MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇓⇓ $\mathcal{H}2$

IME I PREZIME: ELENA BEG **BROJ INDEKSA:** 17-2-0181-2012

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^3 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-3}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-3x-(3+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf)
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 3 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti: $\lim_{x\to -1} \dfrac{|x|}{x}$. 5

Ukupno: 71

(1.) $z^4 - (4-i)^3 = 0$

$z^4 = (4-i)^3$

$|r| = \sqrt{x^2+y^2}$

$|r| = \sqrt{16+1} = \sqrt{17}$ ✓

$\operatorname{tg}\varphi = -\frac{1}{4}$ ✓

$\varphi = -0{,}2449 + 2\pi$

$\varphi = 6{,}0382$

$(4-i)^3 = (\sqrt{17})^3 (\cos 6{,}0382\cdot 3 + i\sin 6{,}0382\cdot 3)$

$(4-i)^3 = (\sqrt{17})^3 (\cos 18{,}1146 + i\sin 18{,}1146)$

$(4-i)^3 = 17\sqrt{17}(\cos 18{,}1146 + i\sin 18{,}1146)$ ✓

$z^4 = \sqrt[4]{17\sqrt{17}} \left(\cos \frac{18{,}1146 + 2k\pi}{4} + i\sin \frac{18{,}1146 + 2k\pi}{4}\right)$

za $k=0$

$z_1 = \sqrt[4]{17\sqrt{17}} (\cos 4{,}528 + i\sin 4{,}528)$ ✓

za $k=1$

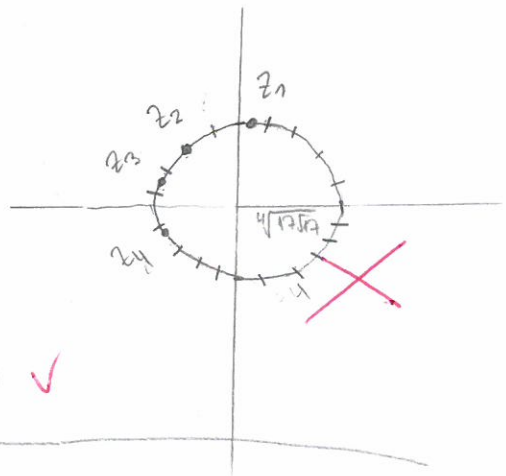
$z_2 = \sqrt[4]{17\sqrt{17}} (\cos 6{,}099 + i\sin 6{,}099)$ ✓

za $k=2$

$z_3 = \sqrt[4]{17\sqrt{17}} (\cos 7{,}6702 + i\sin 7{,}6702)$ ✓

za $k=3$

$z_4 = \sqrt[4]{17\sqrt{17}} (\cos 9{,}2410 + i\sin 9{,}2410)$ ✓



(2) $f(x) = x - \sqrt{x^2-3}$

$x^2 - 3 \geq 0$

$x^2 \geq 3$

$|x| \geq \sqrt{3}$

$D_f: \langle -\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty\rangle$ ✓

$f(-\sqrt{3}) = -\sqrt{3} - \sqrt{(\sqrt{3})^2 - 3} = -\sqrt{3}$

$f(\sqrt{3}) = \sqrt{3} - \sqrt{(\sqrt{3})^2 - 3} = \sqrt{3}$

⇒ nema vertikalnih a.!

H.A.

$$\lim_{x\to+\infty} x-\sqrt{x^2-3} = \lim_{x\to+\infty} (x-\sqrt{x^2-3})\cdot\frac{x+\sqrt{x^2-3}}{x+\sqrt{x^2-3}} = \lim_{x\to+\infty} \frac{x^2+x\sqrt{x^2-3}-x\sqrt{x^2-3}-(x^2-3)}{x+\sqrt{x^2-3}} =$$

$$= \lim_{x\to+\infty} \frac{x^2-x^2+3}{x+\sqrt{x^2-3}} \;/:x = \frac{3}{\infty} = 0 \quad +\infty \;\times$$

$$\lim_{x\to-\infty} x-\sqrt{x^2-3} = \lim_{-x\to+\infty} (-x-\sqrt{x^2-3})\cdot\frac{-x+\sqrt{x^2-3}}{-x+\sqrt{x^2-3}} = \lim_{-x\to+\infty} \frac{x^2-(x^2-3)}{-x+\sqrt{x^2-3}} = \lim_{-x\to+\infty} \frac{3}{-x+\sqrt{x^2-3}} \;/:$$

$= -\infty$

- nema horizontalnih a!

K.A

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{x-\sqrt{x^2-3}}{x} \;/:x = \frac{1-1}{1} = \frac{0}{1} = 0 \rightarrow \text{nema desne kose d!}$$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{-x\to+\infty} \frac{-x-\sqrt{x^2-3}}{-x} \;/:x = \frac{-1-1}{-1} = \frac{-2}{-1} = 2\,(k)$$

$$\lim_{x\to-\infty} f(x)-kx = \lim_{-x\to+\infty} -x-\sqrt{x^2-3}-2x = \lim_{-x\to+\infty} (-3x-\sqrt{x^2-3})\cdot\frac{-3x+\sqrt{x^2-3}}{-3x+\sqrt{x^2-3}} =$$

$$= \lim_{-x\to+\infty} \frac{9x^2-(x^2-3)}{-3x+\sqrt{x^2-3}} = \lim_{-x\to+\infty} \frac{8x^2+3}{-3x+\sqrt{x^2-3}} \;/:x^2 = \frac{8}{0} = +\infty$$

$$\boxed{y = 2x} \rightarrow \text{L.K.A.}$$

$$f'(x) = x' - (\sqrt{x^2-3})' = 1 - \frac{1}{2\sqrt{x^2-3}}\cdot 2x$$

$$= 1 - \frac{2x}{2\sqrt{x^2-3}} = \boxed{1-\frac{x}{\sqrt{x^2-3}}}$$

$$f''(x) = (1)' - \left(\frac{x}{\sqrt{x^2-3}}\right)' = -\frac{(x)'(\sqrt{x^2-3})-x(\sqrt{x^2-3})'}{(\sqrt{x^2-3})^2} =$$

$$= -\frac{\sqrt{x^2-3}-x\cdot\frac{1}{2\sqrt{x^2-3}}\cdot 2x}{(\sqrt{x^2-3})^2} = -\frac{\sqrt{x^2-3}-\frac{x^2}{\sqrt{x^2-3}}}{(\sqrt{x^2-3})^2} =$$

$$= -\frac{\frac{x^2-(x^2-3)}{\sqrt{x^2-3}}}{(\sqrt{x^2-3})^2} = \boxed{-\frac{3}{\sqrt{x^2-3}}}$$

$h(x) = 0$

$x^2 - 3x - 4 = 0$

$x_{1,2} = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot (-4)}}{2}$

$x_{1,2} = \frac{3 \pm 5}{2}$

$x_1 = \frac{8}{2} = 4$

$x_2 = -\frac{2}{2} = -1$ > NIČLA TOČKE!

$h(0) = \frac{0 - 3 \cdot 0 - 4}{0 + 1} = -4$ $\quad T(0, -4)$

$$h'(x) = \frac{(x^2 - 3x - 4)' \cdot (x^2 + 1) - (x^2 - 3x - 4)(x^2 + 1)'}{(x^2 + 1)^2} =$$

$$= \frac{(2x - 3)(x^2 + 1) - (x^2 - 3x - 4) \cdot 2x}{(x^2 + 1)^2} =$$

$$= \frac{2x^3 + 2x - 3x^2 - 3 - 2x^3 + 6x^2 + 8x}{(x^2 + 1)^2} = \boxed{\frac{3x^2 + 10x - 3}{(x^2 + 1)^2}}$$ ✓

$$h''(x) = \frac{(3x^2 + 10x - 3)'(x^2 + 1)^2 - (3x^2 + 10x - 3)((x^2 + 1)^2)'}{(x^2 + 1)^4} =$$

$$= \frac{(6x + 10)(x^2 + 1)^2 - (3x^2 + 10x - 3)(2(x^2 + 1) \cdot 2x)}{(x^2 + 1)^4}$$

$$= \frac{(x^2 + 1)\left((6x + 10)(x^2 + 1) - 4x(3x^2 + 10x - 3)\right)}{(x^2 + 1)^4} =$$

$$= \frac{6x^3 + 6x + 10x^2 + 10 - 12x^3 - 40x^2 + 12x}{(x^2 + 1)^3} =$$

$$= \boxed{\frac{-6x^3 - 30x^2 + 18x^2 + 10}{(x^2 + 1)^3}}$$

$h'(x) = 0$

$3x^2 + 10x - 3 = 0$

$x_{1,2} = \frac{-10 \pm \sqrt{100 - 4 \cdot 3 \cdot (-3)}}{6}$

$x_{1,2} = \frac{-10 \pm 2\sqrt{34}}{6}$ ✓

$x_1 = \frac{-5 + \sqrt{34}}{3} \approx 0{,}27$

$x_2 = -\frac{5 - \sqrt{34}}{3} \approx -3{,}61$

$k_1 \; h\left(\frac{-5 + \sqrt{34}}{3}\right) = -4{,}41$

$k_2 \; h\left(\frac{-5 - \sqrt{34}}{3}\right) = 1{,}41$

KRITIČNE TOČKE!

$k_1\left(\frac{-5 + \sqrt{34}}{3}, -4.41\right)$ $\quad k_2\left(\frac{-5 - \sqrt{34}}{3}, 1.41\right)$

x	$-\infty$		$-3{,}61$		$0{,}27$		$+\infty$
$f'(x)$		$+$		$-$		$+$	
$f(x)$		↗	1.41	↘	-4.41	↗	
			lok. max		min		

IME I PREZIME: ELENA BEG BROJ INDEKSA: 17-2-0181-2012

③ $g(x) = \ln(4 - x^2)$

$D_f: \langle -2, 2 \rangle$ ✓

$4 - x^2 > 0$
$-x^2 > -4$
$x^2 < 4$
$|x| < 2$

$g(-x) = \ln(4 - (-x)^2) = \ln(4 - x^2) = g(x) \Rightarrow$ fja je PARNA! ✓

$g'(x) = \frac{1}{4-x^2} \cdot (-2x) = \boxed{\frac{-2x}{4-x^2}}$ ✗

$g''(x) = \frac{(2x)' \cdot (4-x^2) - 2x(4-x^2)'}{(4-x^2)^2} = \frac{2(4-x^2) - 2x \cdot (-2x)}{(4-x^2)^2} =$

$= \frac{8 - 2x^2 + 4x^2}{(4-x^2)^2} = \frac{2x^2 + 8}{(4-x^2)^2}$

$g''(x) = 0$
$2x^2 + 8 = 0$
$2x^2 = -8$
$x^2 \neq -4$
↳ nema tačke infleksije.

x		-2		2
$f''(x)$		$+$		
$f(x)$		$\cup$		

- KONVEKSNA! ✗

---

④ $h(x) = \frac{x^2 - 3x - (3+1)}{x^2+1} = \frac{x^2 - 3x - 3 - 1}{x^2+1} = \boxed{\frac{x^2 - 3x - 4}{x^2+1}}$

$x^2 + 1 \neq 0$
$x^2 \neq -1$

$\boxed{D_f: \mathbb{R}}$

↓ nema vertikalnih asimptota!

