MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ANTE BOTICA BROJ INDEKSA: 17-1-0014-2010	

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0, 3)$. Izračunati $\int_{\partial K} xy \, ds$. ($\phi$) — 20

2. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0, -3)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x + 2) \, dy$. ($18\pi$) — 20

$f_1 = -\frac{1}{2\sqrt{x}} \Rightarrow -\int \frac{1}{2} x^{-\frac{1}{2}} dx = -\frac{1}{2} \int x^{-\frac{1}{2}} dx = -\frac{1}{2} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = -\sqrt{x}$

3. Izračunati $\int_{(3,-2)}^{(3,5)} \frac{dx}{2\sqrt{x}} + dy$ — 10

$f_2 = 1$ (0)

4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\widehat{\partial K}} (3x + 2) \, dx dy$? ($\phi$) — 20

5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = x^2 + y^2$ što leži iznad područja $D \ldots x^2 + y^2 \leq 2$. Nije potrebno računati površinu baze. — 15

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — ~~15~~

$$y'''(t) + 2y''(t) + y'(t) + 2y(t) = 2, \quad y(0) = 2, \; y'(0) = 0, \; y''(0) = 1.$$

Ukupno: 60

(4) $a = 2$

$$\iint_{\partial K} (3x+2)\, dx dy \qquad w \begin{bmatrix} 0 \\ 0 \\ 2x+3 \end{bmatrix} \Rightarrow \operatorname{div} \vec{w} = 0$$

$$\int = \iiint \operatorname{div} \vec{w} \, dx dy dz = \iiint 0 \, dx dy dz = 0$$

(1) K - krug $\quad T(0,3) \quad \int_{\partial K} xy \, ds \quad \|r'(t)\| = \sqrt{4\sin^2 t + 4\cos^2 t} = \sqrt{4} = 2$

$r = 2 \quad t \in [0, 2\pi]$

$x = r\cos t = 2\cos t + 0 \qquad r(t) = \begin{bmatrix} 2\cos t \\ 2\sin t + 3 \end{bmatrix} \qquad r'(t) = \begin{bmatrix} -2\sin t \\ 2\cos t \end{bmatrix}$

$y = r\sin t = 2\sin t + 3$

$f(r(t)) = 2\cos t \cdot (2\sin t + 3) = 4\sin t \cos t + 6\cos t \Rightarrow \int_{t_1}^{t_2} f(r(t)) \cdot \|r'(t)\| =$

$= \int_0^{2\pi} (4\sin t \cos t + 6\cos t) 2 \, dt = \int_0^{2\pi} 8\sin t \cos t + 12\cos t \, dt = 8\int_0^{2\pi} \sin t \cos t \, dt + 12 \int_0^{2\pi} \cos t \, dt$

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(h) $8\int_0^{2\pi} \sin t \cos t \, dt$ $\left| \begin{array}{l} \sin t = s \\ \cos t \, dt = ds \\ t=0 \Rightarrow s=0 \\ t=2\pi \Rightarrow s=0 \end{array} \right|$ $8\int_0^0 s\,ds = 8\frac{s^2}{2}\Big|_0^0 = \emptyset$

$2(3\cos t)+2 = 6\cos t + 2$

(2) K - kug

$\oint_{\partial K} (2x+2)\,dy$

uz $dx = 0$

uz $dy = 2x+2$

$r = 3$

$S(0,-3)$

$t \in [0, 2\pi]$

$x = r\cos t = 3\cos t$

$y = r\sin t = 3\sin t - 3$

$W\begin{bmatrix} 0 \\ 2x+2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6\cos t + 2 \end{bmatrix}$

$\Rightarrow 2x+2 \Rightarrow 6\cos t + 2$ $\quad r\begin{bmatrix} 3\cos t \\ 3\sin t - 3 \end{bmatrix}$

$r'\begin{bmatrix} -3\sin t \\ 3\cos t \end{bmatrix}$

$W \cdot r'(t) = \begin{bmatrix} 0 \\ 6\cos t + 2 \end{bmatrix} \cdot \begin{bmatrix} -3\sin t \\ 3\cos t \end{bmatrix} = \begin{bmatrix} 0 \\ 18\cos^2 t + 6\cos t \end{bmatrix}$

$(6\cos t + 2)(3\cos t) = 18\cos^2 t + 6\cos t$

$\int_0^{2\pi} 18\cos^2 t + 6\cos t \, dt = 18\int_0^{2\pi} \cos^2 t \, dt + \cancel{6\int_0^{2\pi} \cos t \, dt}$ 0

