

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: TONI PERKOVIĆ BROJ INDEKSA: 17201342011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

F4

1. Neka su z_1 i z_2 rješenja kvadratne jednadžbe $z^2 - z + 4 = 0$. Prikaži ih u kompleksnoj ravnini! Dalje izračunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$ i $\text{Im}\left(\frac{z_2}{z_1}\right)$.

~~12+3~~ 5

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

~~10+5~~

$$\begin{aligned} x_1 + x_2 - x_3 - 3x_4 + 4x_5 &= 2 \\ 3x_1 + x_2 - x_3 - x_4 &= 2 \\ 9x_1 + x_2 - 2x_3 - x_4 - 2x_5 &= 5 \\ x_1 - x_2 - x_4 + 2x_5 &= 1 \end{aligned}$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x - 2} - \arctan(4x^2 - x)$.

15

4. Odrediti tok funkcije $f(x) = \frac{x^2 - 4}{x^2 + 4}$

20(graf)

5. Odrediti i provjeriti:

(a) $\lim_{x \rightarrow -4} \frac{x^2 - 4}{x^2 + 8x + 16} =$

~~4+1~~

(b) $\lim_{n \rightarrow \infty} \left(\frac{n+4}{n}\right)^n =$

8+2

6. Grafički približno riješiti: $4 - \ln x > \arctan x$. Ne zaboravi eksplicitno zapisati skup rješenja. *Provjeri približno rješenje jednadžbe vrštavanjem, a također provjeri nekoliko rješenja nejednadžbe vrštavanjem!*

15+5

Ukupno:

48

③ $g(x) = \sqrt{x^2 + x - 2} - \arctan(4x^2 - x)$

$x^2 + x - 2 \geq 0$

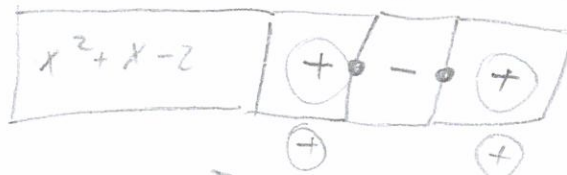
$a=1, b=1, c=-2$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$

$x_1 = -1$

$x_2 = -2$



$D(f) = \langle -\infty, -2 \rangle \cup [1, +\infty)$

✓

④ $f(x) = \frac{x^2-4}{x^2+4}$
 $\rightarrow x^2+4 \neq 0$

①. DOMENA
 $a=1, b=0, c=4$
 $x_{1,2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot 4}}{2}$

$D(f) = \mathbb{R}$

②. PARNOST/NEPARNOST

$f(-x) = \frac{(-x)^2-4}{(-x)^2+4}$

$f(-x) = \frac{x^2-4}{x^2+4} \Rightarrow$ FUNKCIJA PARNA

③. $f(0) = \frac{0-4}{0+4} = -1$

$S(0, -1)$

④. $f(x) = \frac{x^2-4}{x^2+4} /: x^2+4$

$f(x) = x^2-4$

$a=1, b=0, c=-4$

$x_{1,2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot (-4)}}{2}$

$x_1 = 2$
 $x_2 = -2$

$N.T. (2, 0)$
 $(-2, 0)$

⑤. V.A. \rightarrow NEMA

⑥. D.H.A. $\lim_{x \rightarrow +\infty} \frac{x^2-4}{x^2+4} /: x^2 = \frac{1 - \frac{4^0}{x^2}}{1 + \frac{4^0}{x^2}} = \frac{1}{1} = 1$

L.H.A. $\lim_{x \rightarrow -\infty} \frac{x^2-4}{x^2+4} = \lim_{x \rightarrow -\infty} \frac{(-x)^2-4}{(-x)^2+4} = \lim_{x \rightarrow -\infty} \frac{x^2-4}{x^2+4} /: x^2 = \frac{1 - \frac{4^0}{x^2}}{1 + \frac{4^0}{x^2}} = 1$

$x \rightarrow -x$
 $\infty \rightarrow -\infty$

⑦. K.A. $\lim_{x \rightarrow +\infty} \frac{x^2-4}{x^3+4x} /: x^3 = \frac{\frac{x^2-4}{x^3}}{1 + \frac{4}{x^2}}$

$= \lim_{x \rightarrow +\infty} \frac{\frac{1^0}{x} - \frac{4^0}{x^3}}{1 + \frac{4^0}{x^2}} = \frac{0}{1} = 0$

- NEMA K.A.

\rightarrow DRUGI PAPIR....

NASTAVAK (4) ZADATKA

IME I PREZIME: TONI PERKOVIĆ

BROJ INDEKSA: 17201342011

8. $f(x) = \frac{x^2 - 4}{x^2 + 4}$

$$f'(x) = \frac{(x^2 - 4)' \cdot (x^2 + 4) - (x^2 - 4) \cdot (x^2 + 4)'}{(x^2 + 4)^2}$$