$h(-x) = \frac{x^2 + 3x - 4}{x^2+1} \Rightarrow$ NI PARNA, NI NEPARNA
- NIJE PERIODIČNA

H.A.

$\lim_{x \to +\infty} \frac{x^2 - 3x - 4}{x^2+1} \begin{matrix} /:x^2 \\ /:x^2 \end{matrix} = \frac{1}{1} = 1$ $\boxed{y = 1}$ → OBOSTRANA H.A.

$\lim_{x \to +\infty} \frac{x^2 - 3x - 4}{x^2+1} = \lim_{-x \to +\infty} \frac{x^2 + 3x - 4}{x^2+1} \begin{matrix} /:x^2 \\ /:x^2 \end{matrix} \quad \frac{1}{1} = 1$

- nema kosih a!

$$-\frac{9}{34} + 2\cdot\frac{21}{17} - \left(-\frac{72}{17}\right) - \frac{117}{34} = 3$$

$3 = 3$

$$2\cdot\left(-\frac{9}{34}\right) + 5\cdot\left(\frac{21}{17}\right) - \left(-\frac{72}{17}\right) + 2\cdot\left(-\frac{117}{34}\right) = 3$$

$3 = 3$

$$3\cdot\left(-\frac{9}{34}\right) - \frac{21}{17} - 2\cdot\left(-\frac{72}{17}\right) + \left(-\frac{117}{34}\right) = 3$$

$3 = 3$

$$\left(-\frac{9}{34}\right) - \left(\frac{21}{17}\right) + 3\cdot\left(-\frac{72}{17}\right) - 5\cdot\left(-\frac{117}{34}\right) = 3$$

$3 = 3$ ✓

$$x = -\frac{9}{34}$$
$$y = \frac{21}{17}$$
$$z = -\frac{72}{17}$$
$$u = -\frac{117}{34}$$

6.

$$\lim_{x\to -1}\frac{|x|}{x}$$

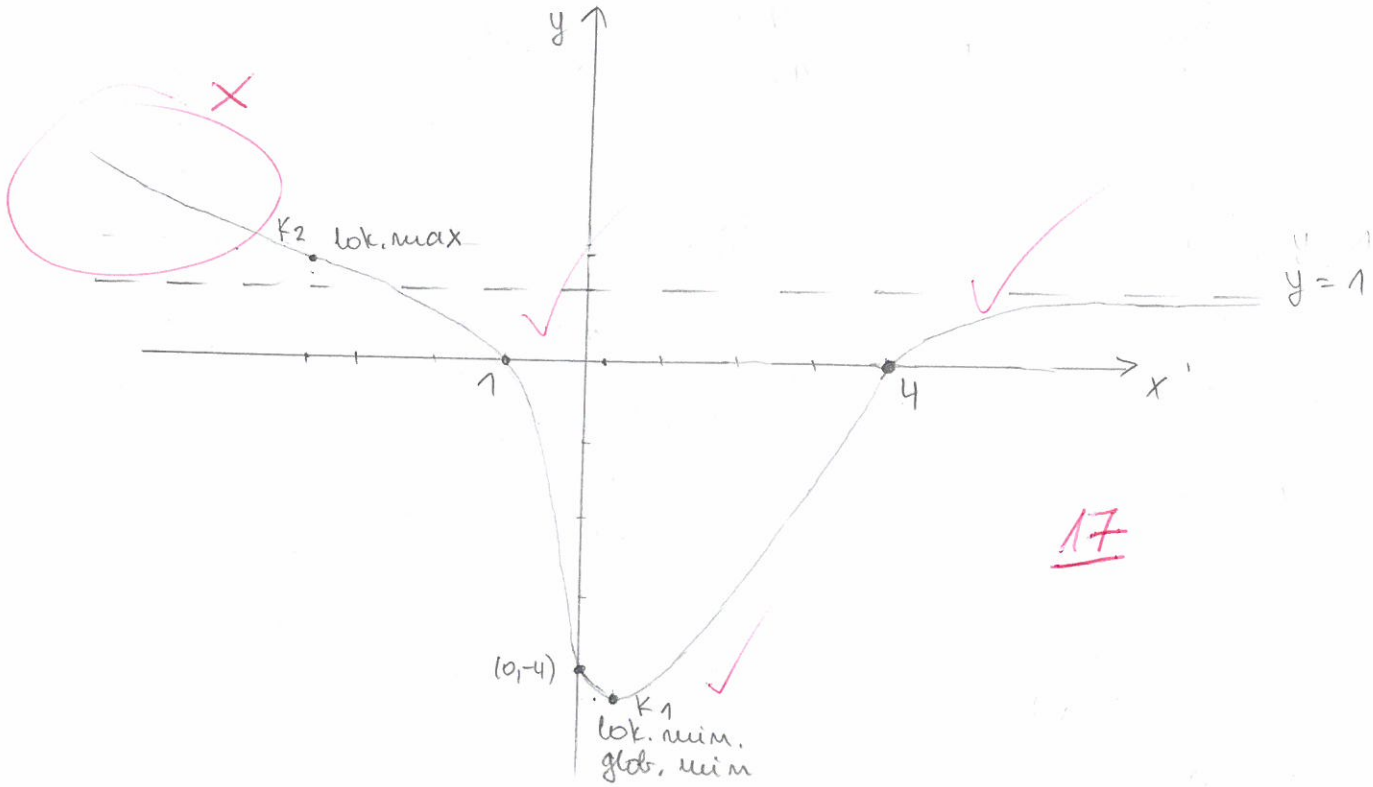
$$\lim_{x\to -1^+} -\frac{x}{x} = \frac{+1}{-1} = -1$$ ✗

$$\lim_{x\to -1^+} \frac{x}{x} = \frac{-1}{-1} = 1$$ ✗

$$\lim_{x\to -1^-} -\frac{x}{x} = \frac{1}{-1} = -1$$

$$\lim_{x\to -1^-} \frac{x}{x} = \frac{-1}{-1} = -1$$

Ime i prezime: ELENA BEG **Broj indeksa:** 17-2-0181-2012



17

5.

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 3 \end{bmatrix} \begin{matrix} \\ \text{II} - 2\text{I} \\ \text{III} - 3\text{I} \\ \text{IV} - \text{I} \end{matrix} \sim \begin{bmatrix} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -7 & 1 & -2 & -6 \\ 0 & -3 & 4 & -6 & 0 \end{bmatrix} \begin{matrix} \text{I} - 2\text{II} \\ \\ \text{III} + 7\text{II} \\ \text{IV} + 3\text{II} \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -27 \\ 0 & 0 & 7 & -6 & -9 \end{bmatrix} \begin{matrix} \\ \\ \text{III} - \text{IV} \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 & -18 \\ 0 & 0 & 7 & -6 & -9 \end{bmatrix} \begin{matrix} \text{I} + 3\text{III} \\ \text{II} - \text{III} \\ \\ \text{IV} - 7\text{III} \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 13 & -45 \\ 0 & 1 & 0 & -4 & 15 \\ 0 & 0 & 1 & 4 & -18 \\ 0 & 0 & 0 & -34 & 117 \end{bmatrix}$$

$-34 x_4 = 117$

$\boxed{x_4 = -\frac{117}{34}} \Rightarrow u$

$x_3 + 4 \cdot \left(-\frac{45}{34}\right) = -18$

$x_3 - \frac{234}{17} = -18$

$x_3 = -18 + \frac{234}{17}$

$\boxed{x_3 = -\frac{72}{17}} \Rightarrow t$

$x_2 - 4 \cdot \left(-\frac{117}{34}\right) = 15$

$x_2 = 15 - \frac{234}{17}$

$\boxed{x_2 = \frac{21}{17}} \Rightarrow y$

$x_1 + 13 \cdot \left(-\frac{45}{34}\right) = -45$

$x_1 = -45 + \frac{1521}{34}$

$\boxed{x_1 = -\frac{9}{34}}$

$\Downarrow x$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ $\mathcal{H}2$

IME I PREZIME: Lavara Navael **BROJ INDEKSA:** 17-2-0097-2021

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

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$4^3 - 4\cdot i^2 \cdot 1 + 3\cdot 1\cdot i^2$

6. Izračunati i provjeriti: $\lim\limits_{x\to -1} \dfrac{|x|}{x}$. 5

Ukupno: 34

(2) a) DOMENA

$x \neq \mathbb{R}$

$x^2 - 3 \geq 0$

$a=1,\ b=0,\ c=-3$

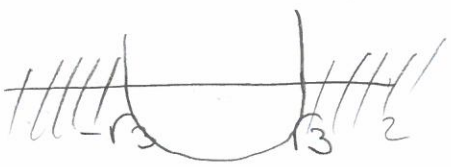
$x_{1/2} = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1/2} = \dfrac{\sqrt{-4\cdot 1\cdot(-3)}}{2}$

$x_{1/2} = \dfrac{2\sqrt{3}}{2}$

$x_1 = \sqrt{3}$

$x_2 = -\sqrt{3}$



$x \in \langle -\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty \rangle$ ✓

b) ASIMPTOTE

(1) V.A.

$\lim\limits_{x\to\sqrt{3}^{\pm}} x - \sqrt{x^2-3} = \sqrt{3}$

⇒ nema vertikalnih asimptota

(2) H.A.

$\lim\limits_{x\to+\infty} x - \sqrt{x^2-3} \cdot \dfrac{x+\sqrt{x^2-3}}{x+\sqrt{x^2-3}} = \lim\limits_{x\to+\infty} \dfrac{(x)^2 - (\sqrt{x^2-3})^2}{x+\sqrt{x^2-3}} =$

$\lim\limits_{x\to+\infty} \dfrac{x^2-x^2+3}{x+\sqrt{x^2-3}} = \lim\limits_{x\to\infty} \dfrac{3\,/:x}{x+\sqrt{x^2-3}\,/:x} = \lim\limits_{x\to\infty} \dfrac{\frac{3}{x}}{1+\sqrt{1-\frac{3}{x^2}}} = \dfrac{0}{2} = 0$

$\lim\limits_{x\to-\infty} x - \sqrt{x^2-3} = \left[\begin{matrix} x \to -\infty \\ -\infty + \infty \end{matrix}\right] = \lim\limits_{x\to\infty} -x - \sqrt{x^2-3} \cdot \dfrac{-x+\sqrt{x^2-3}}{-x+\sqrt{x^2-3}} = \lim\limits_{x\to\infty}$

$\dfrac{(-x)^2 - (\sqrt{x^2-3})^2}{-x+\sqrt{x^2-3}} = \lim\limits_{x\to\infty} \dfrac{x^2-x^2+3}{-x+\sqrt{x^2-3}} = \lim\limits_{x\to\infty} \dfrac{3\,/:x}{-x+\sqrt{x^2-3}\,/:x} = \dfrac{\frac{3}{x}}{-1+\sqrt{1-\frac{3}{x^2}}} = \dfrac{0}{-1} = 0$

OVO JE SAMO DESNA H.A. $y = 0$

(3.) K.A.