$= 18\int_0^{2\pi} \cos^2 t \, dt$ $\left| \begin{array}{l} \cos^2 t = \frac{\cos 2t + 1}{2} \\ = \frac{1}{2}\cos 2t + \frac{1}{2} \end{array} \right|$ $= \cancel{18}^{9}\int_0^{2\pi} \frac{1}{\cancel{2}} (\cos 2t + 1)\,dt$

$9 \cdot 2\pi = 18\pi$

$= 9\int_0^{2\pi} \cos 2t + 1 \, dt = 9\left[\int_0^{2\pi} \cos 2t \, dt + \int_0^{2\pi} dt\right] = 9\int_0^{2\pi} \cos 2t \, dt + 9t\Big|_0^{2\pi}$

$9\int_0^{2\pi} \cos 2t \, dt$ $\left| \begin{array}{l} 2t = s \\ 2dt = ds \\ dt = \frac{1}{2}ds \\ t=0 \Rightarrow s=0 \\ t=2\pi \Rightarrow s=4\pi \end{array} \right|$ $= 9\int_0^{4\pi} \frac{1}{2}\cos s \, ds = \frac{9}{2}\int_0^{4\pi} \cos s \, ds = \cancel{\frac{9}{2}\sin s\Big|_0^{4\pi}}$ 0

$\sin 4\pi = 0$

$\sin 0 = 0$

$I = 18\pi$

IME I PREZIME: ANTE BOTICA **BROJ INDEKSA:** 17-1-2019-2010

(3) $\displaystyle\int_{(3,-2)}^{(3,5)} \frac{dx}{2\sqrt{x}} + dy$

$f_1 = \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{(-\frac{1}{2})}$

$f_2 = 1$

$J = \begin{pmatrix} -f_1 \\ -f_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} x^{(-\frac{1}{2})} \\ -1 \end{pmatrix}$

$\int f_1 dx = -\sqrt{x} + C(y)$

$\frac{\partial(-\sqrt{x})}{\partial y} = \phi + c'(y)$

$\Rightarrow f: -\sqrt{x}$

$0 + C'(y) = -1$

$C'(y) = -1$

$\frac{\partial(-1)}{\partial y} = \phi = C(y)$

$(-\sqrt{3}) - (-\sqrt{3}) = -\sqrt{3} + \sqrt{3} = \phi$

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(6) $y'''(t) + 2y''(t) + y'(t) + 2y(t) = 2$ $\quad y(0)=2,\ y'(0)=0,\ y''(0)=1$

$$s^3F(s) \overset{-2s^2}{-s^2 f(0)} - \cancel{s f'(0)} \overset{-1}{- f''(0)} + 2s^2F(s) \overset{-4s}{-2sf(0)} - \cancel{2f'(0)} + sF(s) \overset{-2}{-f(0)} + 2F(s) = \frac{2}{s}$$

$$s^3F(s) + 2s^2F(s) + sF(s) + 2F(s) - 2s^2 - 1 - 4s - 2 = \frac{2}{s}$$

$$F(s)\left(s^3+2s^2+s+2\right) = \frac{2}{s} + 2s^2+4s+3 = \frac{2s^3+4s^2+3s+2}{s}$$

$$F(s) = \frac{2s^3+4s^2+3s+2}{s(s^3+2s^2+s+2)}$$

$$F(s) = \frac{2s^3+4s^2+3s+2}{s(s+2)(s^2+1)} = \frac{A}{s} + \frac{B}{s+2} + \frac{Cs+D}{s^2+1}$$

$s=0$ → A, $\quad s=-2$ → B, $\quad s^2=-1$ → $Cs+D$

$$\frac{2s^3+4s^2+3s+2}{s(s+2)(s^2+1)} = \frac{A}{s} + \frac{B}{s+2} + \frac{Cs+D}{s^2+1} \quad / \cdot s(s+2)(s^2+1)$$

$$2s^3+4s^2+3s+2 = A(s+2)(s^2+1) + B(s)(s^2+1) + C(s^2)(s+2) + D(s)(s+2)$$

(1) $s=0$

$$0+0+0+2 = A(0+2)(0+1) + 0 + 0 + 0 \Rightarrow 2 = 2A \Rightarrow \underline{\underline{A=1}}$$

