

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

B8

IME I PREZIME: LUKA KNEŽEVIĆ

BROJ INDEKSA: 17-2-1120-2014

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

1. Izračunaj sve kompleksne brojeve  $z$  takve da  $\left|\frac{z}{2}\right|^2 = z + 1 + i$ . Prikaži ih u kompleksnoj ravnini!

10+5

$$5x + 4z + 2t = 3$$

2. Riješiti sustav 4 jednačbe s 4 nepoznane

$$x - y + 2z + t = 1$$

$$4x + y + 2z = 1$$

$$x + y + z + t = 0$$

i obavezno provjeri rješenje:

10+5

3. Za funkciju  $f(x) = \frac{\sin(2x)}{x}$  odrediti koliko iznosi  $f'(\pi)$ .

8+2

4. Za funkciju:  $g(x) = \sqrt{x^2 + 8x + 5}$  treba:

(a) ispitati domenu

(b) pronaći lokalne ekstreme

(c) ispitati asimptote

5

11

14 10

5. Na temelju ispitivanja toka skicirati graf funkcije  $h(x) = \frac{x^2 + 8}{x - 8}$

20 (graf) 7

6. Izračunati i obavezno na neki način provjeriti koliko iznosi  $\lim_{x \rightarrow -1} \left( \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$ .

8+2

3.  $f(x) = \frac{\sin 2x}{x}$

$$f'(x) = \frac{(\sin 2x)' \cdot x - \sin 2x \cdot (x)'}{x^2} = \frac{2\cos 2x \cdot x - \sin 2x}{x^2}$$

$$= \frac{2x \cos 2x - \sin 2x}{x^2}$$

$$f'(\pi) = \frac{2\pi}{\pi^2} = 0.63662 \quad \checkmark$$

Ukupno:

47

4.  $\sqrt{x^2 + 8x + 5}$

a)  $x^2 + 8x + 5 \geq 0$

$$x^2 + 8x + 5 = 0$$

$$x = \frac{-8 \pm \sqrt{64 - 20}}{2} = \frac{-8 \pm \sqrt{44}}{2} = \frac{-8 \pm 2\sqrt{11}}{2} = -4 \pm \sqrt{11}$$

$$x_1 = -7.3166$$

$$x_2 = -0.68$$

$$D_f \left[ -\infty, -7.3166 \right] \cup \left[ -0.68, +\infty \right] \quad \checkmark$$

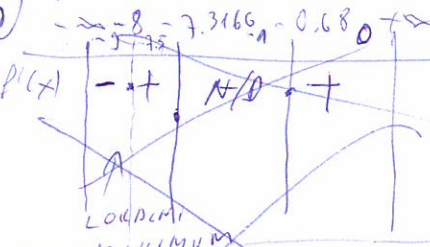
$$-7.316^2 + 8 \cdot (-7.316) + 5 = -0.68^2 - 8 \cdot 0.68 + 5 = 0$$

$$53.5 + 58.5 + 5 = 0 \quad 0.46 - 5.44 + 5 = 0$$

$$-\infty - 7.31 - 7.31 - 0.68 = 0$$

$f(x)$	+	0	+
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~~$-\infty < -7.3166 < 0 < -0.68 < +\infty$~~



$$f(x) = \sqrt{x^2 + 8x + 5} = (x^2 + 8x + 5)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} (x^2 + 8x + 5)^{-\frac{1}{2}} \cdot (2x + 8)$$

$$f'(x) = \frac{x + 4}{\sqrt{x^2 + 8x + 5}}$$

~~$f(-8) = \sqrt{5} = 2.23$  LOKALNI MINIMUM~~  
~~NEMA LOKALNOG MAKSIMUMA~~

$x + 4 = 0$   
 $x = -4$  NIJE UDOMENI

NEMA NI MINIMUM NI MAKSIMUM  
 PAKO ZA  $x \in \mathbb{R}$  ZA  $x \in (-\infty, -7.3166]$   
 PAKO ZA  $x \in \mathbb{R}$  ZA  $x \in [-0.68, +\infty)$

$$\lim_{x \rightarrow -7.3166^+} \sqrt{x^2+8x+5} = \sqrt{(-7.3166)^2 + 8(-7.3166) + 5} = \sqrt{53.29 - 58.4 + 5} = \sqrt{0} = 0$$

$$\lim_{x \rightarrow -7.3166^-} \sqrt{x^2+8x+5} = \sqrt{(-7.3166)^2 + 8(-7.3166) + 5} = \sqrt{53.29 - 58.4 + 5} = \sqrt{0} = 0$$

$$\lim_{x \rightarrow -0.68^+} \sqrt{x^2+8x+5} = \sqrt{(-0.68)^2 + 8(-0.68) + 5} = \sqrt{0.46 - 5.44 + 5} = \sqrt{0.02} \approx 0.14$$

$$\lim_{x \rightarrow -0.68^-} \sqrt{x^2+8x+5} = \sqrt{(-0.68)^2 + 8(-0.68) + 5} = \sqrt{0.46 - 5.44 + 5} = \sqrt{0.02} \approx 0.14$$

$$\lim_{x \rightarrow +\infty} \sqrt{x^2+8x+5} = \lim_{x \rightarrow +\infty} \sqrt{x^2+8x+5} \cdot \frac{\sqrt{x^2+8x+5}}{\sqrt{x^2+8x+5}} = \frac{x^2+8x+5}{\sqrt{x^2+8x+5}} = \frac{1}{0} = +\infty$$

$$\lim_{x \rightarrow -\infty} \sqrt{x^2+8x+5} = \lim_{x \rightarrow -\infty} \sqrt{x^2-8x+5} \cdot \frac{\sqrt{x^2-8x+5}}{\sqrt{x^2-8x+5}} = \frac{x^2-8x+5}{\sqrt{x^2-8x+5}} = \frac{1}{0} = -\infty$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+8x+5}}{x} = \lim_{x \rightarrow +\infty} \frac{1}{1} = 1$$

$$\lim_{x \rightarrow +\infty} (\sqrt{x^2+8x+5} - x) = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+8x+5} - x}{1} = \lim_{x \rightarrow +\infty} \frac{x^2+8x+5 - x^2}{\sqrt{x^2+8x+5} + x} = \frac{8x+5}{\sqrt{x^2+8x+5} + x} = \frac{8}{2} = 4$$

$$\lim_{x \rightarrow -\infty} (\sqrt{x^2+8x+5} - x) = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+8x+5} - x}{1} = \lim_{x \rightarrow -\infty} \frac{x^2+8x+5 - x^2}{\sqrt{x^2+8x+5} - x} = \frac{8x+5}{\sqrt{x^2+8x+5} - x} = \frac{8}{-2} = -4$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+8x+5}}{x} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2-8x+5}}{-x} = -1$$