$$k_1 = \lim_{x\to\infty} \frac{x-\sqrt{x^2-3}}{x} \;/:x = \lim_{x\to\infty} \frac{1-\sqrt{1-\frac{3}{x}}}{1} = \lim_{x\to\infty} = \frac{1-1}{1} = \frac{0}{1} = 0$$

$$k_2 = \lim_{x\to-\infty} \frac{-x-\sqrt{x^2-3}}{-x} \;/:x = \lim_{x\to\infty} \frac{-1-\sqrt{1-\frac{3}{x}}}{-1} = \frac{-2}{-1} = 2$$

$$\left[\begin{matrix} x \to -x \\ -\infty \mapsto +\infty \end{matrix}\right]$$

$$l = \lim_{x\to-\infty} (x-\sqrt{x^2-3}-2x) = \left[\begin{matrix} x \to -x \\ -\infty \mapsto +\infty \end{matrix}\right] = \lim_{x\to\infty} -x-\sqrt{x^2-3}+2x \cdot \frac{-x-\sqrt{x^2-3}-2x}{-x-\sqrt{x^2-3}-2x}$$

$$= \lim_{x\to\infty} \frac{(-x-\sqrt{x^2-3})^2-(2x)^2}{(-x-\sqrt{x^2-3})-2x} = \lim_{x\to\infty} \frac{-x-x^2+3-4x^2}{(-x-\sqrt{x^2-3})-2x} = \frac{-5x^2-x+3 \;/:x^2}{(-x-\sqrt{x^2-3})-2x/:x^2}$$

$$= \lim_{x\to\infty} \frac{-5-\frac{x}{x}+\frac{3}{x^2}}{-\frac{x}{x}-\sqrt{\frac{x^2}{x^4}-\frac{3}{x^4}}-\frac{2x}{x}} = \frac{-5}{0} = -\infty$$

→ 2. DERIVACIJA

$$f(x) = x-\sqrt{x^2-3}$$

$$f'(x) = (x)' - \frac{1}{2\sqrt{x^2-3}} \cdot (x^2-3)' = -\frac{1}{2\sqrt{x^2-3}} \cdot 2x = -\frac{2x}{2\sqrt{x^2-3}} = \frac{-x}{\sqrt{x^2-3}}$$ ✗

$$f''(x) = \frac{(-x)' \cdot (\sqrt{x^2-3}) - (-x)\cdot(\sqrt{x^2-3})'}{(\sqrt{(x^2-3)})^2} = \frac{-1\cdot(\sqrt{x^2-3}) - (-x)\cdot\frac{1}{2\sqrt{x^2-3}}\cdot(x^2-3)'}{x^2-3}$$

$$f''(x) = \frac{-1\cdot(\sqrt{x^2-3}) - (-x)\cdot\frac{2x}{2\sqrt{x^2-3}}}{x^2-3} = \frac{\sqrt{x^2-3}+\frac{2x^2}{2\sqrt{x^2-5}}}{x^2-3}$$

(3) $g(x) = \ln(4-x^2)$

a) DOMENA

$4-x^2 > 0$

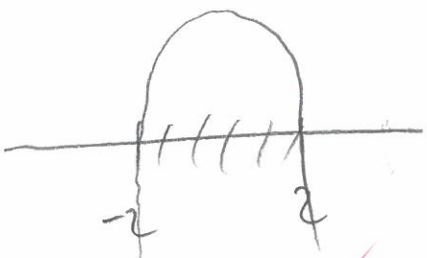
$-x^2 = -4 \;/:(-1)$

$x^2 = 4$

$x = \sqrt{4}$

$x_1 = 2$

$x_2 = -2$



$x \in \langle -2, 2 \rangle$ ✓

b) $g(-x) = \ln(4-x^2)$

⇒ funkcija je parna ✓

IME I PREZIME: Romana Nowsel BROJ INDEKSA:

c) INTERVALI ZAKRIVENOSTI

$f'(x) = \frac{1}{4-x^2} \cdot (4-x^2)' = \frac{-2x}{4-x^2}$ ✓

$f''(x) = \frac{(-2x)' \cdot (4-x^2) - (2x) \cdot (4-x^2)'}{(4-x^2)^2} = \frac{-2(4-x^2) - (2x) \cdot (-2x)}{(4-x^2)^2}$ ✗

$f''(x) = \frac{-8+2x^2+4x^2}{(4-x^2)^2} = \frac{6x^2-8}{(4-x^2)^2}$

$f''(x) = 0$

$6x^2 - 8 = 0$

$6x^2 = 8 / :6$

$x^2 = \frac{4}{3}$

$x_1 = \frac{2\sqrt{3}}{3}$

$x_2 = -\frac{2\sqrt{3}}{3}$

$-1.15 \quad +1.15$

x	$-\infty$	$-\frac{2\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	$+\infty$
$f''(x)$	+	0	−	0 +
$f(x)$	∪	T_1	∩ T_2	∪

→ tocke infleksije

(4) $u(x) = \frac{x^2-3x-(3+1)}{x^2+1} = \frac{x^2-3x-4}{x^2+1}$

① DOMENA

$x^2+1 \neq 0$

$x^2 \neq -1$ $D(u) = \mathbb{R}$

② PARNOST

$u(-x) = \frac{x^2+3x-4}{x^2+1}$ funkcija nije niti parna niti neparna

③ PERIODIČNOST

→ funkcija nije periodična jer nije trigonometrijska

④ NULTOCKE

$x^2-3x-4=0$

$x_{1/2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x_{1/2}=\frac{3\pm\sqrt{9-4\cdot1\cdot(-4)}}{2}$

$x_{1/2}=\frac{3\pm5}{2}$

$x_1=\frac{3+5}{2}=4$ ⟩ nultočke

$x_2=\frac{3-5}{2}=-1$

(5) ASIMPTOTE

a). V.A.

$\lim_{x\to4^+}\frac{x^2-3x-4}{x^2+1}=\frac{0}{17}=0$

$\lim_{x\to-1^+}\frac{x^2-3x-4}{x^2+1}=\frac{0}{2}=0$ } nema V.A.

b.) H.A

$\lim_{x\to\infty}\frac{x^2-3x-4}{x^2+1}\ /:x^2=\frac{1-\frac{3}{x}-\frac{4}{x^2}}{1+\frac{1}{x^2}}=\frac{1}{1}=1$

$\lim_{x\to-\infty}\frac{x^2-3x-4}{x^2+1}=\left[\begin{matrix}x\mapsto-x\\-\infty\mapsto+\infty\end{matrix}\right]=\lim_{x\to\infty}\frac{x^2+3x-4}{x^2+1}\ /:x^2=\lim_{x\to\infty}\frac{1+\frac{3x}{x}-\frac{4}{x^2}}{1+\frac{1}{x^2}}=1$

$y=1$ H.A. ✓

c) K.A

$k_1=\lim_{x\to\infty}\frac{x^2-3x-4}{x^2+1}\cdot\lim_{x\to\infty}\frac{x^2-3x-4}{x(x^2+1)}=\lim_{x\to\infty}\frac{x^2-3x-4\ /:x^3}{x^3+x\ /:x^3}=\frac{0}{1}=0$

nema K.A., ujedno jer ima obe horizontale!

(6) MONOTONOST I EKSTREMI

$u(x)=\frac{x^2-3x-4}{x^2+1}$

$u'(x)=\frac{(x^2-3x-4)'\cdot(x^2+1)-(x^2-3x-4)\cdot(x^2+1)'}{(x^2+1)^2}=\frac{(2x-3)(x^2+1)-(x^2-3x-4)\cdot2x}{(x^2+1)^2}$

$u'(x)=\frac{2x^3+2x-3x^2-3-2x^3+6x+8x}{(x^2+1)^2}=\frac{-3x^2+16x-3}{(x^2+1)^2}$ -1

IME I PREZIME: Romana Wrosl BROJ INDEKSA:

$p'(x) = 0$

$-3x^2+16x-3=0 \,/\cdot(-1)$

$3x^2-16x+3=0$

$x_{1/2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1/2} = \frac{16 \pm \sqrt{256-4\cdot3\cdot3}}{6}$

$x_{1/2} = \frac{16 \pm 2\sqrt{55}}{6}$

$x_1 = \frac{1}{2}\left(\frac{8 \pm \sqrt{55}}{3}\right)$

$x_1 = \frac{8+\sqrt{55}}{3} = 5.1$

$x_2 = \frac{8-\sqrt{55}}{3} = 0.2$

⇒ stacionarne tačke

MONOTONOST

x	$-\infty$		0.2		5.1		$+\infty$
$p'(x)$		$-$	0	$+$	0	$-$	
$p(x)$		↘	$l.min$	↗	$l.max$	↘	

GDJE JE NA SLICI?

$u(x) = \frac{x^2-3x-4}{x^2+1}$

$h(0.2) = -4.4$ $\quad u(5.1) = 0.3$

$m(0.2; -4.4)$ $\quad M(5.1; 0.3)$

GRAF

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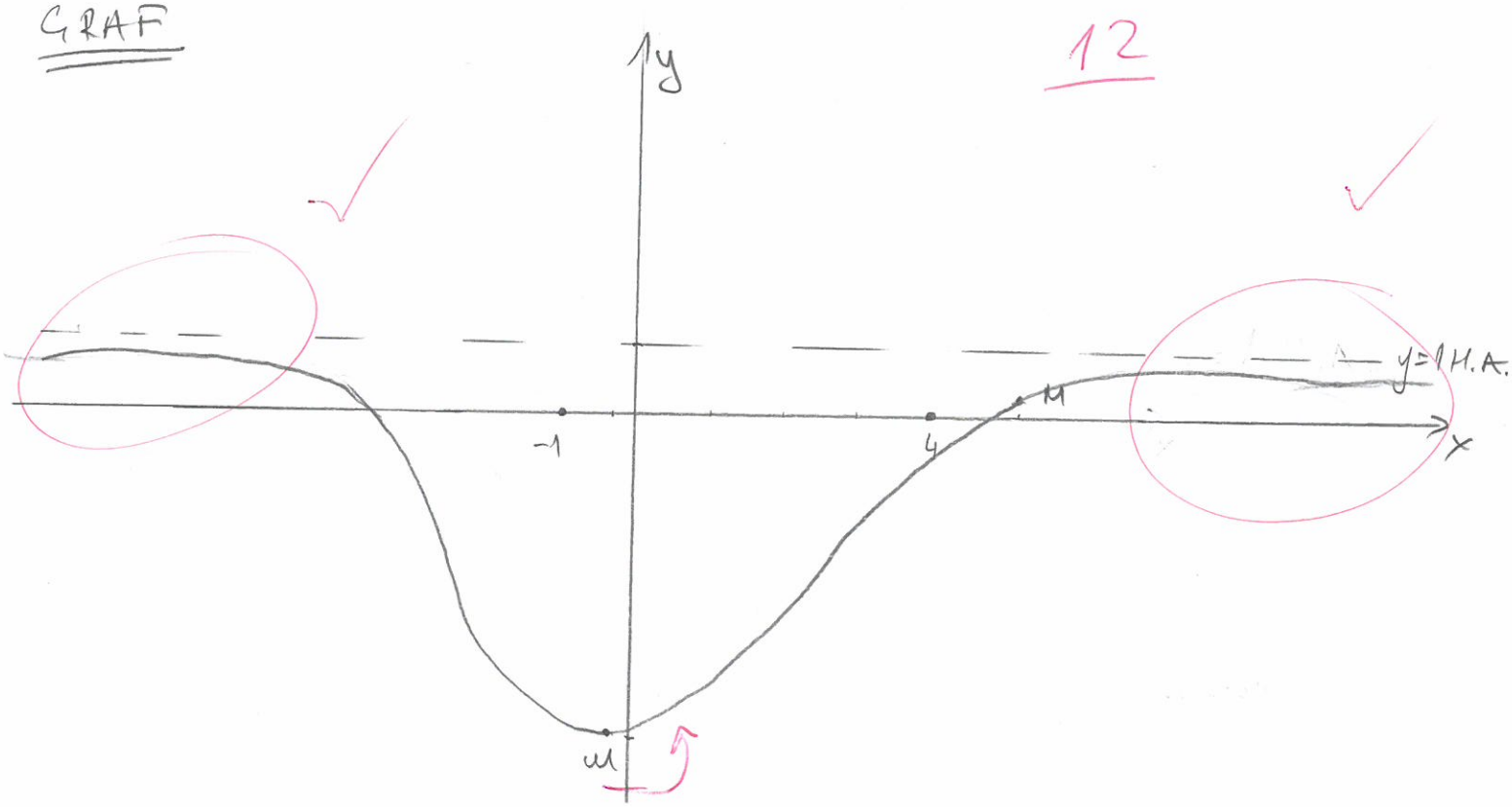