(2) $s=-2$

$$2\cdot(-8) + 4(4) + 3(-2) + 2 = 0 + B(-2)(4+1) + 0 + 0$$

$$\cancel{-16+16} - 6 + 2 = -10B \Rightarrow -10B = -4 \Rightarrow B = \frac{4}{10} = \frac{2}{5} \quad \underline{\underline{B = \frac{2}{5}}}$$

(3) $s=1$

$$2+4+3+2 = 1(1+2)(1+1) + \frac{2}{5}(1)(1+1) + C(1)(1+2) + D(1)(1+2)$$

$$11 = 6 + \frac{4}{5} + 2C + 2D \Rightarrow 2C + 2D = \frac{21}{5} \ /:2 \Rightarrow C + D = \frac{21}{10} \quad (I)$$

(4) $s=-1$

$$-2 + 4 - 3 + 2 = 1(-1+2)(1+1) + \frac{2}{5}(-1)(1+1) + C(1)(-1+2) + D(-1)(-1+2)$$

$$1 = 2 - \frac{4}{5} + C - D \Rightarrow C - D = -\frac{1}{5} \Rightarrow C = D - \frac{1}{5} \quad (II)$$

(II) u (I)

$$D - \frac{1}{5} + D = \frac{21}{10} \Rightarrow 2D = \frac{23}{10} \Rightarrow \underline{\underline{D = \frac{23}{20}}}$$

$$C = \frac{23}{20} - \frac{1}{5} = \frac{19}{20} \Rightarrow C = \frac{19}{20}$$

6. $F(s) = \underbrace{\frac{1}{s}}_{=1} + \frac{2}{5}\left(\frac{1}{s+2}\right) + \frac{19}{20}\left(\frac{s}{s^2+1^2}\right) + \frac{23}{20}\left(\frac{1}{s^2+1^2}\right)$

$\hookrightarrow \frac{1}{s-(-2)} \quad a = (-2)$

$$\boxed{\mathcal{L}^{-1} = 1 + \frac{2}{5} e^{-2t} + \frac{19}{20} \cos t + \frac{23}{20} \sin t}$$

PROVJERA UVRŠTAVANJEM?

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. IME I PREZIME: Andrea Savić BROJ INDEKSA:	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0, 3)$. Izračunati $\int_{\partial K} xy \, ds$. — 20 15

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$$y'''(t) + 2y''(t) + y'(t) + \cancel{+}2y(t) = 2, \quad y(0) = 2, \, y'(0) = 0, \, y''(0) = 1.$$

Ukupno: 15

1.

$$r = \begin{pmatrix} 2\cos t \\ 2\sin t + 3 \end{pmatrix}$$

$$r' = \begin{pmatrix} -2\sin t \\ 2\cos t \end{pmatrix} \quad \|r'(t)\| = \sqrt{4\sin^2 t + 4\cos^2 t}$$

$$\|r'(t)\| = \sqrt{4} = 2$$

$$\int_{\partial k} xy\,ds = \int_0^{2\pi} 2\cos t \cdot (2\sin t + 3)\,2\,dt = \int_0^{2\pi} 8\cos t \sin t + 12\cos t\,dt$$

$2\cos t \sin t = \sin 2t$

$$= \int_0^{2\pi} 4\sin 2t\,dt + 12\int_0^{2\pi}\cos t\,dt = 2\cos 2t\Big|_0^{2\pi} + 12\sin t\Big|_0^{2\pi} =$$

$2t = u$, $2dt = du$, $dt = \frac{du}{2}$

$$= 2\cos 4\pi - 12\sin 2\pi = 2$$

$\cos 4\pi = 1$, $\sin 2\pi = 0$

3. $\displaystyle\int_{(3,-2)}^{(3,5)} \frac{dx}{2\sqrt{x}} + dy$

$$\begin{pmatrix} \frac{1}{2\sqrt{x}} \\ 1 \end{pmatrix} = -\operatorname{grad} f = \begin{pmatrix} -\frac{\partial f}{\partial x} \\ -\frac{\partial f}{\partial y} \end{pmatrix}$$

$$\frac{\partial f}{\partial x} = -\frac{1}{2\sqrt{x}} \Rightarrow f = -\frac{1}{2}\int \frac{1}{\sqrt{x}}\,dx = -\frac{1}{2}\ln|\sqrt{x}| + C(y)$$

$$\frac{\partial f}{\partial y} = -1 \Rightarrow \cancel{\ldots}$$

$-\frac{1}{2}\ln|\sqrt{x}| + c(y)$

$\frac{\partial f}{\partial y} = -1$

$-\frac{1}{2}\frac{1}{\sqrt{x}} \cdot \left(-\frac{1}{2}\cdot\frac{1}{\sqrt{x^3}}\right) + c'(y) = -1$