$$\begin{array}{l} \text{②} \left[ \begin{array}{ccc|ccc} 5 & 0 & 4 & 2 & 1 & 3 \\ 1 & -1 & 2 & 1 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_1} \left[ \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 1 & 1 \\ 5 & 0 & 4 & 2 & 1 & 3 \\ 4 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{array} \right] \xrightarrow{R_3 - 4R_1, R_4 + R_1} \left[ \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 1 & 1 \\ 5 & 0 & 4 & 2 & 1 & 3 \\ 0 & 5 & -6 & -3 & -3 & -3 \\ 2 & 0 & 3 & 2 & 1 & 1 \end{array} \right] \xrightarrow{R_2 - 5R_1, R_4 + 2R_1} \left[ \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 1 & 1 \\ 0 & 5 & -6 & -3 & -3 & -3 \\ 0 & 5 & -6 & -3 & -3 & -3 \\ 4 & -2 & 7 & 4 & 3 & 3 \end{array} \right] \xrightarrow{R_4 - R_2} \left[ \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 1 & 1 \\ 0 & 5 & -6 & -3 & -3 & -3 \\ 0 & 5 & -6 & -3 & -3 & -3 \\ 0 & 3 & 13 & 7 & 6 & 6 \end{array} \right] \xrightarrow{R_3 - R_2} \left[ \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 1 & 1 \\ 0 & 5 & -6 & -3 & -3 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 13 & 7 & 6 & 6 \end{array} \right] \xrightarrow{R_2 \cdot \frac{1}{5}, R_4 \cdot \frac{1}{3}} \left[ \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 1 & 1 \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{13}{3} & \frac{7}{3} & 2 & 2 \end{array} \right] \xrightarrow{R_4 - R_2} \left[ \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 1 & 1 \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{29}{15} & \frac{28}{15} & \frac{13}{5} & \frac{13}{5} \end{array} \right] \xrightarrow{R_4 \cdot \frac{15}{29}} \left[ \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 1 & 1 \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{28}{29} & \frac{13}{29} & \frac{13}{29} \end{array} \right] \xrightarrow{R_1 + R_2, R_4 + \frac{6}{5}R_2} \left[ \begin{array}{ccc|ccc} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{28}{29} & \frac{13}{29} & \frac{13}{29} \end{array} \right] \xrightarrow{R_1 - \frac{4}{5}R_4} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{5} - \frac{4}{5} \cdot \frac{28}{29} & \frac{2}{5} - \frac{4}{5} \cdot \frac{13}{29} & \frac{2}{5} - \frac{4}{5} \cdot \frac{13}{29} \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{28}{29} & \frac{13}{29} & \frac{13}{29} \end{array} \right] \xrightarrow{R_1 \cdot \frac{5}{29}} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{29} & \frac{2}{29} & \frac{2}{29} \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{28}{29} & \frac{13}{29} & \frac{13}{29} \end{array} \right] \xrightarrow{R_1 + \frac{6}{5}R_2} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{29} & \frac{2}{29} & \frac{2}{29} \\ 0 & 1 & 0 & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{28}{29} & \frac{13}{29} & \frac{13}{29} \end{array} \right] \xrightarrow{R_1 + \frac{3}{5}R_2} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{29} & \frac{2}{29} & \frac{2}{29} \\ 0 & 1 & 0 & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{28}{29} & \frac{13}{29} & \frac{13}{29} \end{array} \right] \xrightarrow{R_1 + \frac{3}{5}R_2} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{29} & \frac{2}{29} & \frac{2}{29} \\ 0 & 1 & 0 & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{28}{29} & \frac{13}{29} & \frac{13}{29} \end{array} \right] \xrightarrow{R_1 + \frac{3}{5}R_2} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{29} & \frac{2}{29} & \frac{2}{29} \\ 0 & 1 & 0 & -\frac{3}{5} & -\frac{3}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{28}{29} & \frac{13}{29} & \frac{13}{29} \end{array} \right] \end{array}$$

$$\begin{array}{l} x = 1 \\ y = -1 \\ z = -1 \\ t = 1 \end{array}$$

ODGOVARA ✓

# LUKA KNEŽEVIĆ

6.  $\lim_{x \rightarrow -1} \left( \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right) = \frac{\sqrt{16} - 3 - 1}{1 - 4 + 3} = \frac{4 - 4}{4 - 4} = \frac{0}{0}$  NEODREĐENI OBLIK.

IME I PREZIME:

BROJ INDEKSA:

L. HOPITAL  
 $\frac{\frac{1}{2}(x^2 - 8x + 7)^{-1/2} \cdot (2x - 8) + 1}{2x + 4} = \frac{x - 3}{\sqrt{x^2 - 8x + 7} + 2x + 4} = \frac{-4}{4 - 2 + 4} = \frac{-4}{6} = -\frac{2}{3}$

PROVERKA  
 $\lim_{x \rightarrow -1} \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} = \frac{4^+ - 3 + 1 \cdot 1^+}{0^-} = 0^-$

5. GROF FUNKCIJE  $f(x) = \frac{x^2 + 8}{x - 8}$

$1. 0^+ < x < 8, 8 > 0 < x < 8, +\infty$

$\lim_{x \rightarrow 8^+} \frac{x^2 + 8}{x - 8} = \frac{72}{0^+} = +\infty$

$\lim_{x \rightarrow 8^-} \frac{x^2 + 8}{x - 8} = \frac{72}{0^-} = -\infty$  8 = V.A

$\lim_{x \rightarrow \infty} \frac{x^2 + 8}{x - 8} = \frac{+\infty}{+\infty}$  NEMA H.O.

K.A  
 $\lim_{x \rightarrow \infty} \frac{x^2 + 8}{x(x - 8)} = \frac{x^2 + 8}{x^2 - 8x} \cdot \frac{1}{x} = 1$

$\lim_{x \rightarrow \infty} \frac{x^2 + 8}{x - 8} - x = \frac{x^2 + 8 - x(x - 8)}{x - 8} = \frac{x^2 - x^2 + 8x + 8}{x - 8} = \frac{8x + 8}{x - 8} = 8$  D.K.A  $y = x + 8$

$\lim_{x \rightarrow -\infty} \frac{x^2 + 8}{x - 8} = \lim_{x \rightarrow +\infty} \frac{x^2 + 8}{-x - 8} = \frac{x^2 + 8}{x(-x - 8)} = \frac{x^2 + 8}{-x^2 - 8x} = -1$

$\lim_{x \rightarrow -\infty} \frac{x^2 + 8}{x - 8} = \lim_{x \rightarrow +\infty} \frac{x^2 + 8}{-x - 8} + x = \frac{x^2 + 8 - x^2 - 8x}{-x - 8} = \frac{-8x + 8}{-x - 8} \cdot \frac{1}{x} = 8$

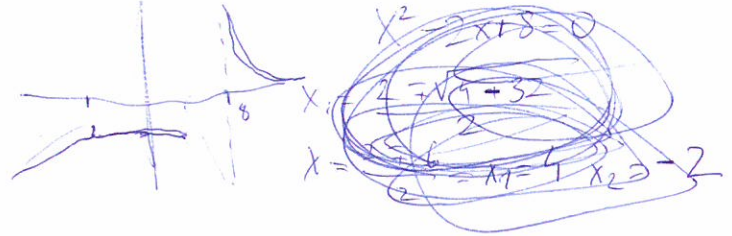
L. K.A  $y = -x + 8$

3.  $f(-x) = f(x)$   $f(-x) = -f(x)$   
 $\frac{x^2 + 8}{-x - 8}$  NIJE PARNA  $\frac{x^2 + 8}{-x - 8} = -\left(\frac{x^2 + 8}{x - 8}\right)$  NIJE NEPARNA

NIJE PERIODIČNA JER NE SADRŽI TRIGONOMETRIJSKE FUNKCIJE.

4.  $f(x) = 0$

$\frac{x^2 + 8}{x - 8} = 0 \Rightarrow x^2 + 8 = 0$   
 $x_{1,2} = \pm \sqrt{0 - 32}$   
 NEMA REŠENJA JER  $x^2 + 8 \neq 0$  NE MOŽE BITI 0.



$f(0) = \frac{8}{-8} = -1$  T(0, -1)

5.  $f(x) = \frac{(x^2 + 8) \cdot (x - 8) - (x^2 + 8) \cdot (x - 8)}{(x - 8)^2} = \frac{2x \cdot (x - 8) - (x^2 + 8)}{(x - 8)^2} = \frac{2x^2 - 16x - 8}{(x - 8)^2}$

~~$f(1) = \frac{2 - 16 - 8}{(-7)^2} = \frac{-22}{49}$~~   
 ~~$f(2) = \frac{8 - 32 - 8}{(-6)^2} = \frac{-32}{36} = -\frac{8}{9}$~~

$$5. f'(x) = \frac{x^2 - 16x - 8}{(x-8)^2} =$$

$$x^2 - 16x - 8 = 0$$

$$x = \frac{16 \pm \sqrt{256 + 32}}{2} = \frac{16 \pm \sqrt{288}}{2} = \frac{16 \pm 16.97}{2}$$

$$x_1 = 0.48$$

$$x_2 = 16.485$$

	$-\infty$	0.5	0.48	2	8	16.45	$+\infty$
$f'(x)$	-	-	-	-	-	+	
$f(x)$	↘	↘	↘	↘	↘	↗	

LOKALNI MINIMUM

$$f(16.45) = \frac{278.60}{16.45 - 8} = \frac{278.6}{8.45} = 32.98$$

$$f''(x) = \frac{(x^2 - 16x - 8)' \cdot (x-8)^2 - (x^2 - 16x - 8) \cdot (x-8)'}{(x-8)^4} = \frac{2x - 16 \cdot (x-8)^2 - (x^2 - 16x - 8) \cdot 2(x-8)}{(x-8)^4}$$

$$= \frac{2x - 16 \cdot (x^2 - 16x + 64) - (x^2 - 16x - 8) \cdot (2x - 16)}{(x-8)^4} = \frac{144x - 1152}{(x-8)^4}$$

