MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: Domagoj Knežević **BROJ INDEKSA:**	

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (4x+3)\,dxdydz$. — 20

2. Izračunati dvostruki integral: $\iint\limits_S x + y\,dxdy$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 4 \text{ i } x \ge y\}$. — 20

3. Izračunati $\int\limits_{(-1,-1)}^{(4,3)} (2x + 2y^2)\,dx + (4xy - 3)\,dy$. — 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{4}{2},0)$, $B(4,\frac{4}{2})$ i $C(\frac{4}{2},\frac{3}{2})$. Izračunati dvostruki integral — ~~15~~

$$\iint\limits_X y^3\,dxdy.$$

5. Neka je kvadrat $K = \{(x,y,z) : x = 3,\ y \in [-4,4],\ z \in [-4,4]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x\,dS$? — 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — ~~15~~

$$y'''(t) - y''(t) - y'(t) + y(t) = t, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 55

1. $r = 2$ $\quad \iiint (4x+3)\,dxdydz = ?$

$x = r\cos\varphi$
$y = r\sin\varphi$
$z = z$

$x^2 + y^2 + z^2 \le 4$

$r^2 \underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1} + z^2 \le 4$

$r^2 + z^2 \le 4$

$r^2 \le 4 - z^2$

$r \le \sqrt{4 - z^2}$

$\varphi \in [0, 2\pi]$
$r \in [0, \sqrt{4 - z^2}]$
$z \in [-2, 2]$

$$I = \int_{-2}^{2}\int_{0}^{\sqrt{4-z^2}}\int_{0}^{2\pi} (4r^2\cos\varphi + 3r)\,d\varphi\,dr\,dz = \int_{-2}^{2}\int_{0}^{\sqrt{4-z^2}} \Big(\cancel{4r^2 \sin\varphi\Big|_0^{2\pi}} + 3r\varphi\Big|_0^{2\pi}\Big)\,dr\,dz$$

$(\sin 2\pi - \sin 0 = 0 - 0)$

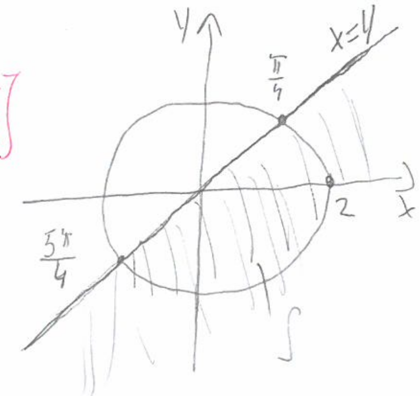
$$= \int_{-2}^{2}\int_{0}^{\sqrt{4-z^2}} 6\pi r\,dr\,dz = 3\pi \int_{-2}^{2} r^2\Big|_0^{\sqrt{4-z^2}} dz = 3\pi \int_{-2}^{2} (4 - z^2)\,dz = 3\pi \left(4z - \frac{1}{3}z^3\right)\Big|_{-2}^{2}$$

$$I = \cancel{3}\pi \frac{32}{\cancel{3}} = 32\pi$$

2. $\iint_S (x+y)\,dx\,dy = ?$ $\quad x^2+y^2 \le 4 \quad x \ge y$

$\varphi \in \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ $\quad \varphi \in \left[\frac{5\pi}{4}, \frac{\pi}{4}\right]$

$r \in [0, 2]$



$x = r\cos\varphi$

$y = r\sin\varphi$

$r^2\underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1} \le 4$

$r^2 \le 4$

$r \le 2$

$$I = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\int_0^2 (r^2\cos\varphi + r^2\sin\varphi)\,dr\,d\varphi = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\left(\frac{1}{3}r^3\cos\varphi + \frac{1}{3}r^3\sin\varphi\right)\Big|_0^2\,d\varphi$$

$$I = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\left(\frac{8}{3}\cos\varphi + \frac{8}{3}\sin\varphi\right)d\varphi = \frac{8}{3}\sin\varphi\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} - \frac{8}{3}\cos\varphi\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} = \frac{8}{3}\left(\sin\frac{5\pi}{4} - \sin\frac{\pi}{4}\right) - \frac{8}{3}\left(\cos\frac{5\pi}{4} - \cos\frac{\pi}{4}\right)$$

$$I = \frac{8}{3}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) - \frac{8}{3}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) = \cancel{-\frac{8\sqrt{2}}{3}} + \cancel{\frac{8\sqrt{2}}{3}} = \boxed{0}$$ ✓

IME I PREZIME: DOMAGOJ KNEŽEVIĆ **BROJ INDEKSA:**

3.
$$\int_{(-1,-1)}^{(4,3)} (2x+2y^2)\,dx + (4xy-3)\,dy$$

$$w = \begin{pmatrix} 2x+2y^2 \\ 4xy-3 \\ 0 \end{pmatrix} = \text{grad } f. = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = 2x+2y^2 \Rightarrow f(x,y) = \int (2x+2y^2)\,dx = x^2+2xy^2+C(y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial[x^2+2xy^2+C(y)]}{\partial y} = 4xy-3$$

$$\cancel{4xy} + \frac{\partial C(y)}{\partial y} = \cancel{4xy} - 3$$

$$C(y) = -\int 3\,dy = -3y + C$$

$$f(x,y) = x^2+2xy^2-3y \quad ✓$$

$$f(4,3) - f(-1,-1) = (16+72-9) - (1-2+3) = 16+72-9 \underbrace{-1+2-3}_{-2} = \boxed{77} \quad ✓$$

7 5

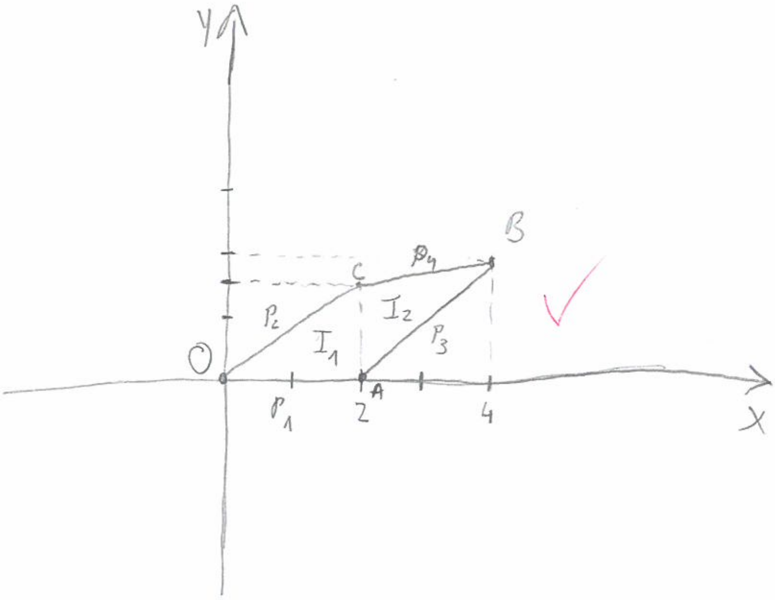
4. $O(0,0)$

$A(2,0)$

$B(4,2)$

$C\left(2,\frac{3}{2}\right)$

$\iint_X y^3\,dx\,dy = ?$



$O(0,0)$, $A(2,0)$: $\quad p_1 \dots\ y = 0$

$O(0,0)$, $C\left(2,\frac{3}{2}\right)$: $\quad p_2 \dots\ y = \frac{\frac{3}{2}}{2}x$, $\quad y = \frac{3}{4}x$