$$f'(x) = \frac{2x \cdot (x^2 + 4) - (x^2 - 4) \cdot 2x}{(x^2 + 4)^2}$$

$$f'(x) = \frac{\cancel{2x^3} + 8x - \cancel{2x^3} + 8x}{(x^2 + 4)^2}$$

$$f'(x) = \frac{16x}{(x^2 + 4)^2} \rightarrow 16x = 0$$

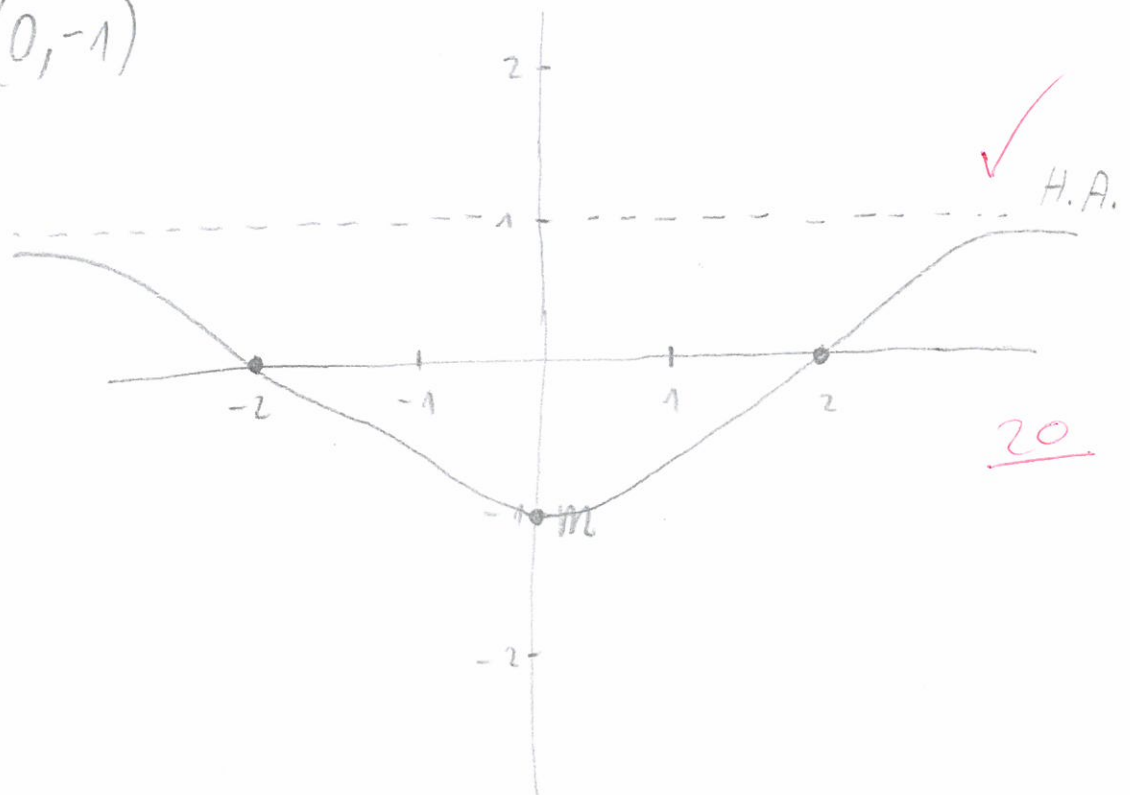
$$x = \frac{0}{16}$$

$$x = 0$$

	$-\infty$	-2	0	2	$+\infty$
$f'(x)$	-	-	+	+	
$f(x)$	\searrow	\searrow	\nearrow	\nearrow	

m

m(0, -1)



$$\begin{array}{l}
 2. \\
 \left| \begin{array}{ccccc|c}
 1 & 1 & -1 & -3 & 4 & 2 \\
 3 & 1 & -1 & -1 & 0 & 2 \\
 9 & 1 & -2 & -1 & -2 & 5 \\
 1 & -1 & 0 & -1 & 2 & 1
 \end{array} \right| \begin{array}{l}
 / \cdot (-3) \quad / \cdot (-9) \quad / \cdot (-1) \\
 \leftarrow + \quad \quad \quad \leftarrow + \\
 \leftarrow + \quad \quad \quad \leftarrow + \\
 \leftarrow +
 \end{array} = \left| \begin{array}{ccccc|c}
 1 & 1 & -1 & -3 & 4 & 2 \\
 0 & -2 & 2 & 8 & -12 & -4 \\
 0 & -8 & 7 & 26 & -38 & -13 \\
 0 & -2 & 1 & 2 & -2 & -1
 \end{array} \right| \begin{array}{l}
 / \cdot (-4) \quad / \cdot (-1) \\
 \leftarrow + \quad \quad \quad \leftarrow + \\
 \leftarrow +
 \end{array}
 \end{array}$$

$$= \left| \begin{array}{ccccc|c}
 1 & 1 & -1 & -3 & 4 & 2 \\
 0 & -2 & 2 & 8 & -12 & -4 \\
 0 & 0 & -1 & 6 & 10 & 3 \\
 0 & 0 & -1 & -6 & 10 & 3
 \end{array} \right| \begin{array}{l}
 / \cdot (-1) \\
 \leftarrow +
 \end{array} = \left| \begin{array}{ccccc|c}
 1 & 1 & -1 & -3 & 4 & 2 \\
 0 & -2 & 2 & 8 & -12 & -4 \\
 0 & 0 & -1 & 6 & 10 & 3 \\
 0 & 0 & 0 & -12 & 0 & 0
 \end{array} \right| \begin{array}{l}
 \\
 \\
 \\
 \\
 \end{array}$$

~~GUSTAV
 NEMA
 RJESENA!!~~

$x_4 = 0$

$x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2$
 $-x_2 + 2x_3 + 8x_4 - 12x_5 = -1$
 $-x_3 + 6x_4 + 10x_5 = 3$
 $-12x_4 = 0$

1. $z^2 - z + 4 = 0$

$a=1, b=-1, c=4$

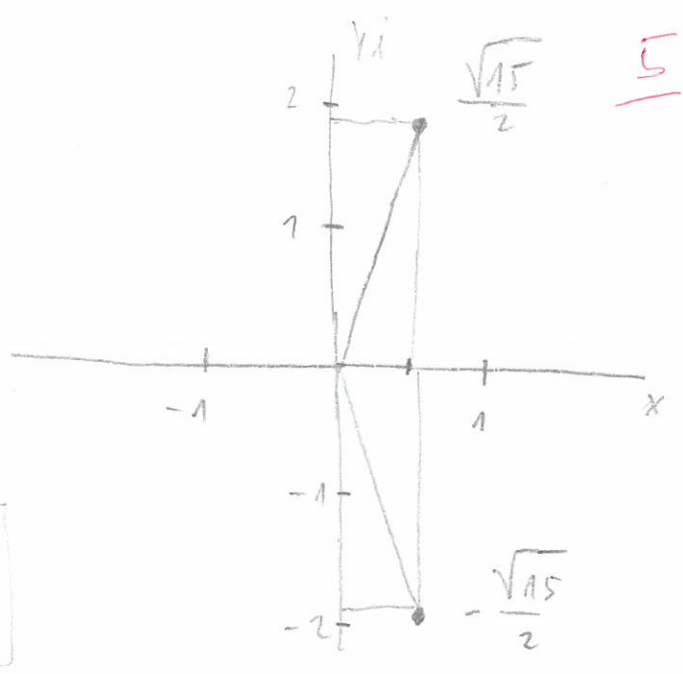
$x_{1,2} = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 4}}{2}$

$x_{1,2} = \frac{1 \pm \sqrt{1-16}}{2}$

$x_{1,2} = \frac{1 \pm \sqrt{-15}}{2}$

$z_1 = \frac{1 + i\sqrt{15}}{2} = \frac{1}{2} + \frac{\sqrt{15}}{2}i$

$z_2 = \frac{1 - i\sqrt{15}}{2} = \frac{1}{2} - \frac{\sqrt{15}}{2}i$



↓
 DRUGI PAPIR.

IME I PREZIME: TONI PERKOVIĆ

BROJ INDEKSA: 17201342011

NASTAVAK (1.) ZADATKA

$$\begin{aligned} \left(\frac{z_2}{z_1} \right) &= \frac{\frac{1}{2} - \frac{\sqrt{15} \cdot i}{2}}{\frac{1}{2} + \frac{\sqrt{15} \cdot i}{2}} = \frac{\frac{1}{2} - \frac{\sqrt{15} \cdot i}{2}}{\frac{1}{2} + \frac{\sqrt{15} \cdot i}{2}} \cdot \frac{\frac{1}{2} + \frac{\sqrt{15} \cdot i}{2}}{\frac{1}{2} + \frac{\sqrt{15} \cdot i}{2}} \\ &= \frac{\left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{15} \cdot i}{2}\right)^2}{\frac{1}{4} + \frac{\sqrt{15} \cdot i}{2}} \\ &= \frac{\frac{1}{4} + \frac{\sqrt{15} \cdot i}{2}}{\frac{1}{4} + \frac{\sqrt{15} \cdot i}{2}} \cdot \frac{\frac{1}{2} + \frac{\sqrt{15} \cdot i}{2}}{\frac{1}{2} + \frac{\sqrt{15} \cdot i}{2}} \\ &= \frac{\frac{1}{4} + \frac{\sqrt{15} \cdot i}{2}}{\frac{1}{4} + \frac{\sqrt{15} \cdot i}{2}} \cdot \frac{\left(\frac{\sqrt{15} \cdot i}{2}\right) + \left(\frac{\sqrt{15} \cdot i}{2}\right)^2}{\left(\frac{\sqrt{15} \cdot i}{2}\right) + \left(\frac{\sqrt{15} \cdot i}{2}\right)^2} \quad \times \end{aligned}$$