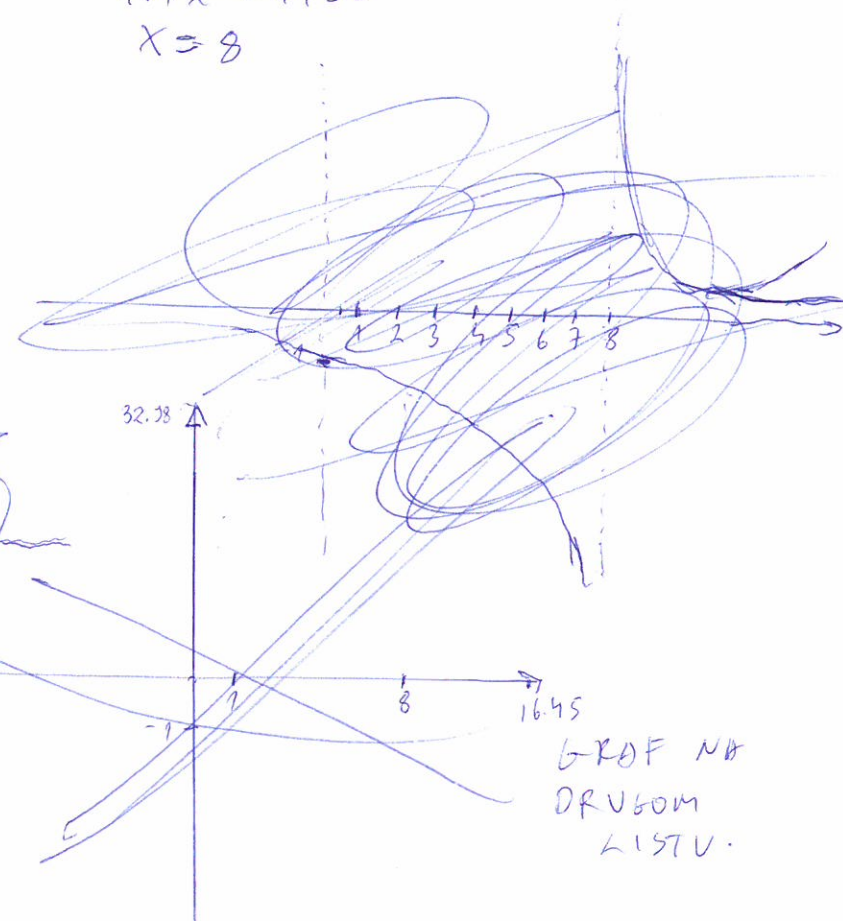
$$144x - 1152 = 0$$

$$x = 8$$

	$-\infty$	0	0.48	8	16.45	$+\infty$
$f''(x)$	-	-	-	+	+	
$f(x)$	↘	↘	↘	↗	↗	

KONKAVNO      KONVEKSNANO

KONVEKSNANO ZA  $x \in (8, +\infty)$   
 KONKAVNO ZA  $x \in (-\infty, 8)$



GRAF NA DRUGOM LISTU.

$$\textcircled{1} \left(\frac{z}{2}\right)^2 = z + 1 + i$$

$$\frac{z^2}{4} = z + 1 + i \quad | \cdot 4$$

$$z^2 = 4z + 4 + 4i$$

$$z^2 - 4z - 4 = 4i$$

$$z + \bar{z} - 4z = 4 + 4i$$

$$\sqrt{x^2 + y^2} = \sqrt{17} \Rightarrow \sqrt{x^2 + y^2} - 4x - 4 = 4 + 4i$$

$$\sqrt{x^2 + y^2} - 4x - 4 = 4 + 4i$$

$$\sqrt{x^2 + y^2} - 3x - 4 = 4 + 4i$$

$$x + 16 - 3x - 4 = 0 \Rightarrow -2x = -12 \Rightarrow x = 6$$

$$\sqrt{x^2 + y^2} - 4x - 4 = 4 + 4i \Rightarrow y = 4$$

$z = 6 + 4i$

$x$

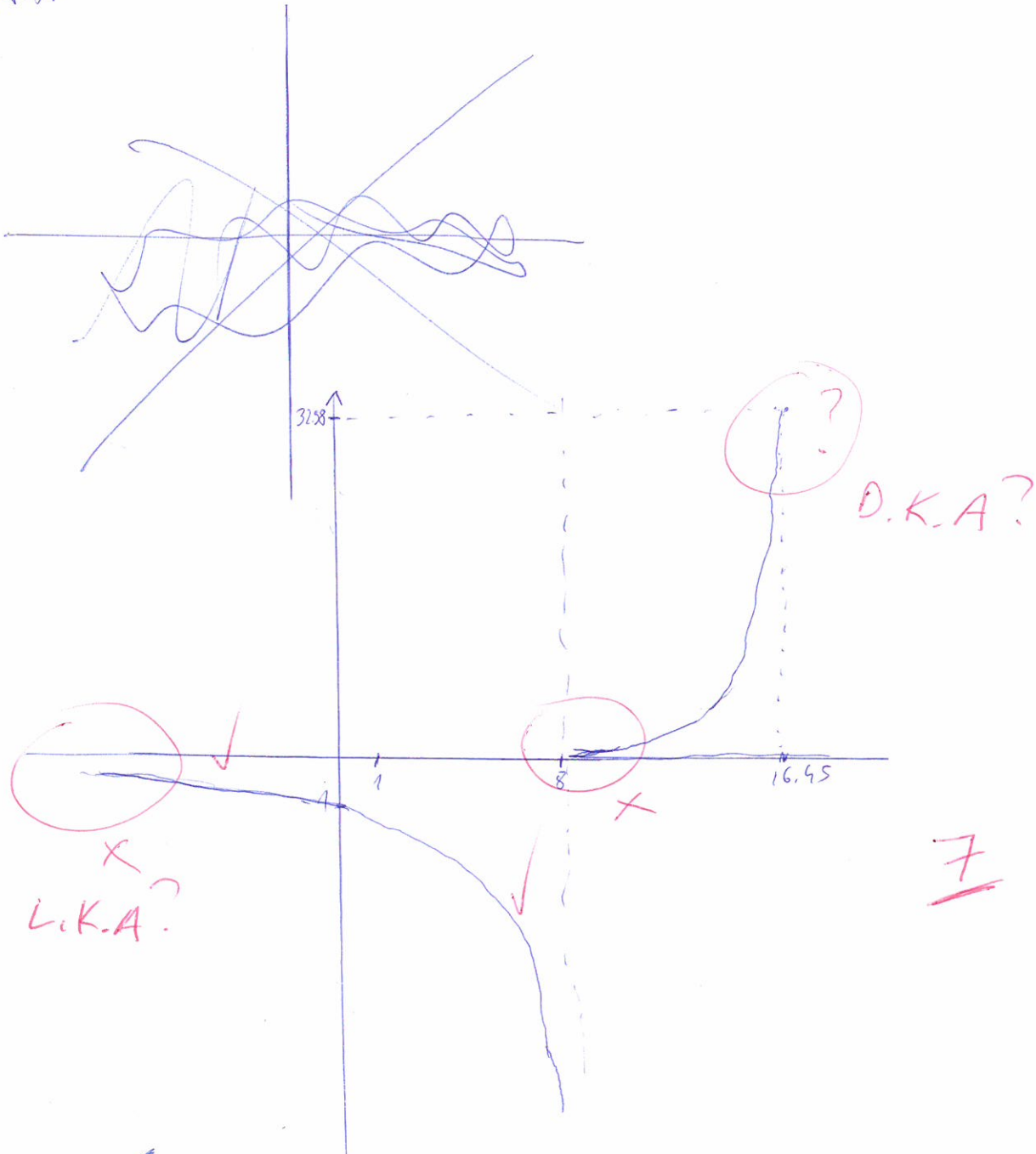
UVRSTI I PROVERI!