$A(2,0)$, $B(4,2)$: $\quad p_3 \dots\ y = \frac{2-0}{4-2}(x-2)$, $\quad y = 2-x$

$B(4,2)$, $C\left(2,\frac{3}{2}\right)$: $\quad p_4 \dots\ y-2 = \frac{\frac{3}{2}-2}{2-4}(x-4)$, $\quad y = \frac{1}{4}x+1$

$$I = I_1 + I_2 = \frac{81}{160} - \frac{3097}{240} = \boxed{-\frac{5951}{480}}$$

$$I_1 = \int_0^2\int_0^{\frac{3}{4}x} y^3\,dy\,dx = \int_0^2 \frac{1}{4}y^4\Big|_0^{\frac{3}{4}x}dx = \int_0^2 \frac{81}{1024}x^4\,dx = \frac{81}{1024}\cdot\frac{1}{5}x^5\Big|_0^2 = \frac{81}{1024}\cdot\frac{32}{5} = \boxed{\frac{81}{160}}$$

$$I_2 = \int_2^4\int_{2-x}^{\frac{1}{4}x+1} y^3\,dy\,dx = \frac{1}{4}\int_2^4 y^4\Big|_{2-x}^{\frac{1}{4}x+1}dx = \frac{1}{4}\int_2^4\left[\left(\frac{1}{16}x^2+1\right)\left(\frac{1}{16}x^2+\frac{1}{2}x+1\right)-\left(4-x^2\right)\left(4-8x+x^2\right)\right]d$$

$$I_2 = \frac{1}{4}\int_2^4\left(\frac{1}{256}x^4+\frac{1}{32}x^3+\frac{1}{16}x^2+\frac{1}{16}x^2+\frac{1}{2}x+1-16+32x\cancel{-4x^2}\cancel{+4x^2}-8x^3+x^4\right)dx$$

$$I_2 = \frac{1}{4}\int_2^4\left(\frac{257}{256}x^4-\frac{255}{32}x^3+\frac{1}{8}x^2+\frac{65}{2}x-15\right)dx = \frac{1}{4}\left(\frac{257}{1280}x^5-\frac{255}{128}x^4+\frac{1}{24}x^3+\frac{65}{4}x^2-15x\right)\Big|_2^4$$

$$I_2 = \frac{1}{4}\left(\frac{7967}{40}-\frac{3825}{8}+\frac{7}{3}+195+30\right) = -\frac{3097}{240}$$

IME I PREZIME: Domagoj Knežević **BROJ INDEKSA:**

6. $y'''(t) - y''(t) - y'(t) + y(t) = t$

$y(0) = 1$

$y'(0) = 0$

$y''(0) = 1$

$$\underbrace{s^3Y(s)} \overset{-s^2}{- s^2y(0)} - s\overset{0}{\cancel{y'(0)}} - \overset{-1}{y''(0)} - s^2Y(s) + \overset{+s}{sy(0)} + \overset{0}{\cancel{y'(0)}} - sY(s) + \overset{+1}{y(0)} + Y(s) = \frac{1}{s^2}$$

$$Y(s)(s^3 - s^2 - s + 1) - s^2 - \cancel{1} + s \cancel{+1} = \frac{1}{s^2}$$

$$Y(s)(s^3 - s^2 - s + 1) = \frac{s^4 - s^3 + 1}{s^2} ✓$$

$s^3 - 2s^2 + s + s^2 - 2s + 1 = s^3 - s^2 - s + 1$

$(s-1)(s+1)$

$s^2 + s - (s-1)$

$$Y(s) = \frac{s^4 + s^2 + 1}{s^2(s^3 - s^2 - s + 1)} ✗ = \frac{s^4 + s^2 + 1}{s^2(s-1)^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s-1)} + \frac{D}{(s-1)^2} + \frac{E}{(s+1)} \Big/ \cdot s^2(s-1)^2(s+1)$$

$$s^4 - s^3 + 1 = As(s^2 - 2s + 1)(s+1) + B(s^2 - 2s + 1)(s+1) + Cs^2(s^2 - 1) + Ds^2(s+1) + Es^2(s^2 - 2s + 1)$$

$$s^4 - s^3 + 1 = As^4 - As^3 - As^2 + As + Bs^3 - Bs^2 - Bs + B + Cs^4 - Cs^2 + Ds^3 + Ds^2 + Es^4 - 2Es^3 + Es^2$$

(0) $\boxed{B = 1}$

(1) $A - 1 = 0$

$\boxed{A = -1}$

(2) $\cancel{A} \cancel{-1} - C + D + E = 0$

$-C + D + E = 0$

$-(2 - E) + D + E = 0$

$-2 + E + D + E = 0$

$D + \cancel{2E} = 2$

$D - \cancel{2E} = -3$

$2D = -1$

$\boxed{D = -\frac{1}{2}}$

(3) $1 + 1 + D - 2E = -1$

$D - 2E = -3$

$-\frac{1}{2} - 2E = -3$

$\boxed{E = \frac{5}{4}}$

(4) $-1 + C + E = 1$

$C + E = 2$

$C = 2 - E$

$\boxed{C = \frac{3}{4}}$

$$Y(s) = -\frac{1}{s} + \frac{1}{s^2} + \frac{3}{4}\frac{1}{s-1} - \frac{1}{2}\frac{1}{(s-1)^2} + \frac{5}{4}\frac{1}{s+1}$$

$$\boxed{Y(s) = -1 + t + \frac{3}{4}e^t + \frac{1}{2}te^t + \frac{5}{4}e^{-t}}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Siniša Cvar

BROJ INDEKSA: 17-2-0066-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (4x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20 15

2. Izračunati dvostruki integral: $\iint\limits_S x + y\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y) \in \mathbb{R}^2 \,:\, x^2+y^2 \leq 4 \text{ i } x \geq y\}$. 20

3. Izračunati $\int\limits_{(-1,-1)}^{(4,3)} (2x+2y^2)\,\mathrm{d}x + (4xy-3)\,\mathrm{d}y$. 15 12

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{4}{2},0)$, $B(4,\frac{4}{2})$ i $C(\frac{4}{2},\frac{3}{2})$. Izračunati dvostruki integral 15
$$\iint\limits_X y^3\mathrm{d}x\mathrm{d}y.$$

5. Neka je kvadrat $K = \{(x,y,z) \,:\, x = 3,\ y \in [-4,4],\ z \in [-4,4]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x\mathrm{d}S$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15
$$y'''(t) - y''(t) - y'(t) + y(t) = t, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

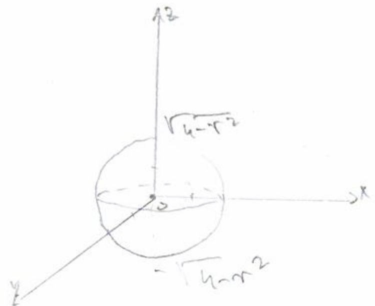
Ukupno: 47

1.

$r = 2$

$T(0,0,0)$

$\iiint\limits_K (4x+3)\,dx\,dy\,dz$



$r \in [0,2]$

$\varphi \in [0, 2\pi]$

$z \in [-\sqrt{4-r^2}, \sqrt{4-r^2}]$

$x = r\cos\varphi + x_0 = r\cos\varphi$

$y = r\sin\varphi + y_0 = r\sin\varphi$

$z = z$

$dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$x^2+y^2+z^2 = R^2$

$x^2+y^2+z^2 = 2^2$

$x^2+y^2+z^2 = 4$

$x^2+y^2 = r^2$

$r^2+z^2 = 4$

$z^2 = 4-r^2 \;/\sqrt{\ }$

$z = \pm\sqrt{4-r^2}$

$z \in [-\sqrt{4-r^2}, \sqrt{4-r^2}]$

$$\iiint (4x+3)\,dx\,dy\,dz = \int_0^{2\pi}\int_0^2\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} (4\cdot(r\cos\varphi)+3)\,r\,dz\,dr\,d\varphi$$