(5) $x+2y-z+u=3$

$2x+5y-z+2u=3$

$3x-y-2z+u=3$

$x-y+3z-5u=3$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -7 & 1 & -2 & -6 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -27 \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] \sim$$

$1.r. \cdot(-2)+2.r.$
$1.r. \cdot(-3)+3.r.$
$1.r. \cdot(-1)+4.r.$

$2.r. \cdot(-2)+1.r.$
$2.r. \cdot(7)+3.r.$
$2.r. \cdot(3)+4.r.$

$4.r. \cdot(-1)+3.r.$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 & -18 \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 13 & -45 \\ 0 & 1 & 0 & -4 & 15 \\ 0 & 0 & 1 & 4 & -18 \\ 0 & 0 & 0 & -34 & 117 \end{array}\right] /:(-34) \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 13 & -45 \\ 0 & 1 & 0 & -4 & 15 \\ 0 & 0 & 1 & 4 & -18 \\ 0 & 0 & 0 & 1 & -34/117 \end{array}\right] \sim$$

$3.r. \cdot 3+1.r.$
$3.r. \cdot(-1)+2.r.$
$3.r. \cdot(-7)+4.r.$

$4.r. \cdot(-13)+1.r.$
$4.r. \cdot(4)+2.r.$
$4.r. \cdot(-4)+3.r.$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -37\,1/9 \\ 0 & 1 & 0 & 0 & \\ & & & & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1/4 & -27/8 \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/4 & -9/8 \\ 0 & 1 & 0 & 1/4 & 3/8 \\ 0 & 0 & 1 & -1/4 & -27/8 \\ 0 & 0 & 0 & -17/4 & 117/8 \end{array}\right] /:(-17/4)$$

$3.r. \cdot 3+1.r.$
$3.r. \cdot(-1)+2.r.$
$3.r. \cdot(-7)+4.r.$

PROVJERA?

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/4 & -9/8 \\ 0 & 1 & 0 & 1/4 & 3/8 \\ 0 & 0 & 1 & -1/4 & -27/8 \\ 0 & 0 & 0 & 1 & -117/34 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 315/136 \\ 0 & 1 & 0 & 0 & 29/17 \\ 0 & 0 & 1 & 0 & -72/17 \\ 0 & 0 & 0 & 1 & -117/34 \end{array}\right] \times$$

$4.r. \cdot(-1/4)+1.r.$
$4.r. \cdot(-1/4)+2.r.$
$4.r. \cdot(1/4)+3.r.$

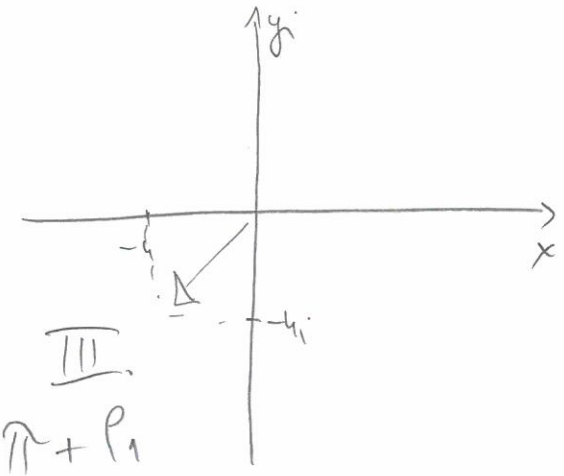
IME I PREZIME: Romana Nowsel BROJ INDEKSA:

① $z^4-(4-i)^3=0$ (binom $(a-b)^3$)

$z^4-(-4-4i)=0$ ×

$z^4+4+4i=0$

$z^4=-4-4i$



III.

$\pi+\varphi_1$

$|-4-4i|=\sqrt{x^2+y^2}$

$=\sqrt{(-4)^2+(-4)^2}$

$=4\sqrt{2}\approx 5.66$

$\operatorname{tg}\varphi=\frac{y}{x}=\frac{-4}{-4}=1=\frac{1}{4}\pi$

$\varphi=\pi+\frac{1}{4}\pi$

$\varphi=\frac{5}{4}\pi$

$z=\sqrt[4]{5.66}\left(\cos\frac{\frac{5}{4}\pi+k\cdot 2\pi}{4}+i\sin\frac{\frac{5}{4}\pi+k\cdot 2\pi}{4}\right)$ $k=0,1,2,3$

$k=0$

$z_0=1.54\left(\cos\frac{\frac{5}{4}\pi+0\cdot 2\pi}{4}+i\sin\frac{\frac{5}{4}\pi+0\cdot 2\pi}{4}\right)$

$z_0=1.54\left(\cos\frac{5}{16}\pi+i\sin\frac{5}{16}\pi\right)$

$z_0=1.54\left(0.555+0.831i\right)$

$z_0=0.855+1.279i$

$k=1$

$z_1=1.54\left(\cos\frac{\frac{5}{4}\pi+1\cdot 2\pi}{4}+i\sin\frac{\frac{5}{4}\pi+1\cdot 2\pi}{4}\right)$

$z_1=1.54\left(\cos\frac{13}{16}\pi+i\sin\frac{13}{16}\pi\right)$

$z_1=1.54\left(-0.831+0.555i\right)$

$z_1=-1.279+0.855i$

$$z_2 = 1.54\left(\cos\frac{\frac{5}{4}\pi + 2\cdot 2\pi}{4} + i\sin\frac{\frac{5}{4}\pi + 2\cdot 2\pi}{4}\right)$$

$$z_2 = 1.54\left(\cos\frac{21}{16}\pi + i\sin\frac{21}{16}\pi\right)$$

$$z_2 = 1.54(-0.555 - 0.831i)$$

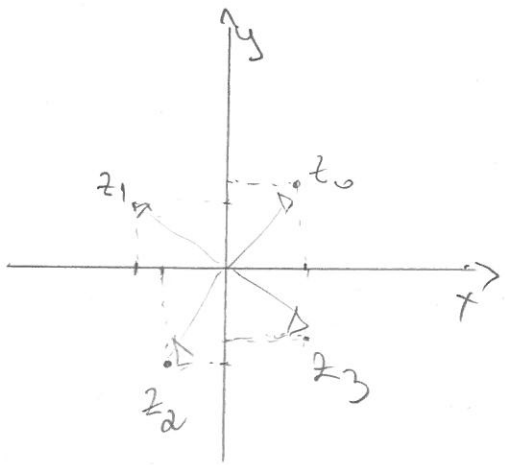
$$z_2 = -0.855 - 1.279i$$

$$z_3 = 1.54\left(\cos\frac{\frac{5}{4}\pi + 3\cdot 2\pi}{4} + i\sin\frac{\frac{5}{4}\pi + 3\cdot 2\pi}{4}\right)$$

$$z_3 = 1.54\left(\cos\frac{29}{16}\pi + i\sin\frac{29}{16}\pi\right)$$

$$z_3 = 1.54(0.831 - 0.555i)$$

$$z_3 = 1.279 - 0.855i$$



MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod ⇓⇓ $\mathcal{H}2$

IME I PREZIME: Tomislav Glavan **BROJ INDEKSA:** 17-0115-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Riješiti jednadžbu: $z^4 - (4-i)^3 = 0$. *Prikaži rješenja u kompleksnoj ravnini!* 12+3
2. Odrediti domenu, sve asimptote i drugu derivaciju funkcije $f(x) = x - \sqrt{x^2-3}$. 5+15+5
3. Ispitati domenu, (ne)parnost i zakrivljenost grafa funkcije $g(x) = \ln(4-x^2)$. 5+5+10
4. Na temelju ispitivanja toka funkcije napraviti skicu grafa funkcije $h(x) = \dfrac{x^2-3x-(3+1)}{x^2+1}$. Ne treba ispitivati zakrivljenost jer se izraz komplicira. 20(graf) 19
5. Gaussovom metodom riješiti matrični sustav i obavezno provjeri rješenje: 15

$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 3 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti: $\lim\limits_{x\to -1} \dfrac{|x|}{x}$. 5

Ukupno: 56

5)

$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 3 \\ x-y+3z-5u &= 3 \end{aligned} \Rightarrow \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right) \begin{matrix} \\ /\cdot(-2) \\ /\cdot(-3) \\ /\cdot(-1) \end{matrix} \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -7 & 1 & -2 & -6 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right) \begin{matrix} \\ \\ /\cdot(7) \\ /\cdot(3) \end{matrix} \sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -27 \\ 0 & 0 & 7 & -6 & -9 \end{array}\right) /\cdot\left(-\tfrac{7}{8}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -27 \\ 0 & 0 & 0 & -\frac{17}{4} & \frac{117}{8} \end{array}\right)$$

$$\begin{aligned} 1x+2y-1z+1u &= 3 \\ 1y+1z &= -3 \\ 8z-2u &= -27 \\ -\tfrac{17}{4}u &= \tfrac{117}{8} \end{aligned} \Rightarrow$$

$$\Rightarrow -\tfrac{17}{4}u = \tfrac{117}{8} \;/:\left(-\tfrac{17}{4}\right) \Rightarrow \boxed{u = -\tfrac{117}{34}} \checkmark$$

$$\Rightarrow 8z - 2\cdot\left(-\tfrac{117}{34}\right) = -27$$

$$8z + \tfrac{117}{17} = -27$$

$$8z = -27 - \tfrac{117}{17}$$

$$8z = -\tfrac{576}{17} \;/:8$$

$$z = -\tfrac{\frac{576}{17}}{8} = -\tfrac{576}{17}\cdot\tfrac{1}{8}$$

$$\boxed{z = -\tfrac{72}{17}} \checkmark$$

$$\Rightarrow 1y + 1\cdot\left(-\tfrac{72}{17}\right) = -3$$

$$1y = -3 + \tfrac{72}{17} \quad \boxed{y = \tfrac{21}{17}} \checkmark$$

$$\Rightarrow 1x + 2\cdot\left(\tfrac{21}{17}\right) - 1\cdot\tfrac{72}{17} + 1\cdot\left(-\tfrac{117}{34}\right) = 3$$

$$1x = \boxed{x = -\tfrac{9}{34}} \checkmark$$

$$\begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & 5 & -1 & 2 \\ 3 & -1 & -2 & 1 \\ 1 & -1 & 3 & -5 \end{pmatrix} \cdot \begin{pmatrix} -\frac{9}{34} \\ \frac{21}{17} \\ -\frac{72}{17} \\ -\frac{117}{34} \end{pmatrix} = \begin{pmatrix} 3 \checkmark \\ 3 \checkmark \\ 3 \checkmark \\ 3 \checkmark \end{pmatrix}$$