LUKA KNEŽEVIĆ

IME I PREZIME:

BROJ INDEKSA:

GRAF



4. b)

	$\infty$	$-7.316$	$-4$	$-0.68$	$+\infty$
		$-10$	$-5$	$-1$	$0$
$f'(x)$		$-$	$+$	$+$	
$f(x)$		$\searrow$	$\searrow$	$\nearrow$	$\nearrow$
			LOKALNI MINIMUM	MINIMUM	

$f(-4) = \sqrt{16 - 32 + 5} = \sqrt{-11}$  NIJE MINIMUM

NEMA LOKALNOG MAKSIMUMA

~~FUNKCIJA U TOČKI -4 IMA LOKALNI MINIMUM~~

NEMA LOKALNOG MINIMUMA JER TOČKA -4 NIJE U DOMENI FUNKCIJE



**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
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bodova

IME I PREZIME: Mevko Biloj

BROJ INDEKSA: 17-1-0061-2011

B8

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

1. Izračunaj sve kompleksne brojeve  $z$  takve da  $\left|\frac{z}{2}\right|^2 = z + 1 + i$ . Prikaži ih u kompleksnoj ravnini! ~~10+5~~

$$5x + 4z + 2t = 3$$

2. Riješiti sustav 4 jednačbe s 4 nepoznanice  $\begin{cases} x - y + 2z + t = 1 \\ 4x + y + 2z = 1 \\ x + y + z + t = 0 \end{cases}$  i obavezno provjeri rješenje: 10+5

$$x + y + z + t = 0$$

3. Za funkciju  $f(x) = \frac{\sin(2x)}{x}$  odrediti koliko iznosi  $f'(\pi)$ . ~~8+2~~

4. Za funkciju:  $g(x) = \sqrt{x^2 + 8x + 5}$  treba:

(a) ispitati domen

~~5~~

(b) pronaći lokalne ekstreme

11

(c) ispitati asimptote

14

5. Na temelju ispitivanja toka skicirati graf funkcije  $e$   $h(x) = \frac{x^2 + 8}{x - 8}$  ~~20 (graf)~~

6. Izračunati i obavezno na neki način provjeriti koliko iznosi  $\lim_{x \rightarrow -1} \left( \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$ . ~~8+2~~

Ukupno:

~~0~~

3.  $f(x) = \frac{\sin(2x)}{x}$

$$f(x)' = \frac{(\sin 2x)' \cdot x - x' \cdot (\sin 2x)}{x^2}$$

$$f(x)' = \frac{\cos 2x \cdot 2 \cdot x - 1 \cdot \sin 2x}{x^2}$$

$$f(x) = \frac{\cos 4x^2 - \sin 2x}{x^2}$$

$$f'(\pi) = \frac{0 \cdot \pi - 0 \cdot (\sin 2\pi)}{0}$$

$$f'(\pi) = 0$$

9. 0)  $Df =$  svi realni brojevi od

0 do  $+\infty$

$$\frac{-8 \pm \sqrt{64 - 4 \cdot 5}}{2} = \frac{-8 \pm \sqrt{44}}{2}$$

$$b) \textcircled{1} \sqrt{x^2 + 8x + 5} = 0 \quad | \quad /^2$$

$$\frac{-8 \pm \sqrt{64}}{2}$$

=

$$x_1 = 14,6 //$$

$$x_2 = 1,36 //$$

$$f_x = 0$$



$$\textcircled{2} f(0) = \sqrt{0^2 + 8 \cdot 0 + 5} =$$

$$f(0) = \sqrt{5} //$$



$$c) f(x) = \sqrt{x^2 + 8x + 5}$$

$$f'(x) = \frac{1}{2\sqrt{x^2 + 8x + 5}} = \frac{1}{2\sqrt{0 + 0 + 5}} = \frac{1}{2\sqrt{5}} =$$

$$d) = f''(x) = \frac{(1)' \cdot (2\sqrt{x^2 + 8x + 5}) - 1 \cdot (2\sqrt{x^2 + 8x + 5})'}{(2\sqrt{x^2 + 8x + 5})^2}$$

$$= \frac{\frac{1}{-2\sqrt{x^2 + 8x + 5}}}{(2\sqrt{x^2 + 8x + 5})^2} = \frac{1}{2\sqrt{x^2 + 8x + 5}^3}$$



(2.)

$$\begin{bmatrix} 5 & + & 0 & - & 4 & + & 2 & | & 3 \\ 1 & - & 1 & + & 2 & + & 1 & | & 1 \\ 4 & + & 1 & - & 2 & + & 0 & | & 1 \\ 1 & + & 1 & + & 1 & + & 1 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & - & 1 & + & 2 & + & 1 & | & 1 \\ 5 & 0 & - & 4 & 2 & | & 3 \\ 4 & 1 & 2 & 0 & | & 1 \\ 1 & 1 & 1 & 1 & | & 0 \end{bmatrix} \begin{matrix} -5 & -4 & -1 \\ \swarrow \\ \swarrow \\ \swarrow \end{matrix}$$

$$\begin{bmatrix} 1 & - & 1 & + & 2 & + & 1 & | & 1 \\ 0 & 5 & 2 & 6 & - & 3 & | & - & 2 \\ 0 & - & 5 & - & 2 & 6 & - & 4 & | & - & 3 \\ 0 & - & 2 & - & 1 & 0 & | & - & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & - & 1 & + & 2 & 1 & | & 1 \\ 0 & 5 & - & 6 & - & 3 & | & - & 2 \\ 0 & 0 & 0 & - & 1 & - & 1 & | & 0 \\ 0 & 2 & - & 1 & 0 & | & - & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & - & 1 & + & 2 & 1 & | & 1 \\ 0 & 1 & - & 4 & - & 3 & | & 0 \\ 0 & 0 & 0 & - & 1 & - & 1 & | & - & 1 \\ 0 & 2 & - & 1 & 0 & | & - & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & - & 1 & + & 2 & 1 & | & 1 \\ 0 & 1 & - & 4 & - & 3 & | & 0 \\ 0 & 0 & 0 & - & 1 & - & 1 & | & - & 1 \\ 0 & 0 & 2 & 6 & 1 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & - & 1 & 2 & 1 & | & 1 \\ 0 & 1 & - & 4 & - & 3 & | & 0 \\ 0 & 0 & 2 & 6 & 1 & | & 0 \\ 0 & 0 & 0 & - & 1 & | & - & 1 \end{bmatrix} \begin{matrix} :7 \\ :7 \\ :7 \end{matrix} \rightarrow \begin{bmatrix} 1 & - & 1 & 2 & 1 & | & 1 \\ 0 & 1 & - & 4 & - & 3 & | & 0 \\ 0 & 0 & 1 & \frac{5}{7} & | & 1 \\ 0 & 0 & 0 & - & 1 & | & - & 1 \end{bmatrix} \begin{matrix} + \frac{6}{7} \\ -3 + 1 \end{matrix}$$

$$\begin{bmatrix} 1 & - & 1 & 2 & 0 & | & - & 1 \\ 0 & 1 & - & 4 & 0 & | & 3 \\ 0 & 0 & 1 & 0 & | & \frac{1}{7} \\ 0 & 0 & 0 & - & 1 & | & - & 1 \end{bmatrix} \begin{matrix} -4 \\ -2 \end{matrix} \rightarrow \begin{bmatrix} 1 & - & 1 & 0 & 0 & | & - & \frac{9}{7} \\ 0 & 1 & 0 & 0 & | & \frac{17}{7} \\ 0 & 0 & 1 & 0 & | & \frac{1}{7} \\ 0 & 0 & 0 & - & 1 & | & - & 1 \end{bmatrix} \begin{matrix} +1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & | & \frac{8}{7} \\ 0 & 1 & 0 & 0 & | & \frac{17}{7} \\ 0 & 0 & 1 & 0 & | & \frac{1}{7} \\ 0 & 0 & 0 & 1 & | & - & 1 \end{bmatrix} \quad \times$$

