kosa asimptota. ✓

VESNA ŠARIĆ

④
$$\begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 2 & 4 \\ 0 & 1 & 4 & -2 & 1 \\ 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 2 & 4 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 1 & 0 & -9 & -8 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim$$

$1r \cdot (-2) + 3r$

$2r \cdot (-1) + 3r$

$2r \cdot (-1) + 4r$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 2 & 4 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & -4 & -7 & -9 \\ 0 & 0 & -4 & 4 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 2 & 4 \\ 0 & 1 & 0 & -11 & -8 \\ 0 & 0 & -4 & -7 & -9 \\ 0 & 0 & -4 & 0 & 3 \end{bmatrix} :(-4)$$

$3r \cdot 1 + 2r$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 2 & 4 \\ 0 & 1 & 0 & -11 & -8 \\ 0 & 0 & 1 & 7/4 & 9/4 \\ 0 & 0 & -4 & 0 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 2 & 4 \\ 0 & 1 & 0 & -11 & -8 \\ 0 & 0 & 1 & 7/4 & 9/4 \\ 0 & 0 & 0 & 7 & 12 \end{bmatrix} :7 \sim \begin{bmatrix} 1 & 1 & 0 & 2 & 4 \\ 0 & 1 & 0 & -11 & -8 \\ 0 & 0 & 1 & 7/4 & 9/4 \\ 0 & 0 & 0 & 1 & 12/7 \end{bmatrix}$$

$3r \cdot 4 + 4r$

---

⑦
$$\begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 1 & 0 & 4 & -2 \\ 0 & 1 & 0 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 0 & -5 & 0 \\ 0 & 1 & 4 & -2 & 1 \\ 1 & 0 & -4 & 6 & -3 \\ 0 & 0 & -4 & 4 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & 6 & -3 \\ 0 & 1 & 4 & -2 & 1 \\ 2 & 3 & 0 & -5 & 0 \\ 0 & 0 & -4 & 4 & -3 \end{bmatrix} \sim$$

$2r \cdot (-1) + 3r$

$2r \cdot (-1) + 4r$

$1r \cdot (-2) + 3r$

$$\begin{bmatrix} 1 & 0 & -4 & 6 & -3 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 3 & 8 & -17 & 6 \\ 0 & 0 & -4 & 4 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & 6 & -3 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & -4 & -11 & 3 \\ 0 & 0 & -4 & 4 & -3 \end{bmatrix} :(-4) \sim \begin{bmatrix} 1 & 0 & -4 & 6 & -3 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 1 & 11/4 & -3/4 \\ 0 & 0 & -4 & 4 & -3 \end{bmatrix}$$

$2r \cdot (-3) + 3r$

$3r \cdot 4 + 4r$

$$\sim \begin{bmatrix} 1 & 0 & -4 & 6 & -3 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 1 & 11/4 & -3/4 \\ 0 & 0 & 0 & 15 & -6 \end{bmatrix} :15 \sim \begin{bmatrix} 1 & 0 & -4 & 6 & -3 \\ 0 & 1 & 4 & -2 & 1 \\ 0 & 0 & 1 & 11/4 & -3/4 \\ 0 & 0 & 0 & 1 & -6/15 \end{bmatrix}$$

KOLIKI JE RANG?