$$\left(\frac{z_1 - z_2}{z_2 + 4} \right) = \frac{\frac{1}{2} + \frac{\sqrt{15} \cdot i}{2} - \frac{1}{2} - \frac{\sqrt{15} \cdot i}{2}}{\frac{1}{2} - \frac{\sqrt{15} \cdot i}{2} + 4} = \frac{0}{\frac{1}{2} - \frac{\sqrt{15} \cdot i}{2} + 4} = \frac{0}{\frac{1}{2} - \frac{\sqrt{15} \cdot i}{2} + 4} \quad \times$$

$$\lim_{x \rightarrow -4} \frac{x^2 - 4}{x^2 + 8x - 16} = \frac{(-4)^2 - 4}{(-4)^2 + 8 \cdot (-4) - 16} = \frac{12}{16 - 32 - 16} = \frac{12}{-32} = -\frac{6}{16} = -\frac{3}{8}$$

PROGRESIVO PREPISANO

$$\begin{aligned} \lim_{n \rightarrow \infty} \left(\frac{n+4}{n} \right)^n &= \lim_{n \rightarrow \infty} \left(\frac{n}{n} + \frac{4}{n} \right)^n \\ &= \lim_{n \rightarrow \infty} \left(1 + \frac{n}{4} \cdot \frac{1}{\frac{n}{4}} \right)^{\frac{4}{n}} \\ &= \lim_{n \rightarrow \infty} \left(1 + \frac{4}{\frac{n}{4}} \right)^{\frac{4}{n}} \\ &= \lim_{x \rightarrow +\infty} \left(1 + \frac{4}{x} \right)^{\frac{4}{x}} \left(\frac{4}{x} \cdot \frac{x}{1} = \frac{4x}{x} \right) \\ &= e^4 \quad \checkmark \end{aligned}$$

PROVJERA?

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IME I PREZIME: **KRISTIAN MARTINOVIĆ** BROJ INDEKSA: **17-2-0110-2011**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

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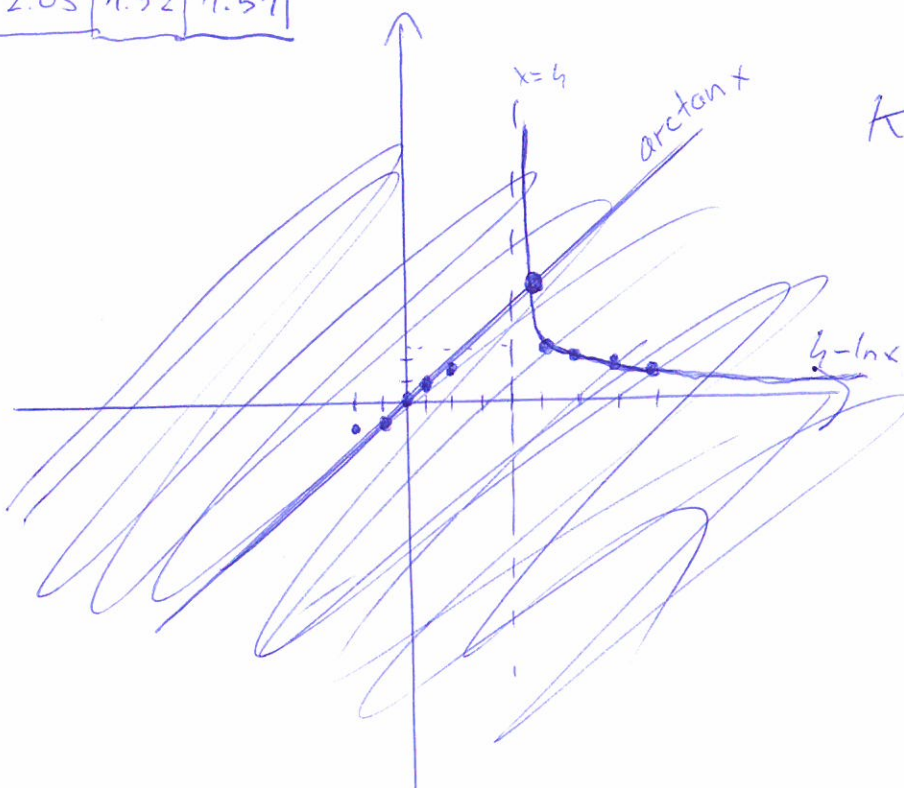
$4 - \ln x > 0$
 $4 > \ln x$, ASIMPTOTA $x = e^4$

$\arctan x = 0$

Ukupno: **40**

x	5	6	7	8	12
f(x)	2.39	2.20	2.05	1.92	1.57

x	1	0	-1	2	-2	5
f(x)	0.78	0	-0.78	1.10	-1.10	1.37



KRIVI

CRTEŽ

NA
DRUGOJ
STRANI

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 9 & 1 & -2 & -1 & -2 & 5 \\ 1 & -1 & 0 & -1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 0 & -1 & 1 & 4 & 8 & 4 \\ 0 & 8 & 7 & 26 & 34 & 23 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 10 & 8 \\ 0 & -1 & 1 & 4 & 6 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 & 4 & 6 & 7 \\ 1 & 1 & -1 & -3 & 4 & 2 \\ 1 & 0 & 0 & 1 & 10 & 8 \\ 0 & -8 & 8 & 32 & 48 & 32 \\ 0 & 8 & 7 & 26 & 34 & 23 \\ 0 & 0 & 15 & 58 & 82 & 55 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -3 & 3 & 9 & 12 & 6 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 0 & -2 & 2 & 8 & 12 & 8 \\ 0 & -1 & 1 & 4 & 6 & 4 \\ -9 & -9 & 9 & 27 & 36 & 18 \\ 9 & 1 & -2 & -1 & -2 & 5 \\ 0 & 8 & 7 & 26 & 34 & 23 \\ -1 & -1 & 1 & 3 & -4 & -2 \\ 1 & -1 & 0 & -1 & 2 & 1 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{bmatrix}$$

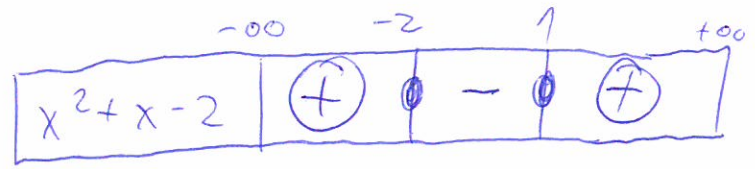
3. $g(x) = \sqrt{x^2 + x - 2} - \arctan(4x^2 - x)$
 $D = \mathbb{R}^+$