PROVJERA?

$$5. \left( \frac{5}{7} - 4 + 2 + 1 \right) =$$

IME I PREZIME: Miroslav Bilušić

BROJ INDEKSA: 17-1-0061-2011

(6.) 
$$\lim_{x \rightarrow -1} \left( \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$$

$$\lim_{x \rightarrow -1} \frac{\sqrt{-1^2 + 8 + 7} - 3 - 1}{-1^2 + 4 + 3} =$$

$$\lim_{x \rightarrow -1} \frac{\sqrt{1 + 8 + 7} - 3 - 1}{1 - 4 + 3} \Rightarrow \frac{0}{0}$$

l'Hôpital

$$\lim_{x \rightarrow -1} \frac{(\sqrt{x^2 - 8x + 7} - 3 + x)'}{(x^2 + 4x + 3)'}$$

$$\lim_{x \rightarrow -1} \frac{1}{2\sqrt{x^2 - 8x + 7} + 1} \cdot (-2x - 8)$$

$$\lim_{x \rightarrow -1} \frac{2x + 4}{1 + 2\sqrt{x^2 - 8x + 7}}$$

$$\lim_{x \rightarrow -1} \frac{1}{1 + 2\sqrt{x^2 - 8x + 7}}$$

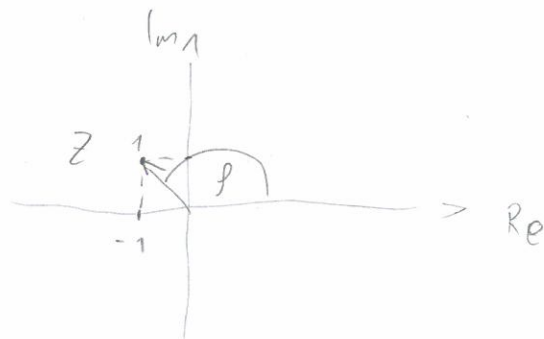
$$\lim_{x \rightarrow -1} \frac{1}{2x + 4} = \frac{1}{2}$$

IME I PREZIME: Marko Biluš

BROJ INDEKSA: 17-1-0061-2011

(1.)  $\left| \frac{z}{2} \right|^2 = z + 1 + i$

$z = \underset{\text{Re } z}{-1} - \underset{\text{Im } z}{i}$



$r = \left| \frac{z}{2} \right|^2 = \sqrt{(-1)^2 + 1^2}$  ~~X~~

$\phi = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$r = \left| \frac{z}{2} \right|^2 = \sqrt{2}$

$k = 0, 1, 2, 3$

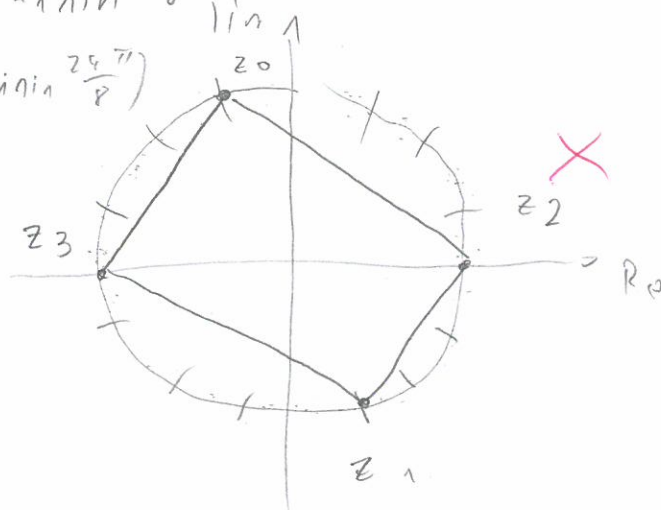
$z_0 = \sqrt[4]{2} \left( \cos \frac{\frac{5\pi}{4} + 2 \cdot 0 \cdot \pi}{2} + i \sin \frac{\frac{5\pi}{4} + 2 \cdot 0 \cdot \pi}{2} \right)$

$z_0 = \sqrt[4]{2} \cdot \left( \cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8} \right)$

$z_1 = \sqrt[4]{2} \cdot \left( \cos \frac{\frac{13\pi}{4} + 2 \cdot 1 \cdot \pi}{2} + i \sin \frac{\frac{13\pi}{4} + 2 \cdot 1 \cdot \pi}{2} \right) = \sqrt[4]{2} \cdot \left( \cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8} \right)$

$z_2 = \sqrt[4]{2} \cdot \left( \cos \frac{\frac{21\pi}{4} + 2 \cdot 2 \cdot \pi}{2} + i \sin \frac{\frac{21\pi}{4} + 2 \cdot 2 \cdot \pi}{2} \right)$

$z_3 = \sqrt[4]{2} \cdot \left( \cos \frac{\frac{29\pi}{4} + 2 \cdot 3 \cdot \pi}{2} + i \sin \frac{\frac{29\pi}{4} + 2 \cdot 3 \cdot \pi}{2} \right)$

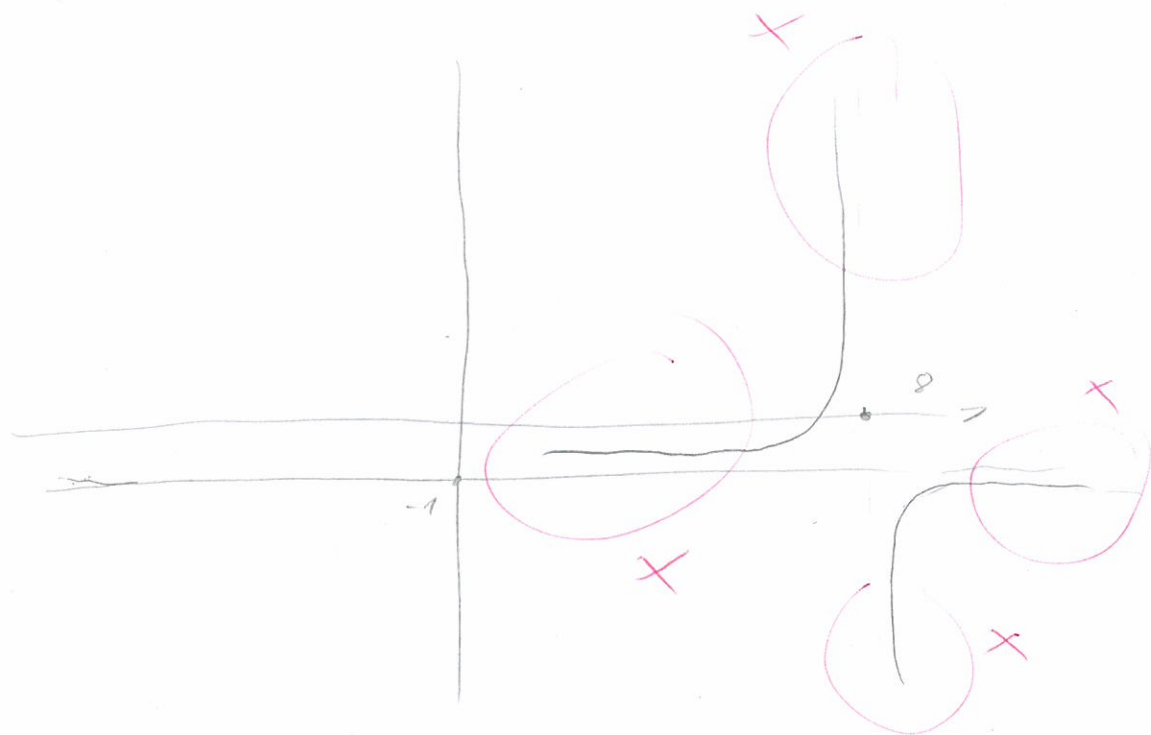


(5)  $Df =$  svi realni osim  $+8$



$$\frac{0^2 + 8}{0 - 8} = \frac{8}{-8} = \underline{\underline{-1}}$$

$$\frac{2x \cdot (x-8) - (x^2+8) \cdot 1}{(x-8)^2} = \frac{2x^2 - 16x - x^2 - 8}{(x-8)^2} = \frac{x^2 - 16x - 8}{(x-8)^2}$$



**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

B8

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

IME I PREZIME: MATE ĆOSIĆ

BROJ INDEKSA: 55924

ZAOKRUŽITI AKO ŽELITE:

ustmeni kod prof. Uglešića

1. Izračunaj sve kompleksne brojeve  $z$  takve da  $\left|\frac{z}{2}\right|^2 = z + 1 + i$ . Prikaži ih u kompleksnoj ravnini!