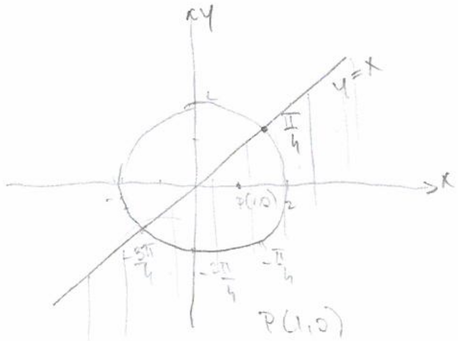
$$= \int_0^{2\pi}\int_0^2\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} (4r\cos\varphi+3)\,r\,dz\,dr\,d\varphi = \int_0^{2\pi}\int_0^2\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} (4r^2\cos\varphi + 3r)\,dz\,dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^2 \left[(4r^2\cos\varphi+3r)\cdot\left(\sqrt{4-r^2}-\sqrt{4-r^2}\right)\right]dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^2 \left(4r^2\sqrt{4-r^2}\cos\varphi - 4r^2\sqrt{4-r^2}\cos\varphi + 3r\cancel{\sqrt{4-r^2}} - 3r\cancel{\sqrt{4-r^2}}\right)dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^2 0\,dr\,d\varphi = 0$$

2. Izračunati dvostruki integral $\iint_S x+y\,dx\,dy$, gdje je $S=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\leq 4 \text{ i } x\geq y\}$



$x^2+y^2\leq 4 \to$ krug

$r^2=4$

$r=2$

$x\geq y$

$y=x \to$ pravac

$T(0,0)$

$r=2$

$r\in[0,2)$ $\quad x=r\cos\varphi$

$y=r\sin\varphi$

$\varphi\in\left[-\frac{3\pi}{4},\frac{\pi}{4}\right]$ $\quad dx\,dy=r\,dr\,d\varphi$

$P(1,0)$

$1\geq 0$ zadovoljava

$\left[-\frac{3\pi}{4},\frac{\pi}{4}\right]$

$$\iint_S x+y\,dx\,dy=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\int_0^2 (r\cos\varphi+r\sin\varphi)\cdot r\,dr\,d\varphi$$

$$=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\int_0^2 (r^2\cos\varphi+r^2\sin\varphi)\,dr\,d\varphi=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\left(\frac{r^3}{3}\cos\varphi+\frac{r^3}{3}\sin\varphi\right)\Big|_0^2 d\varphi$$

$$=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\left(\frac{2^3}{3}\cos\varphi+\frac{2^3}{3}\sin\varphi-\left(\frac{0^3}{3}\cos\varphi+\frac{0^3}{3}\sin\varphi\right)\right)d\varphi$$

$$=\int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}\left(\frac{8}{3}\cos\varphi+\frac{8}{3}\sin\varphi\right)d\varphi=\left[\frac{8}{3}\cdot\sin\varphi+\left(\frac{8}{3}\cdot-\cos\varphi\right)\right]\Big|_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}$$

$$=\frac{8}{3}\sin\varphi-\frac{8}{3}\cos\varphi\Big|_{-\frac{3\pi}{4}}^{\frac{\pi}{4}}=\frac{8}{3}\sin\frac{\pi}{4}-\frac{8}{3}\cos\frac{\pi}{4}-\left(\frac{8}{3}\underbrace{\sin\left(-\frac{3\pi}{4}\right)}_{=-\frac{\sqrt{2}}{2}}-\frac{8}{3}\underbrace{\cos\left(-\frac{3\pi}{4}\right)}_{-\frac{\sqrt{2}}{2}}\right)$$

$$=\frac{8}{3}\cdot\frac{\sqrt{2}}{2}-\frac{8}{3}\cdot\frac{\sqrt{2}}{2}-\left(\frac{8}{3}\cdot\left(-\frac{\sqrt{2}}{2}\right)-\frac{8}{3}\cdot\left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$=\frac{8\sqrt{2}}{6}-\frac{8\sqrt{2}}{6}-\left(-\frac{8\sqrt{2}}{6}+\frac{8\sqrt{2}}{6}\right)$$

$$=0-0=0$$

IME I PREZIME: SILVIJAN CUAR **BROJ INDEKSA:** 17-2-0066-2010

③ Izračunati $\displaystyle\int_{(-1,-1)}^{(4,3)} (2x+2y^2)\,dx + (4xy-3)\,dy$

$$\begin{pmatrix} 2x+2y^2 \\ 4xy-3 \end{pmatrix} = -\text{grad}.\, f = \begin{pmatrix} -\partial x f \\ -\partial y f \end{pmatrix}$$

$$\partial x f = -(2x+2y^2) = -2x-2y^2 \Rightarrow f = \int -2x-2y\,dx = -\cancel{2}\cdot\frac{x^2}{\cancel{2}} - 2xy + c'(y)$$

$$f = -x^2 - 2xy + c'(y)$$

$$\partial y f = -(4xy-3) = -4xy+3 \Rightarrow \cancel{-4xy} + c'(y) = \cancel{-4xy}+3 \Rightarrow c'(y) = 3 \Rightarrow$$

$$\Rightarrow c(y) = \int 3\,dy = 3y + \underset{0}{c} = f$$

$$\boxed{f(x,y) = -x^2 - 2xy + 3y}$$

$+3(-1)$

$$f(-1,-1) - f(4,3) = -(-1)^2 - 2\cdot(-1)\cdot(-1) - (-4^2 - 2\cdot 4\cdot 3 + 3\cdot 3)$$

$$= -1+2-(-16-24+9)$$

$$= -3+16+24-9$$

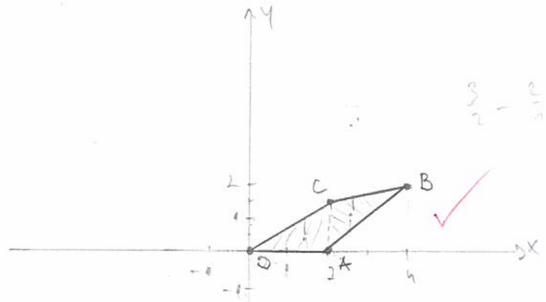
$$= 28$$

---

④ X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{4}{2},0)$, $B(4,\frac{4}{2})$ i $C(\frac{4}{2},\frac{3}{2})$. Izračunati dvostruki integral

$$\iint_X y^3\,dx\,dy$$

$O(0,0)$
$A(2,0)$
$B(4,2)$
$C(2,\frac{3}{2})$



$\overline{OA} \ldots (x_2-x_1)(y-y_1) = (y_2-y_1)(x-x_1)$

$(2-0)(y-0) = (0-0)\cdot(x-0)$

$2\cdot(y-0) = 0\cdot(x-0)$

$2y-0 = 0$

$2y = 0 \;/:2$

$y = 0$

$\overline{AB} \ldots (4-2)(y-0) = (2-0)(x-2)$

$2(y-0) = 2(x-2)$

$2y-0 = 2x-4$

$2y = 2x-4 \;/:2$

$\underline{y = x-2}$

$\overline{BC} \ldots (2-4)(y-2) = (\frac{3}{2}-2)(x-4)$

$-2(y-2) = -\frac{1}{2}(x-4)$

$-2y+4 = -\frac{1}{2}x+2$

$-2y = -\frac{1}{2}x+2-4$

$-2y = -\frac{1}{2}x-2 \;/:(-2)$

$\underline{y = x+1}$

$\overline{OC} \ldots (2-0)(y-0) = (\frac{3}{2}-0)(x-0)$

$2\cdot(y-0) = \frac{3}{2}\cdot(x-0)$

$2y-0 = \frac{3}{2}x-0$

$2y = \frac{3}{2}x \;/:2$

$\boxed{y = \frac{3}{4}x}$

$$\iint_X y^3\,dx\,dy = \int_0^2\int_0^{\frac{3}{4}x} y^3\,dy\,dx + \int_2^4\int_{x-2}^{x+1} y^3\,dy\,dx$$

$$= \int_0^2 \frac{y^4}{4}\Big|_0^{\frac{3}{4}x}\,dx + \int_2^4 \frac{y^4}{4}\Big|_{x-2}^{x+1}\,dx$$

$$= \int_0^2 \frac{(\frac{3}{4}x)^4}{4}\,dx + \int_2^4 \left[\frac{(x+1)^4}{4} - \left(\frac{(x-2)^4}{4}\right)\right]dx$$