$$x^2 + x - 2 \geq 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$$

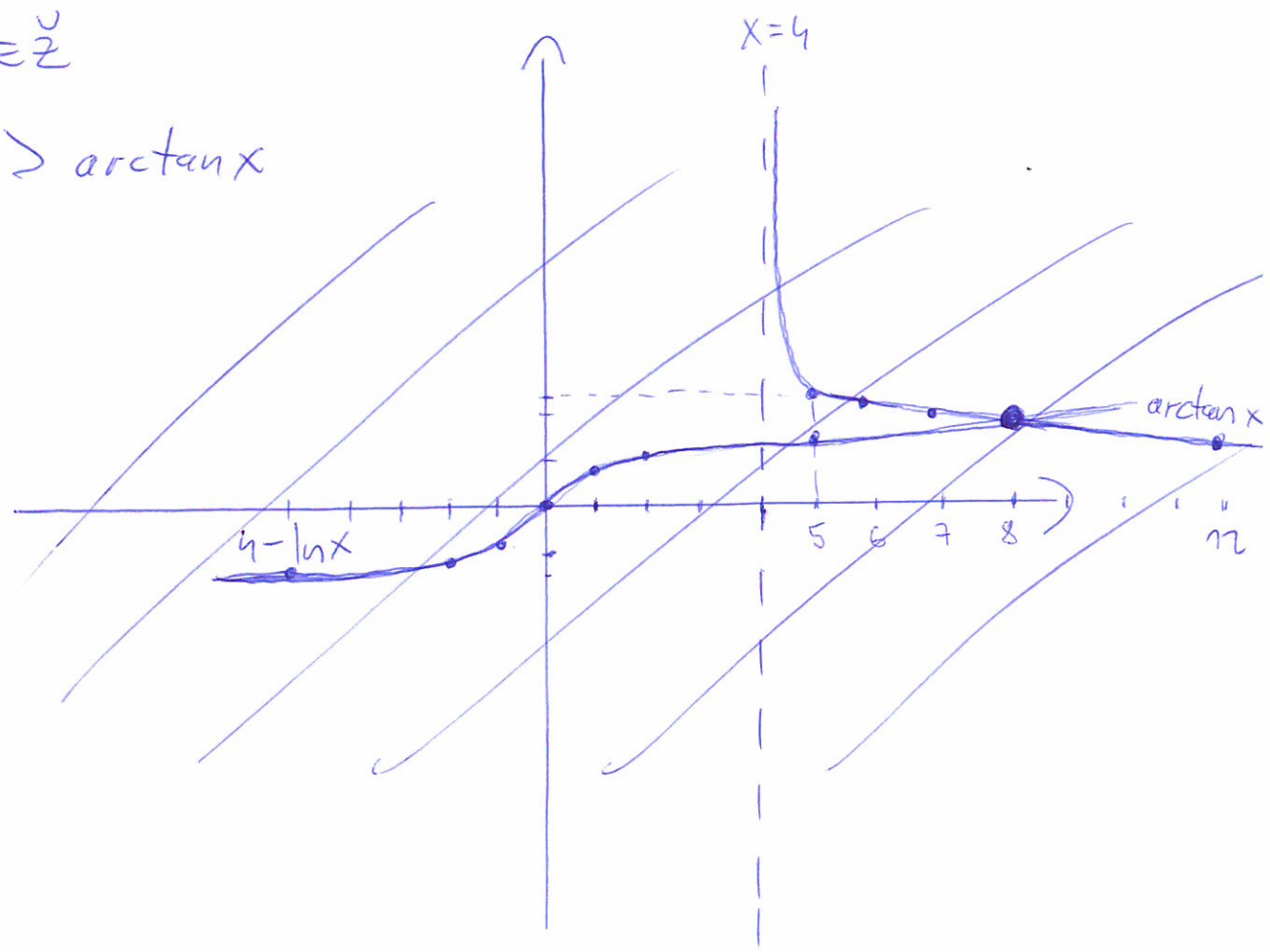
$$x_{1,2} = \frac{1 \pm 3}{2}$$

$$x_1 = 1, x_2 = -2$$



$$Df = \langle -\infty, -2 \rangle \cup [1, +\infty) \checkmark$$

6. CRTEŽ
 $4 - \ln x > \arctan x$



IME I PREZIME: KRISTIAN MARTINOVIĆ

BROJ INDEKSA: 17-2-0110-2017

$$4. f(x) = \frac{x^2 - 4}{x^2 + 4}$$

PARNOST/NEPARNOST

DOMENA

$$x^2 + 4 \neq 0$$

$$D = \mathbb{R}$$

~~$$x^2 \neq -4$$~~

$$f(-x) = \frac{(-x)^2 - 4}{(-x)^2 + 4} = \frac{x^2 - 4}{x^2 + 4} = f(x)$$

FUNKCIJA JE PARNA
NIJE PERIODIČNA

KULTOČKE

$$f(x) = 0$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x_{1,2} = \pm 2$$

$$f(2) = 0, \text{ NT}(2, 0)$$

$$f(-2) = 0, \text{ NT}(-2, 0)$$

$$f(0) = -1, \text{ NT}(0, -1)$$

> x-os

y-os

ASIMPTOTE

V.A. NEMA

H.A.

$$\lim_{x \rightarrow \infty} \frac{x^2 - 4}{x^2 + 4} = \lim_{x \rightarrow \infty} \frac{x^2 - 4 / : x^2}{x^2 + 4 / : x^2} = \lim_{x \rightarrow \infty} \frac{1 - \frac{4}{x^2} = 1}{1 + \frac{4}{x^2} = 1} = \frac{1}{1} = 1$$

$$y = 1, \text{ H.A.}$$

K.A. NEMA

DERIVACIJE

$$f'(x) = \frac{(x^2 - 4)'(x^2 + 4) - (x^2 - 4)(x^2 + 4)'}{(x^2 + 4)^2}$$

$$f'(x) = \frac{16x}{(x^2 + 4)^2}$$

$$f'(x) = \frac{2x(x^2 + 4) - (x^2 - 4)2x}{(x^2 + 4)^2}$$

$$f'(x) = \frac{2x^3 + 8x - (2x^3 - 8x)}{(x^2 + 4)^2}$$

$$f'(x) = \frac{\cancel{2x^3} + 8x - \cancel{2x^3} + 8x}{(x^2 + 4)^2}$$

EKSTREMUM

$$f(0) = -1, \quad ST(0, -1)$$

$$f'(x) = 0$$

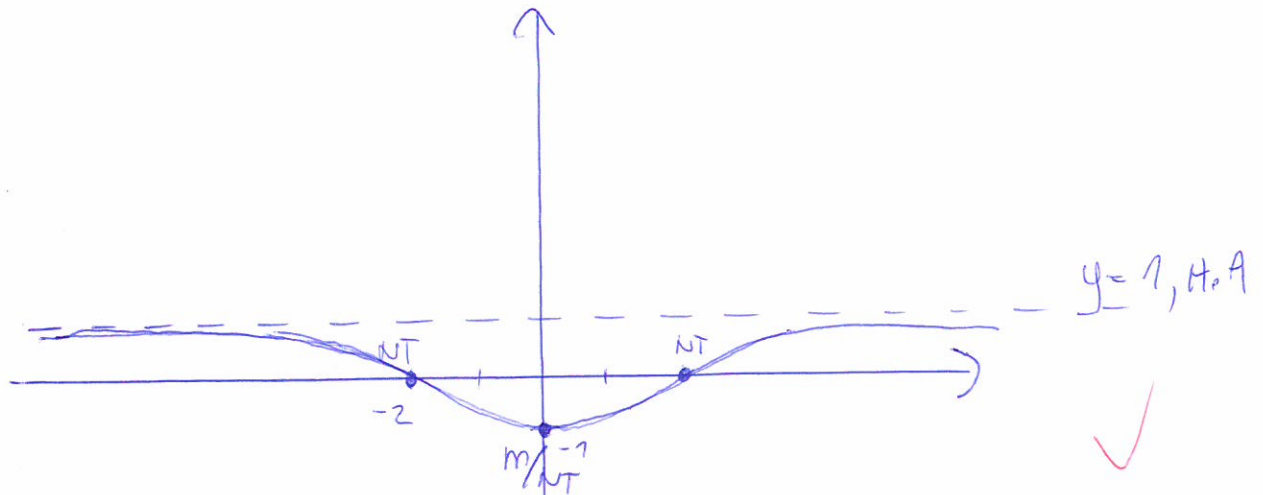
$$16x = 0$$

$$x = 0$$

	$-\infty$	0	$+\infty$
$f'(x)$	-	0	+
$f(x)$	\searrow		\nearrow

$$m(0, -1)$$

m



5.