~~10+5~~

$$5x + 4z + 2t = 3$$

2. Riješiti sustav 4 jednačbe s 4 nepoznanice  $x - y + 2z + t = 1$  i obavezno provjeri rješenje:

10+5

$$4x + y + 2z = 1$$

$$x + y + z + t = 0$$

3. Za funkciju  $f(x) = \frac{\sin(2x)}{x}$  odrediti koliko iznosi  $f'(\pi)$ .

8+2

4. Za funkciju:  $g(x) = \sqrt{x^2 + 8x + 5}$  treba:

- (a) ispitati domenu
- (b) pronaći lokalne ekstreme
- (c) ispitati asimptote

5

11

14

5. Na temelju ispitivanja toka skicirati graf funkcije  $h(x) = \frac{x^2 + 8}{x - 8}$

20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi  $\lim_{x \rightarrow -1} \left( \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$ .

8+2

$$1. \left| \frac{z}{2} \right|^2 = z + 1 + i$$

$$\sqrt{x^2 + iy^2} = x + iy + 1 + i$$

$$\frac{x^2}{4} + \frac{iy^2}{4} = x + iy + 1 + i \quad | \cdot 4$$

$$x^2 + iy^2 = 4x + 4iy + 4 + 4i$$

$$x^2 + iy^2 = 4x + 4iy + 4 + 4i$$

$$4iy = 4 + 4i \quad | :4$$

$$y = 1$$

$$y = -1$$

$$x^2 + y^2 = 4x + 4$$

$$x^2 + 1 = 4x + 4$$

$$x^2 - 4x - 3 = 0 \quad \times$$

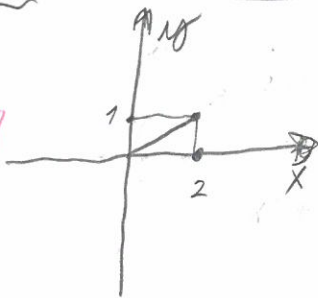
$$x^2 - 4x - 3 = 0$$

$$x_{1,2} = \frac{4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-3)}}{2}$$

$$x_{1,2} = \frac{4 \pm \sqrt{16 - 12}}{2}$$

$$x_{1,2} = 2$$

$$\boxed{\begin{matrix} y = 1 \\ x = 2 \end{matrix}}$$



Ukupno:

30

$$2. \begin{bmatrix} 5 & 0 & 4 & 2 & | & 3 \\ 1 & -1 & 2 & 1 & | & 1 \\ 4 & 1 & 2 & 0 & | & 1 \\ 1 & 1 & 1 & 1 & | & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 2 & 1 & | & 1 \\ 5 & 0 & 4 & 2 & | & 3 \\ 4 & 1 & 2 & 0 & | & 1 \\ 1 & 1 & 1 & 1 & | & 0 \end{bmatrix} \xrightarrow{\begin{matrix} -5 \\ -4 \\ -1 \end{matrix}} \begin{bmatrix} 1 & -1 & 2 & 1 & | & 1 \\ 0 & 5 & -6 & -3 & | & -2 \\ 0 & 5 & -6 & -4 & | & -3 \\ 0 & 2 & -1 & 0 & | & -1 \end{bmatrix} \cdot \frac{1}{5}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 & | & 1 \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & | & -\frac{2}{5} \\ 0 & 5 & -6 & -4 & | & -3 \\ 0 & 2 & -1 & 0 & | & -1 \end{bmatrix} \xrightarrow{\begin{matrix} -1 \\ -5 \\ -2 \end{matrix}} \begin{bmatrix} 1 & 0 & \frac{5}{5} & \frac{2}{5} & | & \frac{3}{5} \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & | & -\frac{2}{5} \\ 0 & 0 & 0 & -1 & | & -1 \\ 0 & 0 & \frac{2}{5} & \frac{6}{5} & | & -\frac{1}{5} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & \frac{4}{5} & \frac{2}{5} & | & \frac{3}{5} \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & | & -\frac{2}{5} \\ 0 & 0 & \frac{2}{5} & \frac{6}{5} & | & -\frac{1}{5} \\ 0 & 0 & 0 & -1 & | & -1 \end{bmatrix} \xrightarrow{\cdot \frac{5}{2}} \begin{bmatrix} 1 & 0 & \frac{4}{5} & \frac{2}{5} & | & \frac{3}{5} \\ 0 & 1 & -\frac{6}{5} & -\frac{3}{5} & | & -\frac{2}{5} \\ 0 & 0 & 1 & \frac{6}{2} & | & -\frac{1}{2} \\ 0 & 0 & 0 & -1 & | & -1 \end{bmatrix} \begin{matrix} \frac{6}{5} \\ -\frac{4}{5} \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{2}{7} & | & \frac{5}{7} \\ 0 & 1 & 0 & \frac{3}{7} & | & -\frac{4}{7} \\ 0 & 0 & 1 & \frac{6}{7} & | & -\frac{1}{7} \\ 0 & 0 & 0 & -1 & | & -1 \end{bmatrix} \xrightarrow{(-1)} \begin{bmatrix} 1 & 0 & 0 & -\frac{2}{7} & | & \frac{5}{7} \\ 0 & 1 & 0 & \frac{3}{7} & | & -\frac{4}{7} \\ 0 & 0 & 1 & \frac{6}{7} & | & -\frac{1}{7} \\ 0 & 0 & 0 & 1 & | & 1 \end{bmatrix} \begin{matrix} \frac{2}{7} \\ -\frac{3}{7} \\ -\frac{6}{7} \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & 0 & | & -1 \\ 0 & 0 & 1 & 0 & | & -1 \\ 0 & 0 & 0 & 1 & | & 1 \end{bmatrix} \quad \begin{matrix} x=1 \\ y=-1 \\ z=-1 \\ t=1 \end{matrix}$$

$$5 \cdot 1 + 0 \cdot (-1) + 4 \cdot (-1) + 2 \cdot 1 = 3$$

$$3 = 3$$

$$1 \cdot 1 + (-1) \cdot (-1) + 2 \cdot (-1) + 1 \cdot 1 = 1$$

$$1 = 1$$

$$4 \cdot 1 + 1 \cdot (-1) + 2 \cdot (-1) + 0 \cdot 1 = 1$$

$$1 = 1$$

$$1 \cdot 1 + 1 \cdot (-1) + 1 \cdot (-1) + 1 \cdot 1 = 0$$

$$0 = 0$$



3.