$$\int (x+1)^4\,dx = \quad x+1=t, \quad dx = dt$$

$$\int t^4\,dt$$

$$\frac{t^5}{5} = \frac{(x+1)^5}{5}$$

$$= \int_0^2 \left(\frac{1}{4}\cdot\frac{3}{4}x^4\right) + \int_2^4 \frac{1}{4}\cdot(x+1)^4 - \left(\frac{1}{4}\cdot(x-2)^4\right)dx$$

$$\int (x-2)^4\,dx = \quad x-2=t, \quad dx = dx$$

$$\int t^4\,dt$$

$$\frac{t^5}{5}$$

$$= \frac{1}{4}\cdot\frac{3}{4}\cdot\frac{x^5}{5}\Big|_0^2 + \left(\frac{1}{4}\cdot\frac{(x+1)^5}{5} - \left(\frac{1}{4}\cdot\frac{(x-2)^5}{5}\right)\right)\Big|_2^4$$

$$= \frac{3}{16}\cdot\frac{2^5}{5} + \left(\frac{1}{4}\cdot\frac{(4+1)^5}{5} - \left(\frac{1}{4}\cdot\frac{(4-2)^5}{5}\right) - \left(\frac{1}{4}\cdot\frac{(2+1)^5}{5} - \left(\frac{1}{4}\cdot\frac{(2-2)^5}{5}\right)\right)\right)$$

$$= \frac{6}{5} + \left(\frac{625}{4} - \frac{8}{5}\right) - \left(\frac{233}{20}\right) = \frac{721}{5} \approx 144{,}2$$

---

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:

$$y'''(t) - y''(t) - y'(t) + y(t) = t \qquad y(0)=1,\ y'(0)=0,\ y''(0)=1$$

$$s^3Y(s) - s^2\underset{1}{y(0)} - \cancel{s\,\underset{0}{y'(0)}} - \underset{1}{y''(0)} - s^2Y(s) - s\,\underset{1}{y(0)} - \cancel{\underset{0}{y'(0)}} - sY(s) - \underset{1}{y(0)} + Y(s) = \frac{1}{s^2}$$

$$s^3Y(s) - s^2 - 1 - s^2Y(s) - s - sY(s) - 1 + Y(s) = \frac{1}{s^2}$$

$$Y(s)(s^3 - s^2 - s + 1) - s^2 - 1 - s - 1 = \frac{1}{s^2}$$

$$Y(s)(s^3 - s^2 - s + 1) = \frac{s^2 + s + 2}{s^2} \quad \Big/ : (s^3 - s^2 - s + 1)$$

$$Y(s) = \frac{\frac{s^2+s+2}{s^2}}{s^3 - s^2 - s + 1}$$

$$Y(s) = \frac{s^2+s+2}{s^2\cdot(s^3 - s^2 - s + 1)} = \frac{s^2+s+2}{(s^5 - s^4 - s^3 + s^2)} = \frac{s^2+s+2}{s^2(s^3 - s^2 - s + 1)}$$

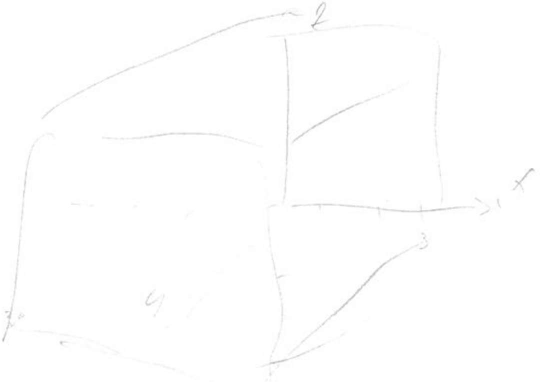
$$Y(s) = \frac{s^2+s+2}{s^2\cdot(s-1)(s^2-1)} \Big/ s^2(s-1)(s^2-1)$$

$$x^3 - x^2 - x + 1 = x^2(x-1) - 1(x-1) =$$
$$= (x-1)(x^2-1)$$
$$= (x-1)^2(x+1)$$

$$s^2 + s + 2 = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{Ds+E}{s^2-1}$$

$\rightarrow$

Ime i prezime: SILVIJAN CVAR **Broj indeksa:** 17-2-0266-2010



$$y(t) = 3+2t-\frac{7}{2}e^t+\frac{1}{2}+\frac{1}{2}te^t-\frac{5}{2}te^t$$

NASTAVAK LAPLACE

(5.)

$$s^2+s+2 = As(s-1)(s^2-1)+B(s-1)(s^2-1)+Cs^2(s^2-1)+(Ds+E)\cdot s^2(s-1)$$

$$s^2+s+2 = As(s^3-s-s^2+1)+B(s^3-s-s^2+1)+Cs^4-Cs^2+(Ds+E)\cdot(s^3-s^2)$$

$$s^2+s+2 = As^4-As^2-As^3+\boxed{As}+Bs^3-\boxed{Bs}-Bs^2+B+Cs^4-Cs^2+Ds^4-Ds^3+Es^3-Es^2$$

$$s^2+s+2 = (A+C+D)s^4+(-A+B-D+E)s^3+(-A-B-C-E)s^2+(A-B)s+B$$

$A+C+D=0$

$-A+B-D+E=0$

$-A-B-C-E=1$

$A-B=1 \rightarrow A-2=1$

$\boxed{B=2}$ $\quad A=1+2$

$\boxed{A=3}$

$-A+B-D+E=0 \quad /\cdot(-1)$

$-A-B-C-E=1$

$A-B+D-E=0$

$-A-B-C-E=1$

$-2B-2E=1$

$-2\cdot 2-2E=1$

$-4-2E=1$

$-2E=1+4$

$-2E=5 \quad /:(-2)$

$\boxed{E=-\frac{5}{2}}$

$-A-B-C-E=1$

$-3-2-C+\frac{5}{2}=1$

$-5-C+\frac{5}{2}=1$

$-C=1+5-\frac{5}{2}$

$-C=6-\frac{5}{2}$

$-C=\frac{7}{2} \quad /:(-1)$

$\boxed{C=-\frac{7}{2}}$

$A+C+D=0$

$3-\frac{7}{2}+D=0$

$-\frac{1}{2}+D=0$

$\boxed{D=\frac{1}{2}}$

$$Y(s)=\frac{3}{s}+\frac{2}{s^2}+\frac{-\frac{7}{2}}{s-1}+\frac{\frac{1}{2}s-\frac{5}{2}}{s^2-1}$$

$$Y(s)=3\cdot\frac{1}{s}+2\cdot\frac{1}{s^2}-\frac{7}{2}\cdot\frac{1}{s-1}+\frac{1}{2}\frac{s}{(s-1)^2}-\frac{5}{2}\cdot\frac{1}{(s-1)^2}$$

$$y(t)=3+2t-\frac{7}{2}e^t+\frac{1}{2}(1+t)e^t-\frac{5}{2}\cdot(te^t)$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: DOMAGOJ NEKIĆ **BROJ INDEKSA:** 17-2-0028-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (4x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20

2. Izračunati dvostruki integral: $\iint\limits_S x+y\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y)\in\mathbb{R}^2 : x^2+y^2 \le 4 \text{ i } x \ge y\}$. 20

3. Izračunati $\int\limits_{(-1,-1)}^{(4,3)} (2x+2y^2)\,\mathrm{d}x + (4xy-3)\,\mathrm{d}y$. 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{4}{2},0)$, $B(4,\frac{4}{2})$ i $C(\frac{4}{2},\frac{3}{2})$. Izračunati dvostruki integral 15
$$\iint\limits_X y^3\,\mathrm{d}x\mathrm{d}y.$$

5. Neka je kvadrat $K = \{(x,y,z) : x = 3,\ y\in[-4,4],\ z\in[-4,4]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x\,\mathrm{d}S$? 15