b)

$$\lim_{n \rightarrow \infty} \left(\frac{n+4}{n} \right)^n = \lim_{n \rightarrow \infty} \left(\frac{n}{n} + \frac{4}{n} \right)^n = \lim_{n \rightarrow \infty} (1 + 0)^n$$

$$= 1^n = 1^{\infty} = \infty$$

a)

$$\lim_{x \rightarrow -4} \frac{x^2 - 4}{x^2 + 8x + 16} \stackrel{/:x^2}{=} \lim_{x \rightarrow -4} \frac{1 - \frac{4}{x^2}}{1 + \frac{8}{x} + \frac{16}{x^2}} = \frac{1}{7} = 1$$

IME I PREZIME: KRISTIAN MARTINOVIĆ

BROJ INDEKSA: 17-2-0110-2017

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 9 & 1 & -2 & -1 & 2 & 5 \\ 1 & -1 & 0 & -1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ 3 & 1 & -2 & -1 & 2 & 5 \\ 3 & 1 & -1 & -1 & 0 & 2 \\ 1 & -1 & 0 & -1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & 2 \\ \hline 0 & -8 & 4 & 26 & -34 & -13 \\ 0 & 1 & -1 & 4 & 6 & 3 \\ 0 & -2 & 1 & 2 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 0 & -1 & 2 & -2 & 3 \\ 0 & -8 & 7 & 26 & -34 & -13 \\ 0 & 0 & -1 & 58 & 14 & 11 \\ 0 & 0 & 3 & 18 & -26 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -7 & -2 & -1 \\ 0 & -1 & 0 & 54 & 8 & 8 \\ 0 & 0 & -1 & 58 & 14 & 11 \\ 0 & 0 & 0 & 1 & 1/12 & 1/8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 17/12 & -1/8 \\ 0 & -1 & 0 & 0 & 7/2 & 59/4 \\ 0 & 0 & -1 & 0 & 55/6 & 73/4 \\ 0 & 0 & 0 & 1 & 1/12 & 1/8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 17/12 & -1/8 \\ 0 & 1 & 0 & 0 & -7/2 & -59/4 \\ 0 & 0 & 1 & 0 & -55/6 & -73/4 \\ 0 & 0 & 0 & 1 & 1/12 & 1/8 \end{bmatrix}$$

SUSTAV IMA
BESKONAČNO MNOGO
RJEŠENJA

$$x_5 \rightarrow \lambda$$

$$\lambda \in \mathbb{R}$$

$$x_1 + \frac{17}{12} \lambda = -1/8$$

$$x_2 - \frac{7}{2} \lambda = -59/4$$

$$x_3 - \frac{55}{6} \lambda = -73/4$$

$$x_4 + \frac{1}{12} \lambda = 1/8$$



PROVJERA ZA $\lambda=0$:

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 \\ 3 & 1 & -1 & -1 & 0 \\ 9 & 1 & -2 & -1 & 2 \\ 1 & 1 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -1/8 \\ -59/4 \\ -73/4 \\ 1/8 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 5 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 2 \\ 2 \\ 5 \\ 1 \end{bmatrix}$$

$$-\frac{1}{8} - \frac{59}{4} + \frac{73}{4} - \frac{3}{8} = \frac{-1}{8} + \frac{14}{4} = \frac{6}{2} = 3$$

$$\begin{array}{r}
 -9 -9 +9 +27 -36 -18 \\
 9 \ 1 \ -2 \ -1 \ 2 \ 5 \\
 \hline
 0 \ -18 \ 11 \ -28 \ 38 \ 23 \\
 0 \ -8 \ 7 \ 26 \ -34 \ -13
 \end{array}$$

$$\begin{array}{r}
 3 \ 13 \ -3 \ -9 \ 12 \ 6 \\
 3 \ 4 \ -1
 \end{array}$$

$$\begin{array}{r}
 -3 \ -3 \ 3 \ 9 \ -12 \ -6 \\
 3 \ 1 \ -1 \ -1 \ 0 \ 2 \\
 \hline
 0 \ -2 \ 2 \ -8 \ -12 \ -6 \\
 0 \ -1 \ 1 \ -4 \ -6 \ -3
 \end{array}$$

$$\begin{array}{r}
 -1 \ -1 \ +1 \ +3 \ -4 \ -2 \\
 1 \ -1 \ 0 \ -1 \ 2 \ 1 \\
 \hline
 0 \ -2 \ 1 \ 2 \ -2 \ -1 \\
 8 \ 8 \ -8 \ -24 \ 32 \ 16 \\
 0 \ -8 \ 7 \ 26 \ -34 \ -13
 \end{array}$$

$$\begin{array}{r}
 0 \ -8 \ 7 \ 26 \ -34 \ -13 \\
 0 \ 8 \ -8 \ 32 \ 48 \ 24 \\
 \hline
 0 \ 0 \ -1 \ 58 \ 14 \ 11
 \end{array}$$

$$\begin{array}{r}
 8 \ 0 \ -1 \ 2 \ -2 \ 3 \\
 0 \ -8 \ 7 \ 26 \ -34 \ -13 \\
 0 \ 8 \ -4 \ -8 \ 8 \ 4 \\
 \hline
 0 \ 0 \ 3 \ 18 \ -26 \ -9 \\
 0 \ 0 \ 1 \ 6 \ 26/3 \ -3
 \end{array}$$

$$\begin{array}{r}
 -8 \ 0 \ 1 \ -2 \ 2 \ -3 \\
 0 \ 0 \ -1 \ 58 \ 14 \ 11 \\
 \hline
 -8 \ 0 \ 0 \ 56 \ 16 \ 8 \\
 -1 \ 0 \ 0 \ 7 \ 2 \ 1 \\
 1 \ 0 \ 0 \ -4 \ -2 \ -1
 \end{array}$$

$$\begin{array}{r}
 0 \ 0 \ -7 \ 406 \ 98 \ 77 \\
 0 \ -8 \ 7 \ 26 \ -34 \ -13 \\
 \hline
 0 \ -8 \ 0 \ 432 \ 64 \ 64 \\
 0 \ -1 \ 0 \ 54 \ 8 \ 8
 \end{array}$$

$$\begin{array}{r}
 0 \ 0 \ -3 \ 174 \ 42 \ 33 \\
 0 \ 0 \ 3 \ 18 \ -26 \ -9 \\
 \hline
 0 \ 0 \ 0 \ 192 \ 16 \ 24 \\
 0 \ 0 \ 0 \ 1 \ 1/12 \ 1/8
 \end{array}$$

$$\begin{array}{r}
 0 \ 0 \ 0 \ -58 \ -28/6 \ 29/4 \\
 0 \ 0 \ -1 \ 58 \ 14 \ 11 \\
 \hline
 0 \ 0 \ -1 \ 0 \ 55/6 \ 73/4 \\
 0 \ 0 \ 0 \ -54 \ -9/2 \ 27/4 \\
 0 \ -1 \ 0 \ 54 \ 8 \ 8 \\
 \hline
 0 \ -1 \ 0 \ 0 \ 7/2 \ 59/4
 \end{array}$$

$$\begin{array}{r}
 0 \ 0 \ 0 \ 7 \ 7/12 \ 7/8 \\
 1 \ 0 \ 0 \ -7 \ -2 \ -1 \\
 \hline
 1 \ 0 \ 0 \ 0 \ -14/12 \ -1/8
 \end{array}$$