$$f(x) = \frac{\sin(2x)}{x}$$

$$f'(x) = \frac{\cos 2x \cdot 2 \cdot x - \sin(2x) \cdot 1}{x^2}$$

$$f'(x) = \frac{2x \cos(2x) - \sin 2x}{x^2}$$

$$f'(\pi) = 0,637 \quad \checkmark$$

4. DOME NA

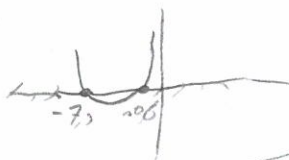
$$g(x) = \sqrt{x^2 + 8x + 5}$$

$$x^2 + 8x + 5 \geq 0$$

$$x_{1,2} = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 5}}{2} =$$

$$x_1 = -0,68$$

$$x_2 = -7,32$$



$$Dx = x \in \left[-\infty, -7,32\right] \cup \left[0,68, +\infty\right) \quad \checkmark$$

$$g''(x) = \frac{1 \cdot \sqrt{x^2 + 8x + 5} - (x+4) \cdot \frac{1}{2\sqrt{x^2 + 8x + 5}} \cdot 2x+8}{(x^2 + 8x + 5)^2}$$

$$g''(x) = \frac{\sqrt{x^2 + 8x + 5} - \frac{(x+4)(x+4)}{2\sqrt{x^2 + 8x + 5}}}{x^2 + 8x + 5}$$

LOR. EKSTRA

$$g(x) = \sqrt{x^2 + 8x + 5}$$

$$g'(x) = \frac{1}{2\sqrt{x^2 + 8x + 5}} \cdot 2x + 8$$

$$g'(x) = \frac{2x + 8}{2\sqrt{x^2 + 8x + 5}} = \frac{2(x+4)}{2\sqrt{x^2 + 8x + 5}}$$

~~WEMA~~ X

$$g'(x) = \frac{x+4}{\sqrt{x^2 + 8x + 5}}$$

ASIMPTOTE ?

N.T

$$x = -4$$

$$\begin{bmatrix} 5 & 0 & 4 & 2 & 1 & 3 \\ 1 & -1 & 2 & 1 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 & 1 \\ 5 & 0 & 4 & 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 & 1 \\ 0 & -3 & -2 & -4 & 1 & -3 \\ 0 & -5 & -1 & -3 & 1 & -2 \end{bmatrix} \cdot \frac{1}{2}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & | & -\frac{1}{2} \\ 0 & -3 & -2 & -4 & | & -3 \\ 0 & -5 & -1 & -3 & | & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & \frac{3}{2} & 1 & | & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & | & -\frac{1}{2} \\ 0 & 0 & -\frac{7}{2} & -4 & | & -\frac{9}{2} \\ 0 & 0 & -\frac{7}{2} & -3 & | & -\frac{9}{2} \end{bmatrix} \cdot \left(-\frac{7}{2}\right)$$

$$\begin{bmatrix} 1 & 0 & \frac{3}{2} & 1 & | & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & 0 & | & -\frac{1}{2} \\ 0 & 0 & 1 & 14 & | & \frac{63}{4} \\ 0 & 0 & -\frac{7}{2} & -3 & | & -\frac{9}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -20 & | & -\frac{125}{8} \\ 0 & 1 & 0 & 2 & | & \frac{63}{8} \\ 0 & 0 & 1 & 14 & | & \frac{63}{4} \\ 0 & 0 & 0 & -\frac{21}{2} & | & \frac{405}{8} \end{bmatrix} \cdot \left(-\frac{21}{2}\right)$$

$$\begin{bmatrix} 1 & 0 & 0 & -20 & | & -\frac{185}{8} \\ 0 & 1 & 0 & 2 & | & \frac{63}{8} \\ 0 & 0 & 1 & 14 & | & \frac{63}{4} \\ 0 & 0 & 0 & 1 & | & -\frac{8505}{16} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & 0 & | & 1 \\ 0 & 0 & 1 & 0 & | & 1 \\ 0 & 0 & 0 & 1 & | & 1 \end{bmatrix}$$



**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
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bodova

IME I PREZIME:

JOSIP MARIĆ

BROJ INDEKSA:

17-2-0227-2012

B8

ZAKRUŽITI AKO ŽELITE:

ustmeni kod prof. Uglešića

1. Izračunaj sve kompleksne brojeve  $z$  takve da  $\left|\frac{z}{2}\right|^2 = z + 1 + i$ . Prikaži ih u kompleksnoj ravnini! 10+5

2. Riješiti sustav 4 jednačbe s 4 nepoznanice  $\begin{cases} 5x + 4z + 2t = 3 \\ x - y + 2z + t = 1 \\ 4x + y + 2z = 1 \\ x + y + z + t = 0 \end{cases}$  i obavezno provjeri rješenje: 10+5

3. Za funkciju  $f(x) = \frac{\sin(2x)}{x}$  odrediti koliko iznosi  $f'(\pi)$ . 8+2

4. Za funkciju:  $g(x) = \sqrt{x^2 + 8x + 5}$  treba:  
(a) ispitati domenu 5  
(b) pronaći lokalne ekstreme 11  
(c) ispitati asimptote 14

5. Na temelju ispitivanja toka skicirati graf funkcije  $e h(x) = \frac{x^2 + 8}{x - 8}$  20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi  $\lim_{x \rightarrow -1} \left( \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$ . 8+2

Ukupno:

15

2.

$$\begin{bmatrix} 5 & 0 & 4 & 2 & | & 3 \\ 1 & -1 & 2 & 1 & | & 1 \\ 4 & 1 & 2 & 0 & | & 1 \\ 1 & 1 & 1 & 1 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 1 & -1 & 2 & 1 & | & 1 \\ 4 & 1 & 2 & 0 & | & 1 \\ 5 & 0 & 4 & 2 & | & 3 \end{bmatrix} \begin{matrix} \\ \text{II} - \text{I} \\ \text{III} - 4\text{I} \\ \text{IV} - 5\text{I} \end{matrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 0 & -2 & 1 & 0 & | & 1 \\ 0 & -3 & -2 & -4 & | & 1 \\ 0 & -5 & -1 & -3 & | & 3 \end{bmatrix} \begin{matrix} \\ \\ \text{II} - \text{III} \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 0 & 1 & 3 & 4 & | & 0 \\ 0 & -3 & -2 & -4 & | & 1 \\ 0 & -5 & -1 & -3 & | & 3 \end{bmatrix} \begin{matrix} \text{I} - \text{II} \\ \\ \text{III} + 3\text{II} \\ \text{IV} + 5\text{II} \end{matrix} \sim \begin{bmatrix} 1 & 0 & -2 & -3 & | & 0 \\ 0 & 1 & 3 & 4 & | & 0 \\ 0 & 0 & 7 & 8 & | & 1 \\ 0 & 0 & 14 & 17 & | & 3 \end{bmatrix} \begin{matrix} \\ \\ \text{III} : 7 \\ \text{IV} - 14\text{III} \end{matrix} \sim \begin{bmatrix} 1 & 0 & -2 & -3 & | & 0 \\ 0 & 1 & 3 & 4 & | & 0 \\ 0 & 0 & 1 & \frac{8}{7} & | & \frac{1}{7} \\ 0 & 0 & 14 & 17 & | & 3 \end{bmatrix} \begin{matrix} \\ \\ \\ \text{IV} - 14\text{III} \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & -3 & | & 0 \\ 0 & 1 & 3 & 4 & | & 0 \\ 0 & 0 & 1 & \frac{8}{7} & | & \frac{1}{7} \\ 0 & 0 & 0 & 1 & | & 1 \end{bmatrix} \begin{matrix} \text{I} + 3\text{IV} \\ \text{II} - 4\text{IV} \\ \text{III} - \frac{8}{7}\text{IV} \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 & | & 13 \\ 0 & 1 & 3 & 0 & | & -4 \\ 0 & 0 & 1 & 0 & | & -1 \\ 0 & 0 & 0 & 1 & | & 1 \end{bmatrix} \begin{matrix} \text{I} + 2\text{III} \\ \text{II} - 3\text{III} \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & 0 & | & -1 \\ 0 & 0 & 1 & 0 & | & -1 \\ 0 & 0 & 0 & 1 & | & 1 \end{bmatrix}$$

x = 1  
y = -1  
z = -1  
t = 1

PROVJERA:

$$5 \cdot 1 + 4 \cdot (-1) + 2 \cdot 1 = 3 \quad \checkmark$$

$$1 - 1 + (-1) + 1 = 0 \quad \checkmark$$

5.  $f(x) = \frac{x^2+8}{x-8}$

1. Domena

$x-8 \neq 0$

$x \neq 8$

DF;  $\mathbb{R} \setminus \{8\}$

2. parnost, nepar.