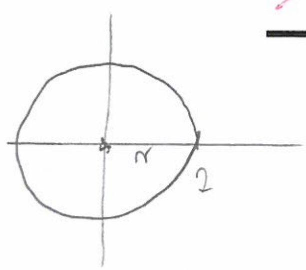
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15
$$y'''(t) - y''(t) - y'(t) + y(t) = t, \quad y(0)=1,\ y'(0)=0,\ y''(0)=1.$$

Ukupno:

1. $\iiint\limits_K (4x+3)\,dx\,dy\,dz$

$r = 2 \quad T(0,0,0)$

$r \in [0,2]$
$\varphi \in [0,2\pi]$
$z \in [0,2]$



$x = r\cos\varphi$
$y = r\sin\varphi$
$z = z$
$dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$$\iiint\limits_K (4x+3)\,dx\,dy\,dz = \int_0^{2\pi}\int_0^2\int_0^2 (4r\cos\varphi+3)\,r\,dr\,dz\,d\varphi$$
$$= \int_0^{2\pi}\int_0^2 (4r^2\cos\varphi + 3r)\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_0^2 \left(4\frac{r^3}{3}\cos\varphi + 3\frac{r^2}{2}\right)\Big|_0^2 dz\,d\varphi$$
$$= \int_0^{2\pi}\int_0^2 \left(\cancel{4}\cdot\frac{\cancel{8}^{2}}{3}\cos\varphi + 3\frac{\cancel{4}}{\cancel{2}}\right)dz\,d\varphi = \int_0^{2\pi}\int_0^2 \left(\frac{2}{3}\cos\varphi + 6\right)dz\,d\varphi = \int_0^{2\pi}\left(\frac{2}{3}2\cos\varphi + 6z\right)\Big|_0^2 d\varphi$$
$$= \int_0^{2\pi}\left(\frac{2}{3}\cdot 2\cos\varphi + 6\cdot 2\right)d\varphi = \int_0^{2\pi}\left(\frac{4}{3}\cos\varphi + 12\right)d\varphi = \frac{4}{3}\sin\varphi + 12\varphi\Big|_0^{2\pi}$$
$$= \left(\frac{4}{3}\cdot\sin 2\pi + 12\cdot 2\pi\right) - \left(\frac{4}{3}\sin 0 + 12\cdot 0\right) = (0+24\pi) - (0+0) = 24\pi$$

(3) $$\int_{(-1,-1)}^{(4,3)} (2x+2y^2)\,dx + (4xy-3)\,dy$$

$$\begin{pmatrix} 2x+2y^2 \\ 4xy-3 \end{pmatrix} = -\text{grad.}\, f = \begin{pmatrix} -\partial x f. \\ -\partial y f. \end{pmatrix}$$

$\partial x f = -(2x+2y^2) \Rightarrow -2x-2y^2 \quad f = \int -2x-2y^2\,dx \Rightarrow -2-2y^2+C(y)$

$\Rightarrow f(x) = -2-2y^2+C(y)$

$\partial y f. -(4xy-3) \Rightarrow -4xy+3 \Rightarrow f = \int -4xy+3 = -4x + \underset{0}{C'(y)}$

$\Rightarrow f(x,y) = -2-2y^2-4x$

$f(-1,-1) - (4,3) = (-2-2\cdot(-1)^2-4\cdot(-1)) - (-2-2\cdot(3)^2-4\cdot 3)$

$= 0+32 = 32$

Ime i prezime: DOMAGOJ NEKIĆ **Broj indeksa:** 17-2-0028-2010

(2) $\iint_S x+y \, dx\, dy$ $\quad S=\{(x,y)\in \mathbb{R}^2 : x^2+y^2\le 4 \ \text{ i } x\ge y\}$

$x^2+y^2\le 4$

x	0	1
y	0	1

$x\ge y$

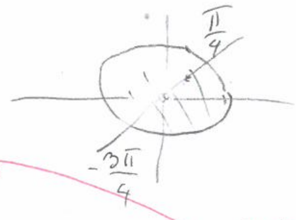
$x^2+y^2=r^2$

$\varphi\in[0,2\pi]$

$r^2=4$

$r\in[0,2]$

$r=2$



$$\iint_S x+y\, dx\, dy = \int_0^2 \int_{-\frac{3}{4}\pi}^{\frac{1}{4}\pi} (x+y)\, dy\, dx = \int_0^2 x+\frac{y^2}{2}\Big|_{-\frac{3}{4}\pi}^{\frac{1}{4}\pi} dx$$

$$=\int_0^2 x+\left(\frac{\frac{1}{16}\pi^2}{\frac{2}{1}} - \frac{\frac{9}{16}\pi^2}{2}\right) dx = \int_0^2 x+\left(\frac{1}{32}\pi^2-\frac{9}{32}\pi^2\right) dx$$

$$=\int_0^2 \left(x-\frac{1}{4}\pi^2\right) dx = \frac{x^2}{2}-\frac{1}{4}x\pi^2\Big|_0^2 = 2-\frac{1}{2}\pi^2$$

(5) $K = \{(x,y,z) : x = 3 \quad y \in [-4,4],\ z \in [-4,4]\}$

$\iint_{\partial K} x \, ds$? Def.

$r = 3$

$x = r \cos t$

$y = r \sin t$

$z = z$

$y \in [-4,4]$

$z \in [-4,4]$

$$r(t) = \begin{pmatrix} 3\cos t \\ \sin t \\ 1 \end{pmatrix} = r'(t) = \begin{pmatrix} -\sin t \\ 3\cos t \\ 1 \end{pmatrix}$$

$$\|r'(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2 + 1}$$

$$= \sqrt{\sin^2 t + \cos^2 t + 1}$$

$$= \sqrt{2}$$

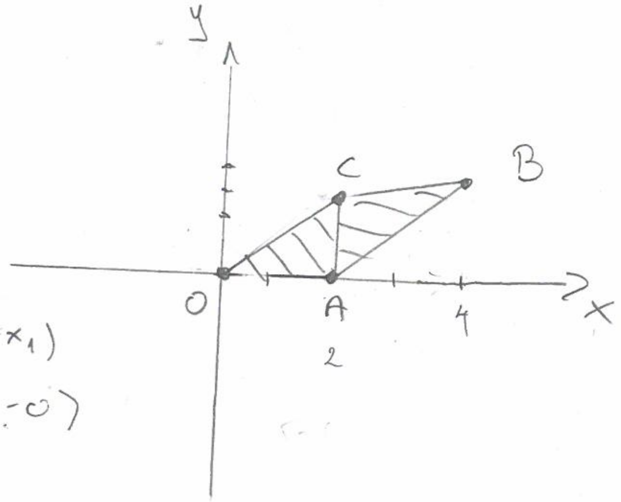
$$\iint_{\partial K} x \, ds = \int_{-4}^{4}\int_{-4}^{4} 3\sqrt{2} \, dz\, dy = \int_{-4}^{4} 3\sqrt{2}\,(4+4)\, dy$$

$$= \int_{-4}^{4} 24\sqrt{2} \, dy = 24\sqrt{2}\,(4+4) = 192\sqrt{2}$$

Ime i prezime: DOMAGOJ NEKIĆ **Broj indeksa:** 17-2-0028-2010

(4) $O(0,0)$, $A(\overset{2}{\frac{4}{2}},0)$, $B(4,\overset{2}{\frac{4}{2}})$ i $C(\overset{2}{\frac{4}{2}},\frac{3}{2})$.

$$\iint_X y^3 \, dx \, dy$$



$O(\overset{x_1}{0},\overset{y_1}{0})$ $A(\overset{x_2}{2},\overset{y_2}{0})$

$\overline{OA}$ ... $(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$$(2-0)(y-0)=(0-0)(x-0)$$

$$2(y-0)=0(x-0)$$

$$y=2$$

$A(\overset{x_1}{2},\overset{y_1}{0})$ $B(\overset{x_2}{4},\overset{y_2}{2})$

$\overline{AB}$ ... $(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$$(4-2)(y-0)=(2-0)(x-2)$$

$$2(y-0)=2(x-2)$$

$$2y=2x-4 \,/:2$$

$$y=x-2$$

$B(\overset{x_1}{4},\overset{y_1}{2})$ $C(\overset{x_2}{2},\overset{y_2}{\frac{3}{2}})$