IME I PREZIME: KRISTIAN MARTINAID BROJ INDEKSA: 17-2-0110-2011

6. $4 - \ln x > \arctan x$

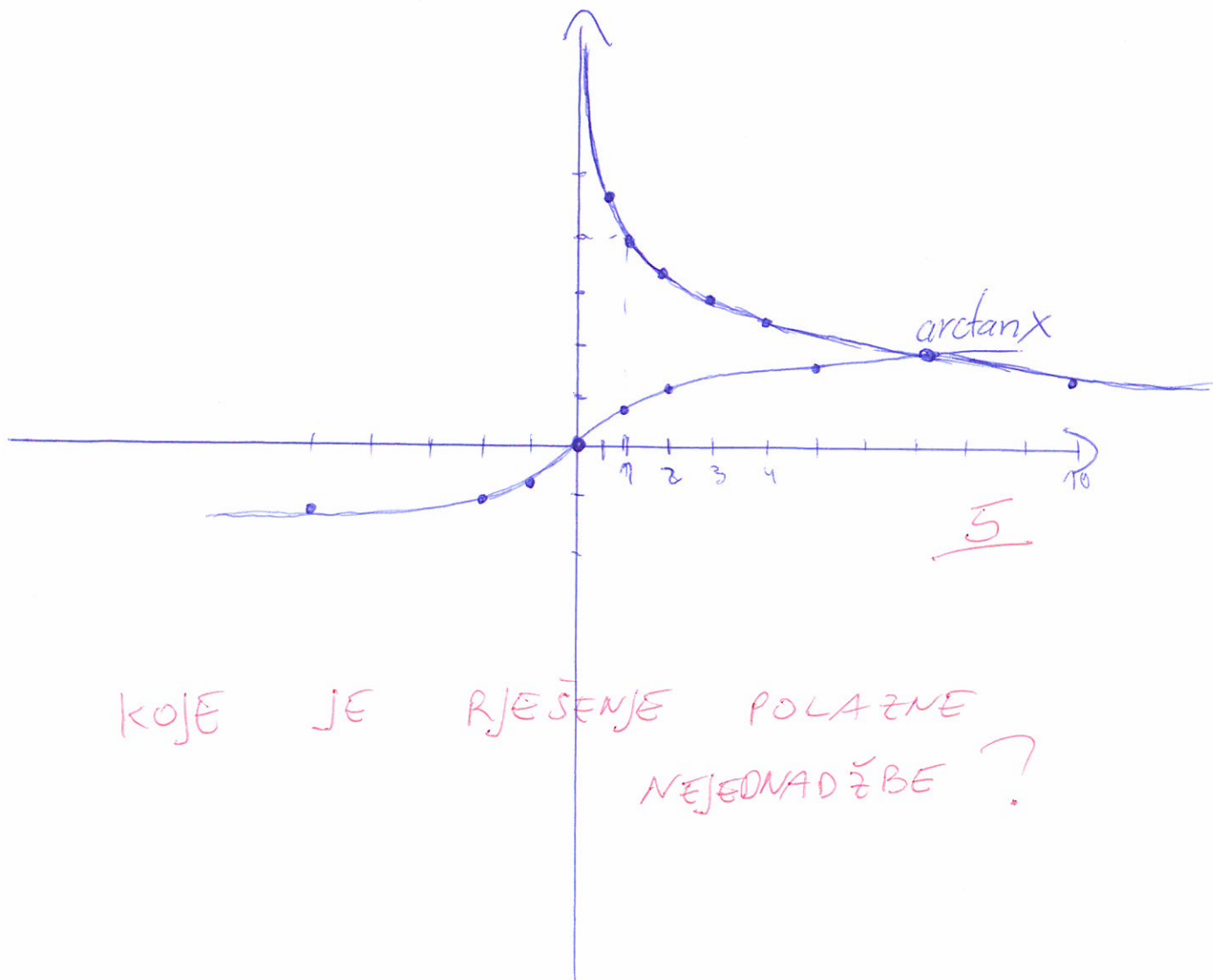
$4 - \ln x > 0$

~~XXXXXXXXXXXXXXXXXXXX~~

$\arctan x = 0$

x	1	0	-1	2	-2	5
f(x)	0.78	0	-0.78	1.10	-1.10	1.37

x	0.5	1	2	3	4	10
$4 - \ln x$	4.69	4	3.30	2.90	2.61	1.69



KOJE JE RJEŠENJE POLAZNE NEJEDNAKOSTI ?

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: SABOLIC BORIS

BROJ INDEKSA: 17-2-0010-2010

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

F4

1. Neka su z_1 i z_2 rješenja kvadratne jednadžbe $z^2 - z + 4 = 0$. Prikaži ih u kompleksnoj ravnini! Dalje izračunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$ i $\text{Im}\left(\frac{z_2}{z_1}\right)$.

12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

10+5

$$\begin{aligned} x_1 + x_2 - x_3 - 3x_4 + 4x_5 &= 2 \\ 3x_1 + x_2 - x_3 - x_4 &= 2 \\ 9x_1 + x_2 - 2x_3 - x_4 - 2x_5 &= 5 \\ x_1 - x_2 - x_4 + 2x_5 &= 1 \end{aligned}$$

3. Odrediti domenu funkcije $g(x) = \sqrt{x^2 + x - 2} - \arctan(4x^2 - x)$.

15

4. Odrediti tok funkcije $f(x) = \frac{x^2 - 4}{x^2 + 4}$

20(graf)

5. Odrediti i provjeriti:

(a) $\lim_{x \rightarrow -4} \frac{x^2 - 4}{x^2 + 8x + 16} =$

4+1

(b) $\lim_{n \rightarrow \infty} \left(\frac{n+4}{n}\right)^n =$

8+2

6. Grafički približno riješiti: $4 - \ln x > \arctan x$. Ne zaboravi eksplicitno zapisati skup rješenja. *Provjeri približno rješenje jednadžbe vrštavanjem, a također provjeri nekoliko rješenja nejednadžbe vrštavanjem!*

15+5

Ukupno:

35

③ $f(x) = \sqrt{x^2 + x - 2} - \arctg(4x^2 - x)$

$x^2 + x - 2 \geq 0$

$x^2 + x - 2 = 0 \Rightarrow a = 1 > 0 \cup$

$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2}$

$x_1 = \frac{-4}{2} = -2$

$x_2 = \frac{2}{2} = 1$

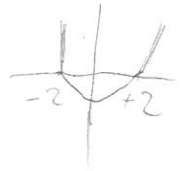
$D_f(x), x \in (-\infty, -2] \cup [1, +\infty)$ ✓

①

④ TOK FUNKCIJE + GRAF

$$f(x) = \frac{x^2 - 4}{x^2 + 4}$$

$$a = 170 \quad \cup$$



a) DOMENA

b) NUL TOČKE

$$x^2 + 4 \neq 0$$

$$x^2 - 4 = 0$$

$$x^2 + 4 = 0$$

$$x^2 = 4$$

$$x^2 \neq -4 \Rightarrow D_{f(x)} = \mathbb{R}$$

$$x_{1,2} = \pm 2$$

c) ASIMPTOTE

$$V.A. \Rightarrow \lim_{x \rightarrow -2} \frac{x^2 - 4}{x^2 + 4} = \lim_{x \rightarrow -2} \frac{(-2)^2 - 4}{(-2)^2 + 4} = \frac{4 - 4}{4 + 4} = \frac{0}{8} = 0$$

$$\lim_{x \rightarrow +2} \frac{x^2 - 4}{x^2 + 4} = \lim_{x \rightarrow +2} \frac{2^2 - 4}{2^2 + 4} = \frac{0}{8} = 0 \Rightarrow \text{NETA V.A.}$$

$$H.A. \Rightarrow \lim_{x \rightarrow \pm\infty} \frac{x^2 - 4}{x^2 + 4} = \lim_{x \rightarrow \pm\infty} \frac{1 - \frac{4}{x^2}}{1 + \frac{4}{x^2}} = \frac{1}{1} = 1 \Rightarrow \boxed{y = 1}$$

d) TOK

$$f'(x) = \left(\frac{x^2 - 4}{x^2 + 4} \right)' = \frac{2x \cdot (x^2 + 4) - (x^2 - 4) \cdot 2x}{(x^2 + 4)^2} = \frac{2x^3 + 8x - 2x^3 + 8x}{(x^2 + 4)^2} = \frac{16x}{(x^2 + 4)^2}$$

$$16x = 0$$

$$\boxed{x = 0}$$

	-∞	-2	0	+2	+∞
$f'(x)$	-	/	+	/	+
$f(x)$	↓	/	↑	/	↑

$$f(0) = \frac{0 - 4}{0 + 4} = \frac{-4}{4} = -1$$

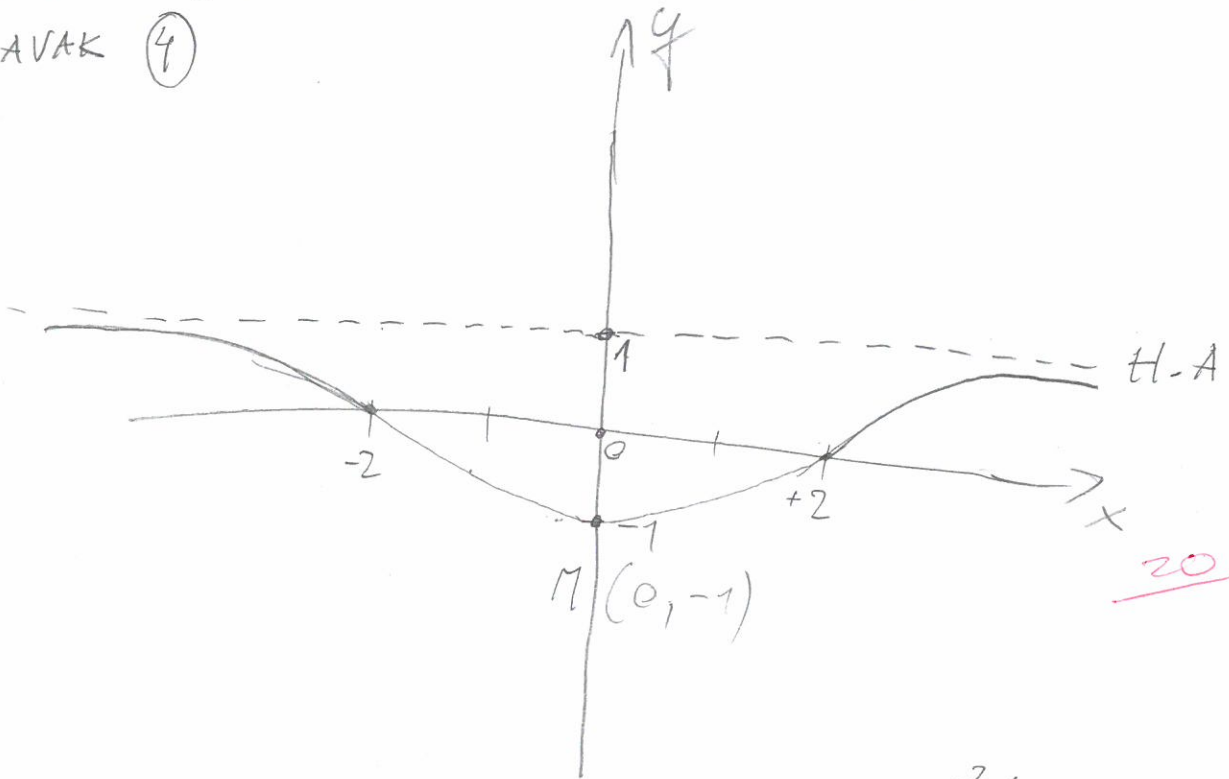
$$M(0, -1)$$

NASTAVAK
NAD DRUGOM
LISTOM

IME I PREZIME: SABOLIC' BORIS

BROJ INDEKSA: 17-2-0010-2010

NASTAVAK (4)



$$K.A. \Rightarrow y = kx + l \quad k = \frac{f(x)}{x} \Rightarrow k = \lim_{x \rightarrow \pm\infty} \left(\frac{\frac{x^2-4}{x^2+4}}{\frac{x}{7}} \right) = \frac{x^2-4}{x^3+4x}$$
$$k = \frac{0-4}{0+4 \cdot 0} = \frac{-4}{0} = \infty \quad \text{NEMA } \underline{\underline{K.A}}$$

(3)

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

F4

IME I PREZIME: FILIP GOSPIĆ

BROJ INDEKSA: 58010

ZAOKRUŽITI AKO ŽELITE:

ustmeni kod prof. Uglešića

1. Neka su z_1 i z_2 rjesenja kvadratne jednadzbe $z^2 - z + 4 = 0$. Prikaži ih u kompleksnoj ravnini! Dalje izracunaj: $\left(\frac{z_1 - z_2}{z_2 + 4}\right)$ i $\text{Im}\left(\frac{z_2}{z_1}\right)$.