$f(-x) = \frac{-x^2+8}{-x-8}$

$f(x) = \frac{x^2+8}{x-8}$

funkcija nije ni parna ni neparna

GRAF?

3. nultočke

$x^2+8=0$

$x(x+8)=0$

$x_1=0, x_2=-8$

4. asimptote

H.D

$\lim_{x \rightarrow +\infty} f(x) = \frac{x^2+8}{x-8} \stackrel{/:x^2}{=} \frac{1+\frac{8}{x^2}}{1-\frac{8}{x}}$

$\lim_{x \rightarrow +\infty} f(x) = \frac{1+0}{1-0} = +\infty$

4.  $g(x) = \sqrt{x^2+8x+5}$

1. Domena

$x^2+8x+5 \geq 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{-8 \pm \sqrt{64-20}}{2}$

$x_1 = \frac{-8-2}{2} = -5$

$x_2 = \frac{-8+2}{2} = -3$

$x_1 = -5$   
 $x_2 = -3$

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

B8

IME I PREZIME: **TOMISLAV PERKOVIĆ**

BROJ INDEKSA: ~~5028~~

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

17-2-0229-2012

- Izračunaj sve kompleksne brojeve  $z$  takve da  $\left|\frac{z}{2}\right|^2 = z + 1 + i$ . Prikaži ih u kompleksnoj ravnini! 10+5
- Riješiti sustav 4 jednačbe s 4 nepoznane  $\begin{cases} 5x + 4z + 2t = 3 \\ x - y + 2z + t = 1 \\ 4x + y + 2z = 1 \\ x + y + z + t = 0 \end{cases}$  i obavezno provjeri rješenje: ~~10+5~~
- Za funkciju  $f(x) = \frac{\sin(2x)}{x}$  odrediti koliko iznosi  $f'(\pi)$ . 8+2
- Za funkciju:  $g(x) = \sqrt{x^2 + 8x + 5}$  treba:
  - ispitati domenu 5
  - pronaći lokalne ekstreme 11
  - ispitati asimptote 14
- Na temelju ispitivanja toka skicirati graf funkcije  $e h(x) = \frac{x^2 + 8}{x - 8}$  20 (graf)
- Izračunati i obavezno na neki način provjeriti koliko iznosi  $\lim_{x \rightarrow -1} \left( \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$ . ~~8+2~~

Ukupno:

**5**

②

$$\begin{bmatrix} 5 & 4 & 0 & 2 & | & 3 \\ 1 & -1 & 2 & 1 & | & 1 \\ 4 & 1 & 2 & 0 & | & 1 \\ 1 & 1 & 1 & 1 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 1 & -1 & 2 & 1 & | & 1 \\ 4 & 1 & 2 & 0 & | & 1 \\ 5 & 4 & 0 & 2 & | & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 0 & -2 & 1 & 0 & | & 1 \\ 0 & -3 & -1 & -1 & | & 1 \\ 0 & -1 & -1 & 1 & | & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & -2 & | & 3 \\ 0 & 0 & 11 & 5 & | & -8 \\ 0 & 0 & 13 & 5 & | & 19 \\ 0 & -1 & -5 & -3 & | & 13 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & -2 & | & 3 \\ 0 & -1 & -5 & -3 & | & 13 \\ 0 & 0 & 13 & 5 & | & 19 \\ 0 & 0 & 11 & 5 & | & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & -2 & | & 3 \\ 0 & -1 & -5 & -3 & | & 13 \\ 0 & 0 & 1 & \frac{5}{13} & | & \frac{19}{13} \\ 0 & 0 & 11 & 5 & | & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -\frac{6}{13} & | & \frac{45}{13} \\ 0 & -1 & 0 & -\frac{14}{13} & | & -\frac{45}{13} \\ 0 & 0 & 1 & \frac{5}{13} & | & \frac{19}{13} \\ 0 & 0 & 0 & 1 & | & -\frac{5}{13} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & | & \frac{65}{13} \\ 0 & -1 & 0 & 0 & | & -\frac{5}{13} \\ 0 & 0 & 1 & 0 & | & \frac{9}{13} \\ 0 & 0 & 0 & 1 & | & -\frac{5}{13} \end{bmatrix}$$

PROVJERA?

④  $g(x) = \sqrt{x^2 + 8x + 5}$

a)  $x^2 + 8x + 5$   
 $a=1 \quad b=8 \quad c=5$   
 $x_{1,2} = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 1 \cdot 5}}{2} = \frac{-8 \pm \sqrt{64 - 20}}{2}$   
 $x_{1,2} = \frac{-8 \pm \sqrt{44}}{2}$   
 $x_1 = \frac{-8 + 6,63}{2} = -0,68 \quad x_2 = \frac{-8 - 6,63}{2} = -7,31$

$D_f \in (-\infty, -7,31) \cup (-0,68, +\infty)$

UKLJUČUJE GRANICE

$$\textcircled{6} \quad \lim_{x \rightarrow -1} \left( \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right) = \frac{\sqrt{(-1)^2 - 8(-1) + 7} - 3 - 1}{(-1)^2 + 4(-1) + 3} = \frac{\sqrt{1 + 8 + 7} - 3 - 1}{1 - 4 + 3} = \frac{4 - 3 - 1}{1 - 4 + 3} = \frac{0}{0}$$

**MATEMATIKA 1:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

B8

IME I PREZIME: TONI UGLEŠIĆ

BROJ INDEKSA: 17-1-0065-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

1. Izračunaj sve kompleksne brojeve  $z$  takve da  $\left|\frac{z}{2}\right|^2 = z + 1 + i$ . Prikaži ih u kompleksnoj ravnini! ~~10+5~~

$$5x + 4z + 2t = 3$$

2. Riješiti sustav 4 jednačbe s 4 nepoznane  $x - y + 2z + t = 1$  i obavezno provjeri rješenje: ~~10+5~~

$$4x + y + 2z = 1$$

$$x + y + z + t = 0$$

3. Za funkciju  $f(x) = \frac{\sin(2x)}{x}$  odrediti koliko iznosi  $f'(\pi)$ . 8+2

4. Za funkciju:  $g(x) = \sqrt{x^2 + 8x + 5}$  treba:

(a) ispitati domenu 5

(b) pronaći lokalne ekstreme 11

(c) ispitati asimptote 14

5. Na temelju ispitivanja toka skicirati graf funkcije  $e$   $h(x) = \frac{x^2 + 8}{x - 8}$  20 (graf)

6. Izračunati i obavezno na neki način provjeriti koliko iznosi  $\lim_{x \rightarrow -1} \left( \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right)$ . ~~8+2~~

Ukupno:

~~0~~

$$2. \left| \begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right| \begin{array}{l} I-III \\ \\ \\ \end{array} \left| \begin{array}{cccc|c} 1 & 1 & 2 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right| \begin{array}{l} \\ \\ \\ \end{array} \left| \begin{array}{cccc|c} 1 & 1 & 2 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & -1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right| \begin{array}{l} \\ \\ III-IV \\ \end{array}$$

$$\left| \begin{array}{cccc|c} 1 & 1 & 2 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & -2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right| \quad \times$$

$$6. \lim_{x \rightarrow -1} \left( \frac{\sqrt{x^2 - 8x + 7} - 3 + x}{x^2 + 4x + 3} \right) = \lim_{x \rightarrow -1} \left( \frac{\sqrt{(-1)^2 - 8(-1) + 7} - 3 + (-1)}{(-1)^2 + 4(-1) + 3} \right) = \lim_{x \rightarrow -1} \left( \frac{4 - 3 - 1}{1 - 4 + 3} \right) =$$

$$= \lim_{x \rightarrow -1} \frac{0}{0} \quad ?$$

$$1. \left| \frac{z}{2} \right|^2 = z + 1 + i$$

$$\left| \frac{1}{2} \right|^2 = 1 + 1 + i$$

$$z = 1$$

$$\frac{1}{4} = 2 + i$$

$$\frac{1}{4} - 2 = i$$

$$i = -\frac{7}{4} \quad \times$$