$\overline{BC}$ ... $(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$$(2-4)(y-2)=(\tfrac{3}{2}-2)(x-4)$$

$$-2(y-2)=-\frac{1}{2}(x-4)$$

$$-2y+4=-\frac{1}{2}x+2$$

$$-2y=\frac{1}{2}x+2-4$$

$$-2y=\frac{1}{2}x-2 \,/:(-2)$$

$$y=-\frac{1}{4}x+1$$

$C(\overset{x_1}{2},\overset{y_1}{\frac{3}{2}})$. $O(\overset{x_2}{0},\overset{y_2}{0})$

$\overline{CO}$ ... $(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

$$(0-2)(y-\tfrac{3}{2})=(0-\tfrac{3}{2})(x-2)$$

$$-2(y-\tfrac{3}{2})=-\frac{3}{2}(x-2)$$

$$-2y+3=-\frac{3}{2}x+3$$

$$-2y=-\frac{3}{2}x+3-3$$

$$-2y=-\frac{3}{2}x \,/:(-2)$$

$$y=\frac{3}{4}$$

$$\iint_X y^3 \, dx\,dy = \int_0^2 \int_0^{\frac{3}{4}} y^3 \, dy\,dx + \int_2^4 \int_{x-2}^{-\frac{1}{4}x+1} y^3 \, dy\,dx$$

$$= \int_0^2 \frac{y^4}{4} \Big|_0^{\frac{3}{4}} dx + \int_2^4 \frac{y^4}{4} \Big|_{x-2}^{-\frac{1}{4}x+1} dx$$

$$= \int_0^2 \left(\frac{\left(\frac{3}{4}\right)^4}{4} - \frac{(0)^4}{4} \right) dx + \int_2^4 \left(\frac{\left(-\frac{1}{4}x+1\right)^4}{4} - \frac{(x-2)^4}{4} \right) dx$$

$$= \int_0^2 \frac{81}{1024} \, dx + \int_2^4 \left(\frac{\left(1-\frac{1}{4}x\right)^2 \left(1-\frac{1}{4}x\right)^2}{4} - \frac{(x-2)^2 (x-2)^2}{4} \right) dx$$

$$= \int_0^2 \frac{81}{1024} \, dx + \int_2^4 \left(\frac{\left(1-\frac{1}{2}x+x^2\right) \cdot \left(1-\frac{1}{2}x+x^2\right)}{4} - \frac{\left(x^2-2x+4\right) \cdot \left(x^2-2x+4\right)}{4} \right) dx$$

$$= \int_0^2 \frac{81}{1024} \, dx + \int_2^4 \left(\frac{1-\frac{1}{2}x+\cancel{x^2}-\frac{1}{2}x+\cancel{1x^2}-\cancel{\frac{1}{2}x^3}+\cancel{x^2}-\cancel{\frac{1}{2}x^3}+\cancel{x^4}}{4} - \frac{\cancel{x^4}-\cancel{2x^3}+\cancel{4x^2}-\cancel{2x^3}+\cancel{4x^2}-8x+\cancel{4x^2}-8x+16}{4} \right) \cancel{dx}$$

$$= \int_0^2 \frac{81}{1024} \, dx + \int_2^4 \left(\frac{x^4-x^3+3x^2-x-1}{4} - \frac{x^4-4x^3+12x^2-16x+16}{4} \right) dx$$

$$= \frac{81}{1024} x \Big|_0^2 + \left(\frac{x^4-x^3+3x^2-x-1}{4} - \frac{x^4-4x^3+12x^2-16x+16}{4} \right) \Big|_2^4$$

$$= \left(\frac{81}{512} \right) + \left(\frac{235}{4} - 19 \right) - \left(\frac{17}{4} - 32 \right)$$

$$= \frac{81}{512} + \frac{159}{4} + \frac{111}{4} = \frac{34641}{512}$$

Ime i prezime: Domagoj Nekić **Broj indeksa:** 17-2-0028-2010

$$y'''(t) - y''(t) - y'(t) + y(t) = t \qquad y(0)=1,\ y'(0)=0,\ y''(0)=1$$

$$s^3Y(s) - s^2y(0) - sy'(0) - y''(0) - \left(s^2Y(s) - sy(0) - y'(0)\right) - \left(sY(0) - y(0)\right) + \frac{Y(s)}{s} = \frac{1}{s^2}$$

$$\cancel{s^3Y(s) - s^2y(0) - \underbrace{sy'(0)}_{0} - y''(0) - s^2Y(s) + sy(0) + \underbrace{y'(0)}_{0} - sY(s) + y(0) + \frac{Y(s)}{s} = \frac{1}{s^2}}$$

$$\cancel{s^3Y(s) - s^2 - 1 - s^2Y(s) + s - sY(s) + 1 + \frac{Y(s)}{s} = \frac{1}{s^2}}$$

$$s^3Y(s) - s^2Y(s) - sY(s) + \frac{Y(s)}{s} = \frac{1}{s^2} + s^2 \cancel{+1} - s \cancel{-1}$$

$$s^3Y(s) - s^2Y(s) - sY(s) + \frac{Y(s)}{s} = \frac{1+s^2-s}{s^2}$$

$$Y(s)\left(s^4 - s^3 - s^2 + 1\right) = \frac{1+s^2-s}{s^2}$$

$$Y(s) = \frac{\frac{1+s^2-s}{s^2}}{(s^4-s^3-s^2+1)} = \frac{s^2-s+1}{s^2(s^4-s^3-s^2+1)} = \frac{s^2-s+1}{s^3(s^3-s^2+s-1)}$$

$$Y(s) = \frac{s^2-s+1}{s^3(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s-1} + \frac{Es+F}{s^2+1} \quad / \cdot s^3(s+1)(s^2+1)$$

$$s^2-s+1 = As^2(s-1)(s^2+1) + Bs(s-1)(s^2+1) + C(s-1)(s^2+1)\ Ds^3(s^2+1) + (Es+F)\cdot s^3\cdot(s-1)$$

$$s^2-s+1 = As^2(s^3+s-s^2-1) + Bs(s^3+s-s^2-1) + C(s^3+s-s^2-1) + Ds^5 + Ds^3 + (Es^4+Fs^3)(s-1)$$

$$s^2-s+1 = As^5 + As^3 - As^4 - As^2 + Bs^4 + Bs^2 - Bs^3 - Bs + Cs^3 + Cs - Cs^2 - C + Ds^5 + Ds^3 + (Es^5 - Es^4 + Fs^4 - Fs^3$$

$$s^2-s+1 = (A+D+E)\cdot s^5 + (-A+B-E+F)\cdot s^4 + (A-B+C+D-F)\cdot s^3 + (-A+B-C)\cdot s^2 + (-B+C)\cdot s - C$$

$$A+D+E=0$$
$$-A+B-E+F=0$$
$$A-B+C-F=0$$
$$-A+B-C=1$$
$$-B+C=-1$$
$$-C=1$$

$C = -1$

$-B + C = -1$
$-B - 1 = -1$
$-B = 0$
$B = 0$

$-A + B - C = 1$
$-A + 0 + 1 = 1$
$-A + 1 = 1$
$-A = 2 /:(-1)$
$A = -2$

$A - B + C - F = 0$
$-2 - 0 - 1 - F = 0$
$-3 - F = 0$
$-F = 3 /:(-1)$
$F = -3$

$-A + B - E + F = 0$
$2 + 0 - E - 3 = 0$
$-1 - E = 0$
$E = -1$

$A + D + E = 0$
$-2 + D - 1 = 0$
$-3 + D = 0$
$D = 3$

$$Y(s) = -\frac{2}{s} + \frac{0}{s^2} - \frac{1}{s^3} + \frac{3}{s+1} + \frac{(-1s - 3)}{s^2+1}$$

$$Y(s) = -2\,\frac{1}{s} + 0 - \frac{2}{s^3} + 3\,\frac{1}{s+1} - \frac{s}{s^2+1} - \frac{3}{s^2+1}$$

$$Y(s) - 2 \cdot 1 + 0 - t^2 + 3 \cdot e^{-t} - \cos t - \sin t$$

$$Y(s) = -2 - t^2 + 3e^{-t} - \cos t - \sin t$$

PROVJERA UVRŠTAVANJEM?