$-\frac{1}{4}\cdot\frac{1}{x^2} + c'(y) = -1$

$c'(y) = \frac{1}{4x^2} - 1$

$c(y) = \frac{y}{4x^2} - y + C$

$\frac{1}{\sqrt{x}} = \ldots = -\frac{1}{2}\frac{1}{x}$

$(x^{-\frac{1}{2}})' = -\frac{1}{2}(x^{-\frac{1}{2}-\frac{2}{2}}) = -\frac{1}{2}x^{-\frac{3}{2}}$

$x^{-\frac{3}{2}}\cdot x^{-\frac{1}{2}} = x^{-\frac{1}{2}-\frac{3}{2}} = x^{-\frac{4}{2}}$

$f(x,y) = -\frac{1}{2}\ln|\sqrt{x}| + \frac{y}{4x^2} - y + C$

$f = f(3,5) - f(3,-2) = -0{,}54 + 0{,}138 - 5 - 0{,}54 - 0{,}125 + 2 = -7{,}759$

6. $y'''(t) + 2y''(t) + y'(t) + 2y(t) = 2$, $y(0)=2$, $y'(0)=0$, $y''(0)=1$

$s^3F(s) - s^2 f(0) - s f'(0) - f''(0) + 2s^2F(s) - sf(0) - f'(0)$

$+ s(F(s) - f(0)) + 2F(s) = \frac{2}{s}$

$s^3F(s) - 2s^2 - 1 + 2s^2F(s) - 2s + sF(s) - 2 + F(s) = \frac{2}{s}$

$F(s)(s^3+2s^2+s+2) = \frac{2}{s} + 2s^2 + 1 + 2s + 2 = \frac{2+2s^3+s+2s^2+2s}{s}$

$F(s) = \frac{2s^3+2s^2+3s+2}{s\cdot(s^3+2s^2+s+2)} = \frac{2s^3+2s^2+3s+2}{s^4+2s^3+s^2+2s}$

$\frac{2s^3+2s^2+3s+2}{s^4+2s^3+s^2+2s} =$

$s(s^3+s^2+s+1)$

$s(s^2(s+1)+\ldots) = s(s^2(s+1)(s^2+1)$

$s^3(s^3+s^2+s+1)$

$s(s^2(s+1)+s+1)$

$s(s^2+1)(s+1)$

Ime i prezime: Andrea Savić **Broj indeksa:**

$$\frac{2s^3+2s^2+3s+2}{s(s^2+1)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2+1}$$

$$Cs^3+Cs^2+Ds^2+Ds$$

$$2s^3+2s^2+3s+2 = As^3+As^2+As+A+Bs^3+Bs+(Cs+D)(s^2+s)$$

$2 = A+C+B$ $\qquad C+B=0 \Rightarrow C=-B$

$2 = A+C+D$ $\qquad 0=C+D \Rightarrow C=-D$

$3 = A+B+D$ $\qquad 3=2-C-C$

$2 = A$ $\qquad 2C=-1$

$C=-\frac{1}{2} \Rightarrow D=\frac{1}{2}$

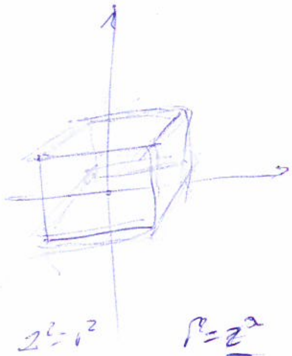
$B=\frac{1}{2}$

$$\frac{2}{s} + \frac{\frac{1}{2}}{s+1} + \frac{-\frac{1}{2}s+\frac{1}{2}}{s^2+1}$$

$$2+\frac{1}{2}e^{-t} - \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t)$$

PROVJERA?