② $f(x) = x - \sqrt{x^2-3}$

1) DOMENA

$x^2 - 3 \geq 0$

$x^2 \geq 3 \mid \sqrt{\ }$

$x \geq \pm\sqrt{3}$

$D_f = x \in \mathbb{R} \setminus \{\pm\sqrt{3}\}$ ✗

2) ASIMPTOTE

V.A nema jer funkcija nije razlomak

H.A

$$\lim_{x\to\infty} \frac{(x-\sqrt{x^2-3})\cdot(x+\sqrt{x^2-3})}{(x+\sqrt{x^2-3})} = \lim_{x\to\infty} \frac{(x)^2 - (\sqrt{(x^2-3)^2})}{(x+\sqrt{x^2-3})} = \lim_{x\to\infty} \frac{x^2 - x^2 - 3}{x+\sqrt{x^2-3}}$$

(+)

$$= \lim_{x\to\infty} \frac{-3}{x+\sqrt{x^2-3}} \begin{matrix} /:x \\ /:x \end{matrix} = \lim_{x\to\infty} \frac{\frac{-3}{x}}{\frac{x}{x}+\frac{\sqrt{x^2-3}}{x}} = \lim_{x\to\infty} \frac{-\frac{3}{x}}{1+\sqrt{\frac{x^2}{x^2}-\frac{3}{x^2}}} = \lim_{x\to\infty} \frac{-\frac{3}{x}}{1+\sqrt{1-\frac{3}{x^2}}}$$

$$= \frac{-\frac{3}{\infty}}{1+\sqrt{1-\frac{3}{\infty}}} = \frac{0}{1+\sqrt{1-0}} = \frac{0}{1+1} = \frac{0}{2} = 0$$ ✓

KOSA

-> nema kose jer smo dobili H.A = 0 LIJEVA KOSA?

3) I. DERIVACIJA

$f(x) = x - \sqrt{x^2-3} \rightarrow x - (x^2-3)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x - (x^2-3)^{-\frac{1}{2}}$ ✗

$f'(x) = \frac{1}{2}x - \frac{1}{\sqrt{x^2-3}}$

II DERIVACIJA

$f'(x) = \frac{1}{2}x - (x^2-3)^{-\frac{1}{2}}$

$f''(x) = \frac{1}{2}\cdot\left(-\frac{1}{2}\right) - (x^2-3)^{-\frac{3}{2}} \cdot 2x$

$f''(x) = -\frac{1}{4} - \frac{1}{\sqrt{(x^2-3)^3}} \cdot 2x$

IME I PREZIME: Tomislav Glavan BROJ INDEKSA: 17-0115-2011

③ $g(x) = \ln(4-x^2)$

*DOMENA

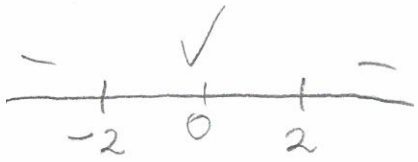
$4-x^2 > 0$

$-x^2 > -4 \;/\cdot(-1)$

$x^2 < 4 \;/\sqrt{}$

$x < \mp\sqrt{4}$

$x < \mp 2$



$Df = x \in \langle -2, 2 \rangle$ ✓

*NE(PARNOST)

$g(x) = \ln(4-x^2) \rightarrow g(-x) = \ln(4-(x)^2) = \ln(4-x^2) \Rightarrow$ PARNA ✓

④ $h(x) \dfrac{x^2-3x-(3+1)}{x^2+1} = \dfrac{x^2-3x-4}{x^2+1}$

1) DOMENA

$x^2+1 \neq 0$

$x^2 \neq -1 \;/\sqrt{}$

$x \neq \pm\sqrt{-1}$

×

$Df = x \in \mathbb{R}$

2) ASIMPTOTE

V.A

$x^2+1 = 0$

$x^2 = -1 \;/\sqrt{}$

nema V.A

⇒

H.A

$$\lim_{x\to\infty} \frac{x^2-3x-4}{x^2+1} \,/:x^2 = \lim_{x\to\infty} \frac{\frac{x^2}{x^2}-\frac{3x}{x^2}-\frac{4}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \lim_{x\to\infty} \frac{1-\frac{3}{x}-\frac{4}{x^2}}{1+\frac{1}{x^2}}$$

$$= \frac{1-\frac{3}{\infty}-\frac{4}{\infty}}{1+\frac{1}{\infty}} = \frac{1-0-0}{1+0} = \frac{1}{1} = 1$$

$y=1$

K.A

nema K.A jer ima H.A

3) Ne(Parnost)

$$h(-x) = \frac{x^2-3x-4}{x^2+1} = \frac{(-x)^2-3\cdot(-x)-4}{(-x^2)+1} = \frac{x^2+3x-4}{x^2+1}$$

ni parna ni neparna; nije periodična jer nije trigonometrijska

4) Nultočke

$$h(x) = \frac{x^2-3x-4}{x^2+1} = 0 \;/\cdot(x^2+1)$$

$$x^2-3x-4=0$$

$$\frac{-b\mp\sqrt{b^2-4a\cdot c}}{2\cdot a} = \frac{3\mp\sqrt{(-3)^2-4\cdot1\cdot(-4)}}{2\cdot1} = \frac{3\mp\sqrt{9+16}}{2} = \frac{3\mp\sqrt{25}}{2} = \frac{3\mp5}{2}$$

$$x_1 = \frac{3-5}{2} = \frac{-2}{2} = -1 \;;\; x_2 = \frac{3+5}{2} = \frac{8}{2} = 4$$

$x = (-1,0), (4,0)$

$$y = \frac{-4}{1} = -4$$

$(0,-4)$

5) 1. derivacija

$$f(x) = \frac{x^2-3x-4}{x^2+1} \Rightarrow f'(x) = \frac{(x^2-3x-4)'\cdot(x^2+1)-(x^2-3x-4)\cdot(x^2+1)'}{(x^2+1)^2}$$

$$f'(x) = \frac{(2x-3)\cdot(x^2+1)-(x^2-3x-4)\cdot(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{2x^3+2x-3x^2-3-2x^3+6x^2+8x}{(x^2+1)^2} = \frac{3x^2+10x-3}{(x^2+1)^2}$$

(4) IME I PREZIME: Tomislav Glavan BROJ INDEKSA: 17-0115-2011

6) KRITIČNE TOČKE

$$f'(x) = \frac{3x^2+10x-3}{(x^2+1)^2} = 0 \;/\cdot (x^2+1)^2$$

$$3x+10x-3=0$$

$$\frac{-b \mp \sqrt{b^2-4a\cdot c}}{2\cdot a} = \frac{-10 \mp \sqrt{(10)^2-4\cdot 3\cdot(-3)}}{2\cdot 3} = \frac{-10 \mp \sqrt{100+36}}{6} = \frac{-10 \mp \sqrt{136}}{6}$$

$$x_1 = \frac{-10-\sqrt{136}}{6} = \frac{-5-\sqrt{34}}{3} \approx -3,6 \; ; \; x_2 = \frac{-10+\sqrt{136}}{6} = \frac{-5+\sqrt{34}}{3} \approx 0,28$$

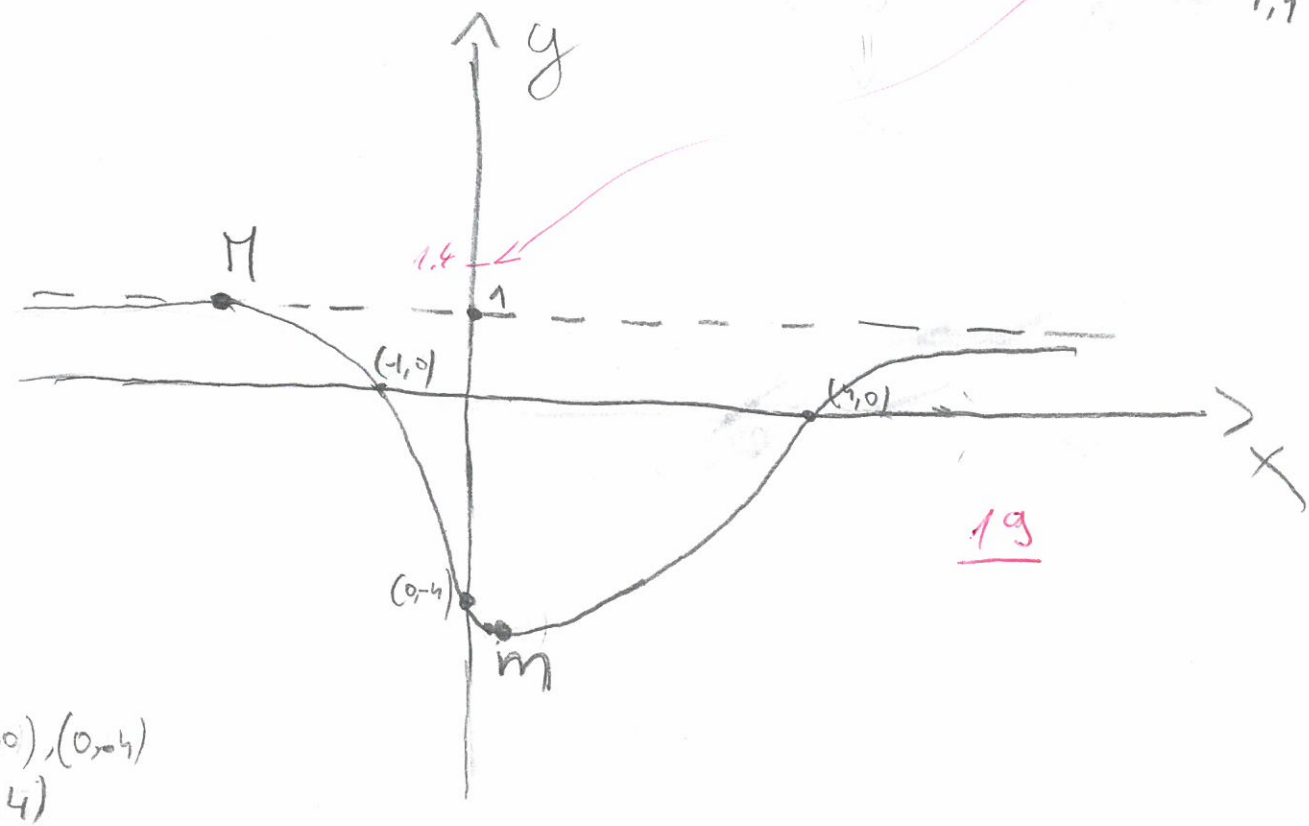
	$-\infty$	-5	$-3,6$	0	$0,28$	$+\infty$
$f'(x)$		+		−		+
$f(x)$		↗	M	↘	m	↗

$$f'(x) = \frac{3\cdot(-5)^2+10\cdot(-5)-3}{((-5)^2+1)^2} = +$$

$$\left(\frac{-5-\sqrt{34}}{3}, 1\right) \quad \left(\frac{-5+\sqrt{34}}{3}, -4,4\right) \quad y = \frac{\left(\frac{-5-\sqrt{34}}{3}\right)^2 - 3\cdot\frac{-5-\sqrt{34}}{3} - 4}{\left(\frac{-5-\sqrt{34}}{3}\right)^2+1} \approx 1,4$$

$$\approx -4,4$$

GRAF



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H.A = $y=1$

nultočke $(-1,0), (4,0), (0,-4)$

$m\left(\frac{5+\sqrt{34}}{6}, -0,4\right)$

(6) $\lim_{x \to -1} \frac{|x|}{x} = \frac{|-1|}{(-1)} = \frac{1}{(-1)} = -1$ ✓

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IME I PREZIME: ANTE VEDRIĆ **BROJ INDEKSA:** 17-2-0198-2012

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

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$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 3 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti: $\lim_{x\to -1} \dfrac{|x|}{x}$. 5

Ukupno: 25

5.)