12+3

2. Riješi sustav Gaussovom metodom i obavezno provjeri rješenje:

~~10+5~~

$$x_1 + x_2 - x_3 - 3x_4 + 4x_5 = 2$$

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$$9x_1 + x_2 - 2x_3 - x_4 - 2x_5 = 5$$

$$x_1 - x_2 - x_4 + 2x_5 = 1$$

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~~15~~

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15+5

Ukupno:

29

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & : & 2 \\ 3 & 1 & -1 & -1 & 0 & : & 2 \\ 9 & 1 & -2 & -1 & -2 & : & 5 \\ 1 & -1 & 0 & -1 & 2 & : & 1 \end{bmatrix} \begin{array}{l} R_2 - 3R_1 \\ R_3 - 9R_1 \\ R_4 - R_1 \end{array} \sim \begin{bmatrix} 1 & 1 & -1 & -3 & 4 & : & 2 \\ 0 & -2 & 2 & 8 & -12 & : & -4 \\ 0 & -8 & 7 & 26 & -38 & : & -13 \\ 0 & -2 & 1 & 2 & -2 & : & -1 \end{bmatrix} \cdot \left(-\frac{1}{2}\right) \sim$$

$$\begin{bmatrix} 1 & 1 & -1 & -3 & 4 & : & 2 \\ 0 & 1 & -1 & -4 & 6 & : & 2 \\ 0 & -8 & 7 & 26 & -38 & : & -13 \\ 0 & -2 & 1 & 2 & -2 & : & -1 \end{bmatrix} \begin{array}{l} R_1 - R_2 \\ R_3 + 8R_2 \\ R_4 + 2R_2 \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & -2 & : & 0 \\ 0 & 1 & -1 & -4 & 6 & : & 2 \\ 0 & 0 & -1 & -6 & 10 & : & 3 \\ 0 & 0 & 0 & -6 & 10 & : & 3 \end{bmatrix} \begin{array}{l} \\ \\ \cdot (-1) \\ \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & -2 & : & 0 \\ 0 & 1 & -1 & -4 & 6 & : & 2 \\ 0 & 0 & 1 & 6 & -10 & : & 3 \\ 0 & 0 & 0 & -6 & 10 & : & 3 \end{bmatrix} \begin{array}{l} \\ R_2 + R_3 \\ \\ \end{array} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 & : & 0 \\ 0 & 1 & 0 & 2 & -16 & : & -1 \\ 0 & 0 & 1 & 6 & -10 & : & -3 \\ 0 & 0 & 0 & -6 & 10 & : & 3 \end{bmatrix} \cdot \left(-\frac{1}{6}\right) \sim \begin{bmatrix} 1 & 0 & 0 & 1 & -2 & : & 0 \\ 0 & 1 & 0 & 2 & -16 & : & -1 \\ 0 & 0 & 1 & 6 & -10 & : & -3 \\ 0 & 0 & 0 & 1 & \frac{5}{3} & : & -\frac{1}{2} \end{bmatrix} \begin{array}{l} R_1 - R_4 \\ R_2 - 2R_4 \\ R_3 - 6R_4 \\ \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{11}{3} & : & \frac{1}{2} \\ 0 & 1 & 0 & 0 & -\frac{38}{3} & : & 0 \\ 0 & 0 & 1 & 0 & -\frac{20}{3} & : & 0 \\ 0 & 0 & 0 & 1 & \frac{5}{3} & : & -\frac{1}{2} \end{bmatrix}$$

RJEŠENJE!

$$3. g(x) = \sqrt{x^2+x-2} - \arctan(4x^2-x)$$

$$x^2+x-2 \geq 0$$

$$x^2+x-2=0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$x_1=1 \quad x_2=-2$$

$$4x^2-x \geq 0$$

$$4x^2-x=0$$

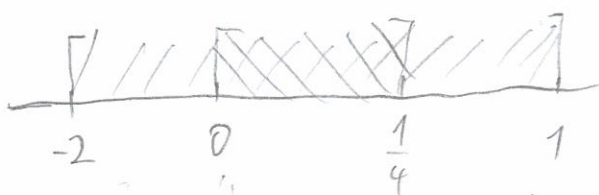
$$x(4x-1)=0$$

$$x_3=0$$

$$x_4=4x-1=0$$

$$4x=1$$

$$x=\frac{1}{4}$$



$$x \in [0, \frac{1}{4}] \quad \times$$

$$5. \lim_{x \rightarrow 4} \frac{x^2-4}{x^2+8x+16} \stackrel{L'H}{=} \lim_{x \rightarrow 4} \frac{2x}{2x+8} = \lim_{x \rightarrow 4} \frac{2 \cdot (-4)}{2 \cdot (-4) + 8} = \lim_{x \rightarrow 4} \frac{-8}{0} = -\infty \quad \checkmark$$

$$b) \lim_{n \rightarrow \infty} \left(\frac{n+4}{n}\right)^n = \quad \times$$

IME I PREZIME: FILIP GOSPIĆ

BROJ INDEKSA: 58040

$$f(x) = \frac{x^2 - 4}{x^2 + 4}$$

1^o DOMENA

$$x^2 + 4 \neq 0$$

$$x^2 \neq -4$$

$x \in \mathbb{R}$

2^o N.T.

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x_1 = -2 \quad x_2 = 2$$

$$x_1(-2, 0)$$

$$x_2(2, 0)$$

3^o H.A.

$$\lim_{x \rightarrow \infty} \frac{x^2 - 4}{x^2 + 4} = \lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{x^2}{x^2} + \frac{4}{x^2}} = \frac{1 - 0}{1 + 0} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 4}{x^2 + 4} = \lim_{x \rightarrow -\infty} \frac{+^2 - 4}{+^2 + 4} = \frac{+^2 + \frac{4}{x^2}}{+^2 + \frac{4}{x^2}} = \frac{-1}{-1} = 1$$

K.A. $y = kx + l$

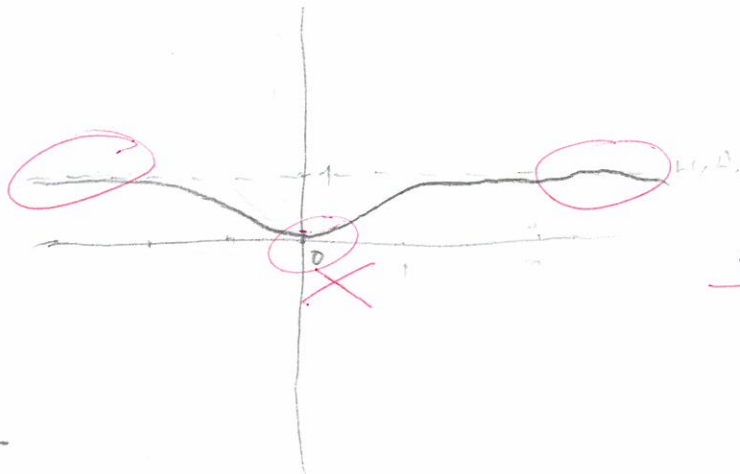
$$k = \lim_{x \rightarrow \infty} \frac{f(x) - l}{x} = \lim_{x \rightarrow \infty} \frac{\frac{x^2 - 4}{x^2 + 4} - 1}{x} = \lim_{x \rightarrow \infty} \frac{x^2 - 4 - (x^2 + 4)}{x(x^2 + 4)} = \lim_{x \rightarrow \infty} \frac{-8}{x^3 + 4x} = 0$$

$$l = \lim_{x \rightarrow \infty} f(x) = 1$$

$$16x = 0$$

$$x = 0$$

	$-\infty$	0	$+\infty$
f(x)	-	+	
f'(x)	\searrow	\nearrow	



15