4.



$$\iint_{\partial K} (3x+z)\,dx\,dy$$

5. $z^2 = r^2$ $\quad r^2 = z^2$ $\quad r = \sqrt{z}$ $\quad z^2 = 2$

ρ

$$P = \int_0^{2\pi}\int_0^{\sqrt{z}}\int_{\sqrt{2}}^{\sqrt{2}} r\,dz\,dr\,d\rho = 2\pi \int_{-\sqrt{2}}^{\sqrt{2}} \frac{r^2}{2}\Big|_0^{\sqrt{z}} dz = 2\pi \int_{-\sqrt{2}}^{\sqrt{2}} \frac{z}{2}\,dz = \pi z^2 \Big|_{-\sqrt{2}}^{\sqrt{2}} = 0$$

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IME I PREZIME: MARIN MAGAŠ **BROJ INDEKSA:** 17-2-0061-2010	Broj ↓ bodova

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Ukupno: 10

6. $s^3F(s) - s^2f(0) - sf'(0) - f''(0) + 2(s^2F - sf(0) - f'(0))$

$+ sF(s) - f(0) + 2 = 2$

$s^3F(s) - 2s^2 - 1 + 2(s^2F - 2s) + sF(s) - 2 + 2 = 2$

$s^3F(s) - 2s^2 - 1 + 2s^2F(s) - 4s + sF(s) - 2 + 2 = 2$

$s^3F(s) + 2s^2F(s) + sF(s) = 2 + 2$

$F(s)(s^3 + 2s^2 + s) = 3 + 2s^2 + 4s$

$F(s) = \frac{2s^2 + 4s +}{s^3 + 2s^2 + s} = \frac{2s^2 + 4s + 3}{s(s+1)^2}$

$\frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2} = A(s^2 + 2s + 1) + Bs(s+1) + Cs$

$As^2 + 2As + A + Bs^2 + Bs + Cs$

→

1. $r=2$ $T(0,3)$ $\int_{\partial t} xy\,ds$

$x = r\cos\varphi$

$y = r\sin\varphi + 3$

$r = \begin{vmatrix} 2\cos\varphi \\ 2\sin\varphi + 3 \end{vmatrix} \Rightarrow r' = \begin{vmatrix} -2\sin\varphi \\ 2\cos\varphi \end{vmatrix}$ ✓

$\|r'\| = \sqrt{4\sin^2\varphi + 4\cos^2\varphi}$

$= \sqrt{4} = 2\,dt$ ✓

$\int 2\,(2\cos\varphi\,(2\sin\varphi + 3))\,dt$ ✗

$\int 2\,(4\cos\varphi\sin\varphi + 6\cos\varphi)\,dt$

$2\int_0^2 4\cos\varphi\sin\varphi\,dt + 2\int_0^2 6\cos\varphi\,dt$ ✗ ✗

$8\cdot\frac{1}{2}\underbrace{\int_0^2 \sin 2\varphi\,dt}_{\mathrm{I}} + 12\int_0^2 \underbrace{\cos\varphi\,dt}_{\mathrm{II}}$

I $\begin{vmatrix} 2\varphi = t \\ 2d\varphi = dt \\ d\varphi = \frac{dt}{2} \end{vmatrix}$ $\Rightarrow 4\int_0^2 \sin t\,dt + 12\int_0^2 \cos\varphi\,dt$

$-4\cos t\Big|_0^2 + 12\sin\varphi\Big|_0^2 = -4\cos 2\varphi\Big|_0^2 + 12\cos\varphi\Big|_0^2$

$-4(\cos 4 - \cos 0) + 12(\cos 2 - \cos 0)$

$= -4(1-1) + 12(1-1) = \emptyset$

Ime i prezime: MARIN MAGAŠ **Broj indeksa:** 17-2-0061-2010

6. $s^3F(s) - 2s^2 - 1 + 2(s^2F(s) - 2s) + sF(s) - 2 + 2F(s) = 2$

$s^3F(s) - 2s^2 - 1 + 2s^2F(s) - 4s + sF(s) - 2 + 2F(s) = 2$

$s^3F(s) + 2s^2F(s) + sF(s) + 2F(s) = 2 + 2 + 4s + 1 + 2s^2$

$F(s)(s^3 + 2s^2 + s + 2) = 2s^2 + 4s + 5$

$F(s)(s^2+1)(s+2) = 2s^2 + 4s + 5$

$$F(s) = \frac{2s^2+4s+5}{(s+2)(s^2+1)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+1}$$