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 2 & 5 & -1 & 2 & 3 \\ 3 & -1 & -2 & 1 & 3 \\ 1 & -1 & 3 & -5 & 3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & -7 & 1 & -2 & -6 \\ 0 & -3 & 4 & -6 & 0 \end{array}\right] \sim$$

$1R\cdot(-2)+2R$
$1R\cdot(-3)+3R$
$1R\cdot(-1)+4R$

$2R\cdot 7+3R$
$2R\cdot 3+4R$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 8 & -2 & -27 \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 & -18 \\ 0 & 0 & 7 & -6 & -9 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 & -18 \\ 0 & 0 & 0 & -34 & 117 \end{array}\right]$$

$3R\cdot(-7)+4R$ $\quad$ $4R:(-34)$

$3R-4R$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 3 \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 & -18 \\ 0 & 0 & 0 & 1 & -\frac{47}{34} \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & \frac{215}{34} \\ 0 & 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 & -\frac{72}{17} \\ 0 & 0 & 0 & 1 & -\frac{117}{34} \end{array}\right]$$

$4R\cdot(-4)+3R$

$3R-1\cdot 1R$

$3R\cdot(-1)+2R$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & \frac{79}{34} \\ 0 & 1 & 0 & 0 & \frac{21}{17} \\ 0 & 0 & 1 & 0 & -\frac{72}{17} \\ 0 & 0 & 0 & 1 & \frac{-117}{34} \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{9}{34} \\ 0 & 1 & 0 & 0 & \frac{21}{17} \\ 0 & 0 & 1 & 0 & -\frac{72}{17} \\ 0 & 0 & 0 & 1 & -\frac{117}{34} \end{array}\right]$$

$2R \cdot (-2) + 1R$

$x = -\frac{9}{34}$ ✓ $z = -\frac{72}{17}$ ✓

$y = \frac{21}{17}$ ✓ $u = -\frac{117}{34}$ ✓

① $-\frac{9}{34} + 2 \cdot \frac{21}{17} + \frac{72}{17} - \frac{117}{34} = 3$

$-\frac{9}{34} + 2 \cdot \frac{42}{34} + \frac{144}{34} - \frac{117}{34} = 3$

$-\frac{9}{34} + \frac{84}{34} + \frac{144}{34} - \frac{117}{34} = 3$

$\frac{102}{34} = 3$

$3 = 3$

② $2 \cdot -\frac{9}{34} + 5 \cdot \frac{21}{17} + \frac{72}{17} + 2 \cdot \left(-\frac{117}{34}\right) = 3$

$-\frac{18}{34} + \frac{105}{17} + \frac{72}{17} - \frac{234}{34} = 3$

$-\frac{18}{34} + \frac{210}{34} + \frac{144}{34} - \frac{234}{34} = 3$

$\frac{102}{34} = 3$ $\quad 3 = 3$

③ $-\frac{9}{34} - \frac{21}{17} + 3 \cdot -\frac{72}{17} - 5 \cdot \left(-\frac{117}{34}\right) = 3$

$-\frac{9}{34} - \frac{42}{34} - \frac{432}{34} + \frac{585}{34} = 3$

$\frac{102}{34} = 3$

$3 = 3$ ✓

3.) $3 \cdot \left(-\frac{9}{34}\right) - \frac{21}{17} - 2 \cdot -\frac{72}{17} - \frac{117}{34} = 3$

$-\frac{27}{34} - \frac{42}{34} + \frac{288}{34} - \frac{117}{34} = 3$

$\frac{102}{34} = 3$

$3 = 3$

IME I PREZIME: ANTE VEDRIĆ BROJ INDEKSA: 17-2-0198-201

2.) $f(x) = x - \sqrt{x^2-3}$

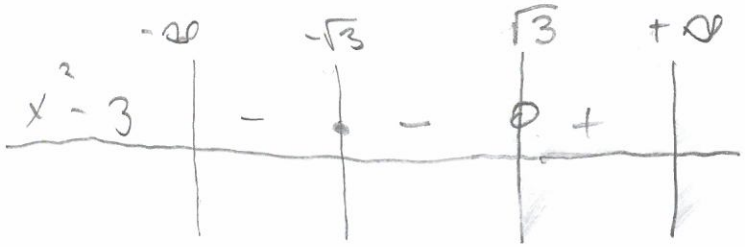
$x^2 - 3 \geq 0$

$x^2 - 3 = 0$

$x^2 = 3$

$x_1 = \sqrt{3}$

$x_2 = -\sqrt{3}$



$Df = \langle -\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty \rangle$ ✓

$\lim_{x \to \sqrt{3}} = \ldots - \sqrt{x^2-3} \ldots \lim_{x \to \sqrt{3}} - \sqrt{(-\sqrt{3})^2 - 3} = \ldots -\sqrt{3-3} =$

$= \lim_{x \to -\sqrt{3}} = \sqrt{3} - 3$

$\lim_{x \to \infty} x - \sqrt{x^2-3} \Big/ \cdot \frac{x - \sqrt{x^2-3}}{x - \sqrt{x^2-3}} : \frac{x^2 \cdot (x^2-3)}{x^2 - x^2 - 3}$?

IME I PREZIME: ANTE VEDRIĆ

BROJ INDEKSA: 17-2-0198-2012

3) $g(x) = \ln(4 - x^2)$

$4 - x^2 > 1$ ✗

$-x^2 > -3$

$x^2 > 3$

$g(-x) = \ln(4 - (-x)^2)$

$g(-x) = \ln(4 - x^2)$ Funkcija je parna ✓

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod↓↓ $\mathcal{H}2$

IME I PREZIME: Ante Podišić **BROJ INDEKSA:**

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

POPUNJAVA NASTAVNIK Broj ↓ bodova

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$$\begin{aligned} x+2y-z+u &= 3 \\ 2x+5y-z+2u &= 3 \\ 3x-y-2z+u &= 3 \\ x-y+3z-5u &= 3 \end{aligned}$$

6. Izračunati i provjeriti: $\lim_{x\to-1} \dfrac{|x|}{x}$. 5

Ukupno: 31

1. $z^4-(4-i)^3=0 \quad i^0=1, i^1=i, i^2=-1, i^3=-i$

$z^4=(4-i)^3$

$a^3-3a^2b+3ab^2-b^3$

$z^4=4^3-3\cdot4^2\cdot i+3\cdot4\cdot i^2-(i)^3$

$z^4=64-48i+12i-(-i)$

$z^4=64-59i$

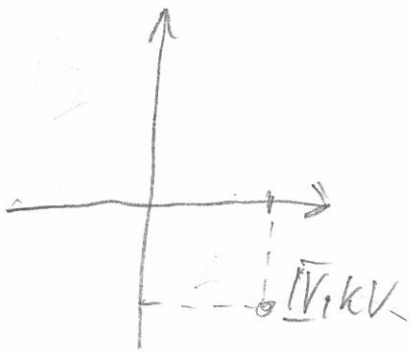
$z=\sqrt[4]{64-59i}$

$r=\sqrt{x^2+y^2}$

$r=\sqrt{64^2+(-59)^2}$

$r=\sqrt{4096+3481}$

$\boxed{r=87,05}$



$tg\,\varphi=\dfrac{y}{x} \Rightarrow tg\,\varphi=\dfrac{-59}{64}=-0,92$

$\varphi=2\pi-0,92$

$\boxed{\varphi=5,36}$

$z=\sqrt[4]{r}\left(\cos\dfrac{5,36+2k\pi}{4}+i\sin\dfrac{5,36+2k\pi}{4}\right)$

$k=0,1,2,3$

$k=0$

$z_1=3,05\left(\cos\dfrac{5,36}{4}+i\sin\dfrac{5,36}{4}\right)$

$z_1=3,05(0,22+0,97i)$

$\boxed{z_1=0,671+2,95i}$

$k=1$

$$z_2 = 3{,}05\left(\cos\frac{5{,}36+2\pi}{4} + i\sin\frac{5{,}36+2\pi}{4}\right)$$

$$z_2 = 3{,}05(-0{,}97 + 0{,}22\,i) \Rightarrow \boxed{z_2 = -2{,}95 + 0{,}671\,i}$$