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.		POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: DANIJEL SORIĆ	**BROJ INDEKSA:**	

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (4x+3)\,dxdydz$. ~~20~~ 15
2. Izračunati dvostruki integral: $\iint\limits_S x + y\,dxdy$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \le 4 \text{ i } x \ge y\}$. ~~20~~
3. Izračunati $\int\limits_{(-1,-1)}^{(4,3)} (2x+2y^2)\,dx + (4xy-3)\,dy$. 15
4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{4}{2},0)$, $B(4,\frac{4}{2})$ i $C(\frac{4}{2},\frac{3}{2})$. Izračunati dvostruki integral ~~15~~
$$\iint\limits_X y^3\,dxdy.$$
5. Neka je kvadrat $K = \{(x,y,z) : x = 3,\ y \in [-4,4],\ z \in [-4,4]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x\,dS$? ~~15~~
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15
$$y'''(t) - y''(t) - y'(t) + y(t) = t, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 15

1. $r=2$

$T[0,0]$

$x^2+\cancel{y^2}+z^2=r^2$

$x=\sqrt{4-z^2}$

$\iiint\limits_K (4x+3)\,dxdydz$

$x = r\cos\varphi$
$y = r\sin\varphi$
$dxdy = r\,dr\,d\varphi$

$$\int_0^{2\pi}\int_{-2}^{2}\int_0^{\sqrt{4-z^2}}\big((4r\cos\varphi)+3\big)r\,dr\,d\varphi\,dz = \int_0^{2\pi}\int_{-2}^{2}\int_0^{\sqrt{4-z^2}}(4r\cos\varphi+3)r\,dr\,d\varphi\,dz$$

$$= \int_0^{2\pi}\int_{-2}^{2}\int_0^{\sqrt{4-z^2}}(4r^2\cos\varphi+3r)\,dr\,d\varphi\,dz = \int_{-2}^{2}\int_0^{\sqrt{4-z^2}}\left[\int_0^{2\pi}(4r^2\cos\varphi+3r)\,d\varphi\right]dr\,dz$$

$$= \int_{-2}^{2}\int_0^{\sqrt{4-z^2}}\left[\int_0^{2\pi}4r^2\cos\varphi\,d\varphi + \int_0^{2\pi}3r\,d\varphi\right]dr\,dz = \int_{-2}^{2}\int_0^{\sqrt{4-z^2}}\left[\underbrace{4r^2\cdot\sin\varphi\Big|_0^{2\pi}}_{0} + 3r\cdot\varphi\Big|_0^{2\pi}\right]$$

$$= \int_{-2}^{2}\int_0^{\sqrt{4-z^2}}3r\cdot 2\pi\,dr\,dz = \int_{-2}^{2}\int_0^{\sqrt{4-z^2}}\left[6\pi r\,dr\right]dz = \int_{-2}^{2}\left[6\pi\cdot\frac{r^2}{2}\Big|_0^{\sqrt{4-z^2}}\right]dz =$$

$$= \int_{-2}^{2} \left[6\pi \cdot \frac{r^2}{2} \Big|_{0}^{\sqrt{4-z^2}} \right] dz = \int_{-2}^{2} \left[6\pi \cdot \left(\frac{\left(\sqrt{4-z^2}\right)^2}{2} - \frac{0^2}{2} \right) \right] dz$$

$$= \int_{-2}^{2} \left[6\pi \cdot \frac{4-z^2}{2} \right] dz = \int_{-2}^{2} 3\pi \cdot (4 - z^2)\, dz = \int_{-2}^{2} (12\pi - z^2)\, dz = \int_{-2}^{2} 12\pi\, dz - \int_{-2}^{2} z^2\, dz$$

$$= 12\pi \int_{-2}^{2} dz - \int_{-2}^{2} z^2\, dz = \left(12\pi \cdot z \Big|_{-2}^{2}\right) - \left(\frac{z^3}{3} \Big|_{-2}^{2}\right) = (12\pi \cdot 2+2) - \left(\frac{2^3}{3} + \frac{2^3}{3}\right)$$

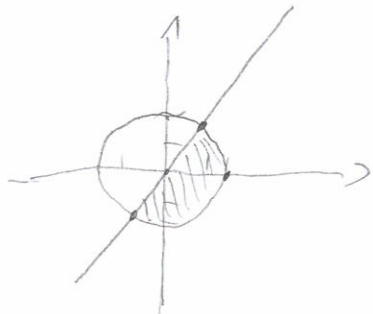
$$= (12\pi \cdot 4) - \left(\frac{8}{3} + \frac{8}{3}\right) = 48\pi - \frac{16}{3} = \frac{128}{3}\pi$$

2. $\iint_S x+y \, dx\, dy$ $\qquad x^2+y^2 \le r^2$

$x^2+y^2 \le 4 \qquad r^2 = 4 \qquad x = r\cos\varphi$

$\qquad r = \sqrt{4} \qquad y = r\sin\varphi$

$x \ge y \qquad r = 2 \qquad dx\,dy = r\,dr\,d\varphi$



$$\int_{0}^{1\ (2)} \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} (r\cos\varphi + r\sin\varphi)\, r\, dr\, d\varphi = \int_{0}^{1} \int_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} \left(r^2(\cos\varphi + \sin\varphi)\right) dr\, d\varphi$$

PAUSE

Ime i prezime: DANIJEL SORIĆ **Broj indeksa:**

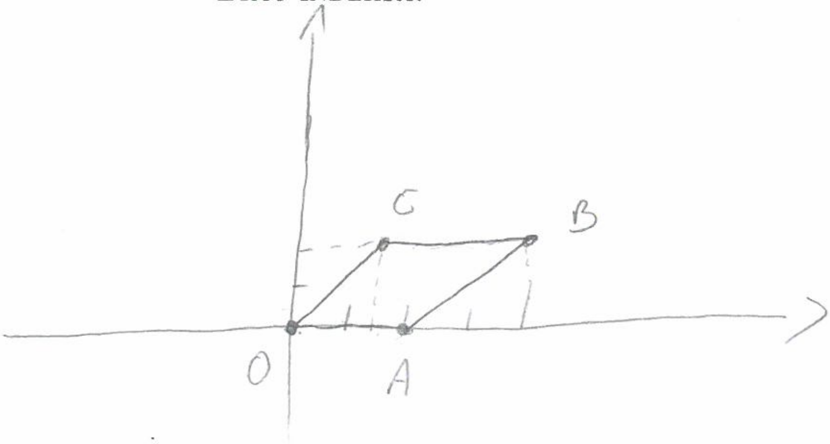
4.