$k=2$

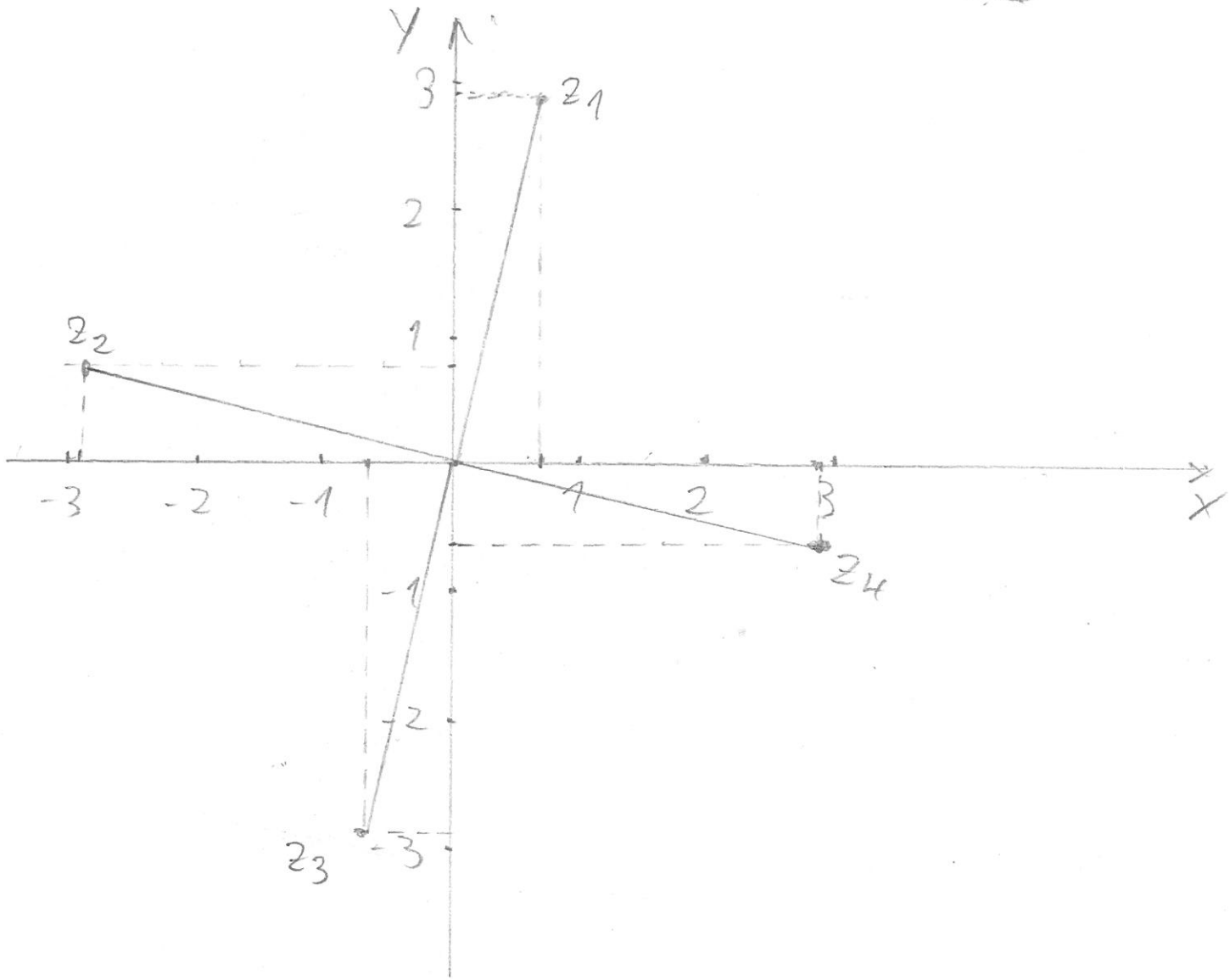
$$z_3 = 3{,}05\left(\cos\frac{5{,}36+4\pi}{4} + i\sin\frac{5{,}36+4\pi}{4}\right)$$

$$z_3 = 3{,}05(-0{,}22 - 0{,}97\,i) \Rightarrow \boxed{z_3 = -0{,}671 - 2{,}95\,i}$$

$k=3$

$$z_4 = 3{,}05\left(\cos\frac{5{,}36+6\pi}{4} + i\sin\frac{5{,}36+6\pi}{4}\right)$$

$$z_4 = 3{,}05(0{,}97 - 0{,}22\,i) \Rightarrow \boxed{z_4 = 2{,}95 - 0{,}67\,i}$$



IME I PREZIME: Ante Podišić BROJ INDEKSA:

2. $f(x) = x - \sqrt{x^2-3}$

1) Domena

$D_f(x): x \in [-3, 3]$ ✗

$x^2 - 3 \geq 0$

$x^2 \geq 3 / \sqrt{\ }$

$\boxed{x = \pm 3}$

2) Asimptote

V.A. $\lim_{x \to 3} x - \sqrt{x^2-3} = 3 - \sqrt{9-3} = \underline{3 - \sqrt{6}}$

Funkcija nema V.A jer nije razlomljena!

H.A. u $\lim_{x \to \infty} x - \sqrt{x^2-3} = \lim_{x \to \infty} \frac{x}{x} - \sqrt{\frac{x^2}{x^2}\left(\frac{3}{x^2}\right)} = 1 - 1 = 0$

$1 \quad 1 \quad 0$

H.A u 0 ! 6

$y = 0$ OVO JE DESNA H.A.!!!

Kosa

$y = kx + l$

$k \Rightarrow \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x - \sqrt{x^2-3}}{x} = \frac{1 - 1}{1} = \frac{0}{1} = 0$

Kosa nema ! 6 ✗

3) Derivacija

$f'(x) = 1 - \frac{1}{2}(x^2-3)^{-\frac{1}{2}} \cdot 2x$ ✓

$f'(x) = 1 - \frac{2x}{2(x^2-3)^{\frac{1}{2}}}$

$f'(x) = 1 - \frac{x}{\sqrt{x^2-3}}$

$f''(x) = -\frac{\frac{x^2-3}{\sqrt{x^2-3}} - x \cdot (x^2-3)^{-\frac{1}{2}}}{x^2-3}$

$\boxed{\quad}$

3. $g(t)=\ln(4-x^2)$

Domena

$x^2-4>0$

$x^2=4 \mid \sqrt{}$

$x=\pm 2$

$(4-x)^2=(2-x)(2+x)$ ✗

	-2		0		2	$+\infty$
$2(-x)$		$-$		$+$		$-$
$2+x$		$-$		$+$		$+$
		$+$		$+$		

$D_f(t): x \in \langle -2, 2 \rangle$ ✓

Parnost/neparnost

$g(-x)=\ln(4-(-x))^2$ $\quad g(x)=g(-x)$,

$g(-x)=\ln(4-x) \rightarrow$ funkcija je neparna ✓

Derivacija

$g(t)=\ln(4-x^2)$

$g'(t)=\frac{1}{4-x^2}\cdot(4-x^2)'$

$g'(t)=\frac{1}{4-x^2}\cdot(-2x)$

$g'(t)=\frac{-2x}{4-x^2}$

$g''(t)=\frac{-2\cdot(4-x^2)-(-2x)\cdot(-2x)}{4-x^2}$ ✓

$g''(x)=\frac{2\cdot(4-x^2)-4x^2}{4-x^2}$ ✗

Zakrivljenost

	-2		0		2
$f''(x)$		$+$		$+$	
$f(x)$		∪		∪	

Funkcija je u cijeloj svojoj domeni konkavna!

✗

4. IME I PREZIME: Ante Podišić BROJ INDEKSA:

$$f(x)=\frac{x^2-3x-(3+1)}{x^2+1}$$

1) Domena

$x^2+1\neq 0$

$x(x+1)=0$

$\boxed{x=0}$ $\boxed{x=-1}$

	$-\infty$		-1		0		$+\infty$
x		$-$		$-$	0	$+$	
$x+1$		$-$	0	$+$		$+$	
		$+$		$-$		$+$	

$D_f(x): x\in\langle-\infty,-1\rangle\cup\langle 0,+\infty\rangle$

2) Asimptote

$$V.A=\lim_{x\to -1}\frac{x^2-3x-(3+1)}{x^2+1}=\lim_{x\to -1}\frac{1+3-4}{1+1}=\frac{0}{1}=0$$

V.A ... $\boxed{x=-1}$ ✗ $\boxed{x=0}$ ✗

$$\lim_{x\to 0}\frac{-3-1}{+1}=\frac{-4}{1}=-4$$

$$H.A\ \lim_{x\to+\infty}\frac{x^2-3x-(3+1)}{x^2+1}\ /L.P$$

$$\lim_{x\to\infty}\frac{2x-3\ /:x}{2x+1\ /:x}=\lim_{x\to\infty}\frac{\frac{2x}{x}-\frac{3}{x}}{\frac{2x}{x}+\frac{1}{x}}=\frac{2}{2}=1$$

($\frac{2x}{x}=2$, $\frac{3}{x}=0$, $\frac{1}{x}=0$)

H.A ... $\boxed{y=1}$ obostrana H.A

Kosa

Kosa nema jer funkcija ima vertikalna i horizontalna asimptota!

(3.) Parnost - neparnost

$f(-x) = \frac{-x^2-3(-x)-(3+1)}{-x^2+1} = \frac{x^2+3x-(3+1)}{x^2+1}$ Niti parna niti neparna!

Funkcija nije trigonometrijska pa nije periodična!

(4.) Nultočke

$x^2-3x-(3+1)=0$

$x^2-3x-4=0$

$x_{1,2} = \frac{3 \pm \sqrt{9+16}}{2}$

$x_{1,2} = \frac{3 \pm 5}{2} \Rightarrow$ $x_1 = 4$, $x_2 = -1$ $\Rightarrow$ T(4, 0)

$h(0) = \frac{16-12-4}{16+1}$ $h(-1) = \frac{1+3-4}{1+1}$

$h(0) = \frac{0}{17} = 0$ // $h(-1) = \frac{0}{2} = 0$ //

(5.) Derivacija

$f'(x) = \frac{(2x-3)(x^2+1)-(x^2-3x-4)\cdot 2x}{(x^2+1)^2}$

$f'(x) = \frac{2x^3+2x-3x^2-3-(2x^3+6x^2-8x)}{(x^2+1)^2}$

$f'(x) = \frac{3x^2+10x-3}{(x^2+1)^2}$

$f''(x) = \frac{(6x+10)\cdot(x^2+1)^2-(3x^2+10x-3)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$

IME I PREZIME: Anto Podišić BROJ INDEKSA:

(6) Ekstremi

$$f'(x) = \frac{3x^2+10x-3}{(x+1)^2} = 0$$

	$-\infty$		-1		0		$+\infty$
$f'(x)$		+	//			+	
$f(x)$		↗	//			↗	

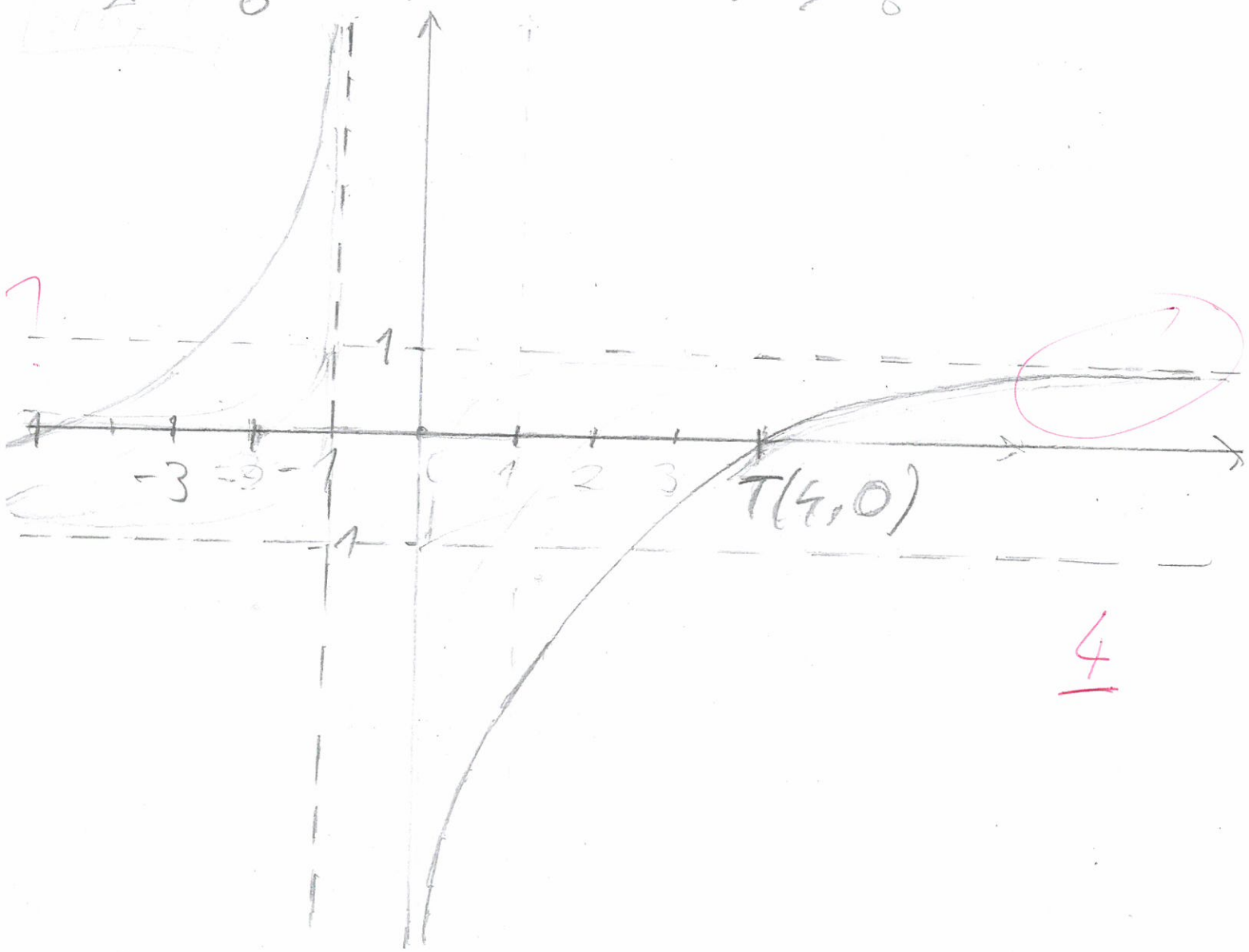
$3x^2+10x-3=0$

$$x_{1/2} = \frac{-10 \pm \sqrt{100+36}}{6}$$

$$x_{1/2} = \frac{-10 \pm 11,66}{6}$$

$$x_1 = \frac{1,66}{6} = 0,27$$

Fankcija nema ekstroma ✓
Nema inflekcije ✓
Fankcija raste uz skok poslije intervala, gdje $x \in \langle -1, 0 \rangle$ ✓



4

6. $\lim_{x \to -1} \frac{|x|}{x} = \lim_{x \to -1} \frac{1}{-1} = -1$ ✓

Provjera

$$\lim_{x \to -\infty} \frac{|x|}{x} = \lim_{x \to -\infty} \frac{\frac{x}{x}}{\frac{x}{x}} = \frac{1}{1} = 1$$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ $\mathcal{H}2$

IME I PREZIME: GORAN BASIOL **BROJ INDEKSA:** 17-1-0031-2010

ZAOKRUŽITI AKO ŽELITE: (ustmeni kod prof. Uglešića)

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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6. Izračunati i provjeriti: $\lim_{x\to -1} \dfrac{|x|}{x}$. 5

Ukupno: 0

6) $\lim_{x\to -1} \frac{|x|}{x} \Rightarrow \lim_{x\to -1} \frac{|-1|}{-1} \Rightarrow \lim_{x\to -1} \frac{1}{-1} = -1$ ✓

2) $f(x) = x - \sqrt{x^2-3}$

$f'(x) = 1 - \sqrt{2x} - 0$ ✗

$f'(x) = 1 - \sqrt{2x}$

$f'(x) = 1 - (2x)^{\frac{1}{2}}$

$f''(x) = -\left(\frac{1}{2}(2x)\right)$

$f''(x) = -x$

MATEMATIKA 1: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇓⇓ H2

IME I PREZIME: Jelena Hateš **BROJ INDEKSA:** 17-2-0103-2011

ZAOKRUŽITI AKO ŽELITE: ustmeni kod prof. Uglešića

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6. Izračunati i provjeriti: $\lim_{x\to -1} \dfrac{|x|}{x}$. 5

Ukupno: 22

1* Svođenje na $z = x+yi$

$z^4 - (4-i)^3 = 0$

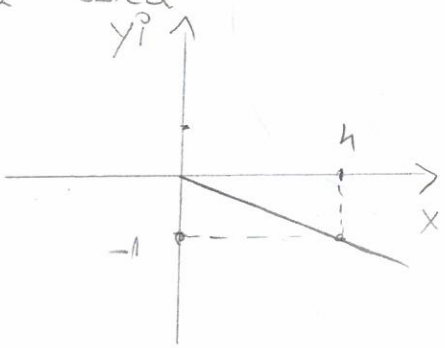
$z^4 = (4-i)^3$

$x = 4$

$y = -1$

$w = 4 - i$

2* skica



3* udaljenost od ishodišta

$r = |r| = \sqrt{x^2+y^2}$

$r = \sqrt{16+1} = \sqrt{17}$

4* $\operatorname{tg}\varphi = \frac{y}{x} = \frac{-1}{4} = \varphi = 34,5^\circ 57' 50''$

3. 345° * 317°

5*

$w = \sqrt{17}(\cos 34,5^\circ 57' 50'' + i \sin 34,5^\circ 57' 50'')$

$w^3 = (\sqrt{17})^3(\cos 3\cdot 34,5^\circ 57' 50'' + i \sin 34,5^\circ 57' 50'')$

$w^3 = 17\sqrt{17}(\cos 317^\circ 53' 50'' + i \sin 317^\circ 53' 50'')$

$\sqrt[4]{17\sqrt{17}}\left(\cos \dfrac{317^\circ 53' 50'' + k\cdot 360^\circ}{4} + i \sin \dfrac{317^\circ 53' 50''}{4}\right)$

6* $k = 0, 1, 2, 3$

$\rightarrow k = 0$

$z = \sqrt[n]{r}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right)$

$z_0 = \sqrt[8]{17^3}\left(\cos 79^\circ 28' 28'' + i\sin 79^\circ 28' 28''\right)$

$\rightarrow k = 1$

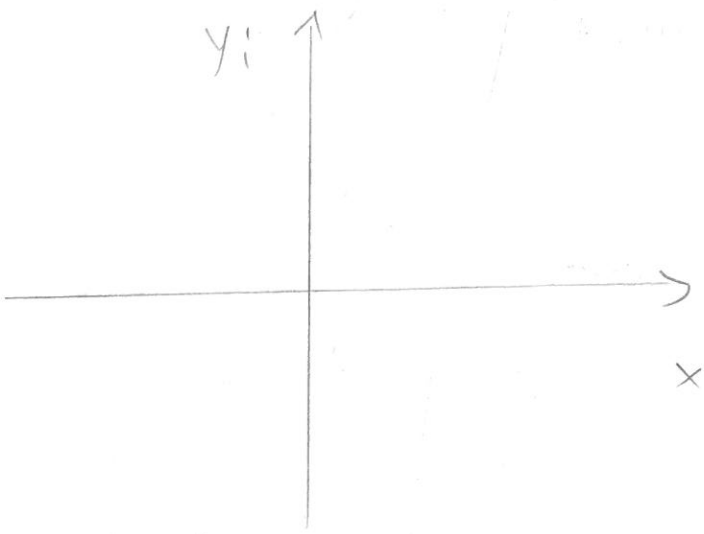
$z_1 = \sqrt[8]{17^3}\left(\cos 169^\circ 28' 28'' + i\sin 169^\circ 28' 28''\right)$

$\rightarrow k = 2$

$z_2 \sqrt[8]{17^3}\left(\cos 259^\circ 28' 28'' + i\sin 259^\circ 28' 28''\right)$

$\rightarrow k = 3$

$z_3 = \sqrt[8]{17^3}\left(\cos 349^\circ 28' 28'' + i\sin 349^\circ 28' 28''\right)$



IME I PREZIME: Jelena Maleš BROJ INDEKSA: 17-2-0103-2011

2. Odrediti domenu, sve asimptote i drugu derivaciju

$f(x) = x - \sqrt{x^2-3}$

1* DOMENA

$x^2 - 3 \geq 0$

$x^2 \geq 3 \ / \sqrt{\ }$

$-\sqrt{3} \geq x \geq \sqrt{3}$ ✗

$D(f): x \in \langle -\infty, -\sqrt{3}] \ [\sqrt{3}, +\infty \rangle$ ✓

2* ASIMPTOTE

V.A. Nema

$\lim_{x \to -\sqrt{3}^-} (x - \sqrt{x^2-3}) = -\sqrt{3} - 0 = -\sqrt{3}$

$\lim_{x \to -\sqrt{3}^+} (x - \sqrt{x^2-3}) = -\sqrt{3}$

$\lim_{x \to \sqrt{3}^-} = (x - \sqrt{x^2-3}) = \sqrt{3}$

$\lim_{x \to \sqrt{3}^+} = (x - \sqrt{x^2-3}) = \sqrt{3}$

H.A. $\lim_{x \to \infty} (x - \sqrt{x^2-3}) \cdot \frac{x + \sqrt{x^2-3}}{x + \sqrt{x^2-3}} = \frac{x^2 - x^2 + 3}{x + \sqrt{x^2-3}} = \lim_{x \to \infty} \frac{3 \ / : x}{x + \sqrt{x^2-3} \ / : x} = \frac{0}{\infty} = 0$

$y = 0$ ✓

OVO JESAMO RADILI H.A.

Kose nema jer postoji horizontalna

$f'(x) = 1 - \frac{1}{2\sqrt{x^2-3}} \cdot 2x = \frac{\sqrt{x^2-3} - x}{\sqrt{x^2-3}}$

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$$x) = \frac{(\sqrt{x^2-3} - x)' \cdot (\sqrt{x^2-3}) - (\sqrt{x^2-3} - x) \cdot (\sqrt{x^2-3})'}{(\sqrt{x^2-3})^2}$$

$$f''(x) = \frac{\left(\frac{1}{2\sqrt{x^2-3}} \cdot 2x - 1\right) \cdot (\sqrt{x^2-3}) - (\sqrt{x^2-3} - x) \cdot \left(\frac{1}{2\sqrt{x^2-3}} \cdot 2x\right)}{x^2-3}$$

$$f''(x) = \frac{\left(\frac{1}{2\sqrt{x^2-3}} \cdot 2x - 1\right)\sqrt{x^2-3} - (\sqrt{x^2-3} - x) \cdot \frac{1}{2\sqrt{x^2-3}} \cdot 2x}{x^2-3}$$

$$f''(x) = \frac{\left(\frac{x}{\sqrt{x^2-3}} - 1\right)\sqrt{x^2-3} - (\sqrt{x^2-3} - x) \cdot \frac{x}{\sqrt{x^2-3}}}{x^2-3}$$

$$f''(x) = \frac{x - \sqrt{x^2-3} - \left(x - \frac{x^2}{\sqrt{x^2-3}}\right)}{x^2-3}$$

$$f''(x) = \frac{x - \sqrt{x^2-3} - x + \frac{x^2}{\sqrt{x^2-3}}}{x^2-3} = \frac{-(x^2-3) + x^2}{\sqrt{x^2-3}} = \frac{x^2 - 3 + x^2}{(x^2-3)\sqrt{x^2-3}} = \frac{3}{(x^2-3)\sqrt{x^2-3}}$$

⑥ $\lim_{x \to -1} \frac{|x|}{x} = \lim_{x \to -1} \frac{-y}{x} = \lim_{x \to -1} (-1) = -1$