$O[0,0]$ $\int\int y^3 dx dy$

$A\left[\frac{4}{2}, 0\right]$

$B\left[4, \frac{4}{2}\right]$

$C\left[\frac{4}{2}, \frac{3}{2}\right]$



$$\int_{x}^{x-2}\int_{0}^{2} y^3 dy dx = \int_{0}^{2}\int_{x}^{x-2} \left[y^3 dy\right] dx = \int_{0}^{2}\left[\frac{y^4}{4}\Big|_{x}^{x-2}\right] dx =$$

5. $x = 3$

$y \in [-4, 4]$

$z \in [-4, 4]$

$\iint x \, ds$

$r = \begin{pmatrix} 3\cos t \\ 3\sin t \end{pmatrix}$

$r' \begin{pmatrix} -3\sin t \\ 3\cos t \end{pmatrix}$

$\|r'(t)\| = \sqrt{(-3\sin t)^2 + (3\cos t)^2} = \sqrt{9\sin^2 t + 9\cos^2 t} = \sqrt{9 \cdot (\underbrace{\sin^2 t + \cos^2 t}_{1})} = \sqrt{9} = 3$

$\iint_{-4}^{4} 3\cos t \cdot 3 \, dt$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MATEJ ČURK **BROJ INDEKSA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (4x+3)\,dxdydz$. ~~20~~ 15

2. Izračunati dvostruki integral: $\iint\limits_S x + y\,dxdy$, gdje je $S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4 \text{ i } x \geq y\}$. 20

3. Izračunati $\int\limits_{(-1,-1)}^{(4,3)} (2x + 2y^2)\,dx + (4xy - 3)\,dy$. 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{4}{2},0)$, $B(4,\frac{4}{2})$ i $C(\frac{4}{2},\frac{3}{2})$. Izračunati dvostruki integral ~~15~~

$$\iint\limits_X y^3 dxdy.$$

5. Neka je kvadrat $K = \{(x,y,z) : x = 3,\ y \in [-4,4],\ z \in [-4,4]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x dS$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - y''(t) - y'(t) + y(t) = t, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

$x = r\cos t$ $y = r\sin t$ $z = z$ $dxdydz = r dr dt dz$

Ukupno: 15

1. $x^2 + y^2 + z^2 = R^2$

$x^2 + y^2 + z^2 = 4$

$x^2 + y^2 = r^2$

$r^2 + z^2 = 4$ $\quad t \in [0, 2\pi]$

$z^2 = 4 - r^2$ $\quad r \in [0, 2]$

$z = \pm\sqrt{4 - r^2}$ $\quad z \in [-\sqrt{4-r^2}, \sqrt{4-r^2}]$

$$\iiint\limits_K (4x+3) = \int_0^{2\pi}\int_0^2\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} (4r\cos t + 3)\, r\,dz\,dr\,dt$$

DALJE...?

4) $O(0,0)$ $A\left(\frac{4}{2},0\right)$ $B\left(4,\frac{4}{2}\right)$, $C\left(\frac{4}{2},\frac{3}{2}\right)$

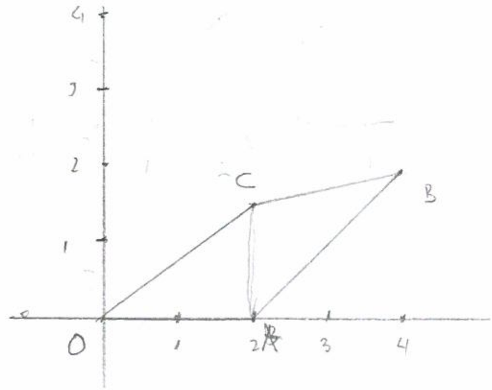
x_1, y_1 $x_2\ y_2$
$(0,0)$ $A\left(\frac{4}{2},0\right)$

$\overline{OA}$... $(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

...$\left(\frac{4}{2}-0\right)(y-0)=(0-0)(x-0)$

$\frac{4}{2}y=0$

$y=-\frac{4}{2}$ ✗



$\overline{AB}$... $\left(4-\frac{4}{2}\right)(y-0)=\left(\frac{4}{2}-0\right)\left(x-\frac{4}{2}\right)$

$=2y=\frac{4}{2}\left(x-\frac{4}{2}\right)$

$2y=\frac{4}{2}x-4 \;/:2$

$y=x-2$ ✓

$\overline{BC}$ $\left(\frac{4}{2}-4\right)\left(y-\frac{4}{2}\right)=\left(\frac{3}{2}-\frac{4}{2}\right)(x-4)$

$-2\cdot\left(y-\frac{4}{2}\right)=-\frac{1}{2}(x-4)$

$-2y+4=-\frac{1}{2}x+2 \;/:(-2)$

$-2y=-\frac{1}{2}x+2-4$

$-2y=-\frac{1}{2}x-2 \;/:(-2)$

$y=\frac{1}{4}x+1$ ✓

CO $\left(0-\frac{4}{3}\right)\left(y-\frac{3}{2}\right)=\left(0-\frac{3}{2}\right)\left(x-\frac{4}{3}\right)$

$-\frac{4}{3}\left(y-\frac{3}{2}\right)=-\frac{3}{2}\left(x-\frac{4}{3}\right)$

$-\frac{4}{3}y+2=-\frac{3}{2}x+2$

$-\frac{4}{3}y=-\frac{3}{2}x+2-2$

$-\frac{4}{3}y=-\frac{3}{2}x \;/:\left(-\frac{4}{3}\right)$

$y=\frac{9}{8}x$

$$\iint_X y^3\,dx\,dy=\int_0^2\int_{-\frac{4}{2}}^{\frac{9}{8}x} y^3\,dx\,dy+\int_2^4\int_{x-2}^{\frac{1}{4}x+1} y^3\,dx\,dy=$$

✗

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: IVAN ŠIKIĆ **BROJ INDEKSA:** 17-1-0014-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K kugla radijusa $r = 2$ sa centrom u ishodištu. Izračunati $\iiint\limits_K (4x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20

2. Izračunati dvostruki integral: $\iint\limits_S x + y\,\mathrm{d}x\mathrm{d}y$, gdje je $S = \{(x,y) \in \mathbb{R}^2 \,:\, x^2+y^2 \leq 4 \text{ i } x \geq y\}$. 20

3. Izračunati $\int\limits_{(-1,-1)}^{(4,3)} (2x+2y^2)\,\mathrm{d}x + (4xy-3)\,\mathrm{d}y$. 15

4. X je zadan kao četverokut s vrhovima $O(0,0)$, $A(\frac{4}{2},0)$, $B(4,\frac{4}{2})$ i $C(\frac{4}{2},\frac{3}{2})$. Izračunati dvostruki integral 15

$$\iint\limits_X y^3\mathrm{d}x\mathrm{d}y.$$

5. Neka je kvadrat $K = \{(x,y,z) \,:\, x=3,\ y\in[-4,4],\ z\in[-4,4]\}$. Kako preko definicije izračunati $\iint\limits_{\partial K} x\mathrm{d}S$? 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) - y''(t) - y'(t) + y(t) = t, \quad y(0)=1,\ y'(0)=0,\ y''(0)=1.$$

Ukupno: 0

1. $r=2$

$$\iiint (4x+3)\,dx\,dy\,dz$$

$$\int_0^{2\pi}\int_0^{2}\int_2^{\sqrt{r-4}} (4x+3)\,dz\,dx\,dy$$

$S=\{(x,y)\in\mathbb{R}^2$

$x^2+y^2\leq 4$

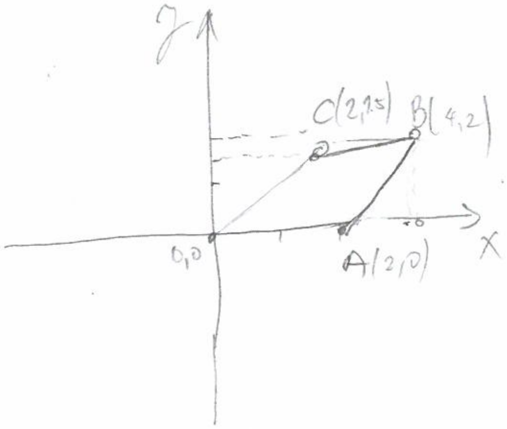
$x\geq y$

2. $\iint x+y\,dx\,dy$

$$\int_{-2}^{2}\int_{x-2}^{0} x+y\,dx\,dy$$

?

4.



(AB) $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{2-0}{4-2}(x-2)$

$y = \frac{2}{2}(x-2)$

$y = x - 2$

(OC) $y - 0 = \frac{1.5-0}{2-0}(x-0)$

$y = \frac{3}{4}x$

$$\int_0^{x-2} dx \int_x^{\frac{3}{4}x} y^3 \, dy$$

5.