$A(s^2+1) + (Bs+C)(s+2) = As^2 + A + Bs^2 + 2Bs + Cs + 2C$

$2 = A + B \Rightarrow A = 1$

$4 = 2B + C \Rightarrow B = 1$

$5 = A + 2C \Rightarrow 5 = 1 + 2C$

$6 = 2C$

$C = 3$

$$\frac{1}{s+2} + \frac{s+3}{s^2+1} = \frac{1}{s+2} + \frac{s}{s^2+1} + \frac{3}{s^2+1}$$

$$= e^{-2t} + \cos t + 3 \cdot \sin t$$

Ime i prezime: MARIN MAGAŠ **Broj indeksa:** 17-2-0061-2010

6. → ~~NASTAVAK~~

$\cancel{2 = A + B} \Rightarrow \cancel{\boxed{B = -1}}$

$\cancel{4 = 2A + B + C} \Rightarrow \cancel{4 = 6 - 1 + C}$

$\cancel{\boxed{3 = A}}$ $\cancel{\boxed{C = -1}}$

$\cancel{3 \cdot \frac{1}{s} - \frac{1}{s+1} - 1\,\frac{1}{(s+1)^2} = 3 - 2e^{-t} - te^{-t}}$

5. $z = x^2 + y^2$ $x^2 + y^2 \le 2$

$z = r^2\cos^2\varphi + r^2\sin^2\varphi = r^2 \cdot 1 \Rightarrow z = r^2$ $r = \sqrt{2}$

$\int_0^{2\pi} d\varphi \int_2^{r\sin\varphi} dz \int_0^{\sqrt{2}} r\,dr$

$r \in [0, \sqrt{2}]$

$\varphi \in [0, 2\pi]$

$z \in [2, r\sin\varphi]$

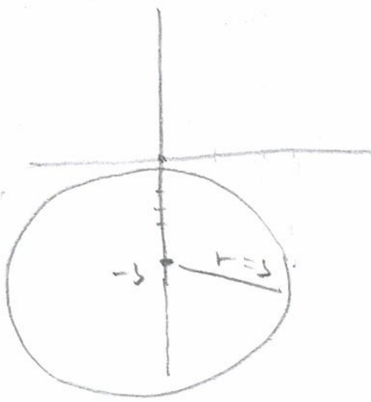
2. $r = 3$ $T(0,-3)$ $\int (2x+2)\,dy$

$x = r\cos\varphi$

$y = r\sin\varphi - 3$

$r\,dr\,d\varphi$

?



3. $w\begin{bmatrix} \frac{1}{2\sqrt{x}} \\ 1 \end{bmatrix} - \text{grad}$

$d_x f = -\frac{1}{2\sqrt{x}} \quad / \int dx$

$f = -\int \frac{1}{2\sqrt{x}}\,dx = -\frac{1}{2}\int x^{-\frac{1}{2}}\,dx$

$= -\frac{1}{2}\,\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C(y)$

$= -\sqrt{x} + C(y)$

$d_y f = -1$

$\frac{\partial f}{\partial y} = \frac{d(-\sqrt{x} + C(y))}{d(y)}$

$\frac{C(y)}{d(y)} = -y$

$F(x,y) = -\sqrt{x} - y$

$F(3,-2) - F(3,5) =$

$-\sqrt{3} + 2 - (-\sqrt{3} - 5)$

$-\sqrt{3} + 2 + \sqrt{3} + 5 = 7$ ✓

4. $\cancel{\iiint 3r^2 \cos\varphi \, dr\, d\varphi\, dz + \iiint 2\, r\, dr\, d\varphi\, dz}$

$\int_0^2 3r^2 dr \underbrace{\int_0^2 \cos\varphi\, d\varphi}_{0} \int_0^2 dz$

$\underbrace{\iiint 2\, r\, dr\, d\varphi\, dz}$

$4\int_0^2 r\,dr \int_0^2 d\varphi$

$4\int_0^2 r\,dr\; 2 = 8\,\frac{r^2}{2}\Big|_0^2$

$= 4r^2\Big|_0^2$

$= 16$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: ANTONIO VUJATOVIĆ **BROJ INDEKSA:** 17-1-0011-201

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0, 3)$. Izračunati $\int_{\partial K} xy\, \mathrm{d}s$. — 20
2. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0, -3)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x + 2)\, \mathrm{d}y$. — 20
3. Izračunati $\displaystyle\int_{(3,-2)}^{(3,5)} \frac{\mathrm{d}x}{2\sqrt{x}} + \mathrm{d}y$ — 10
4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\widehat{\partial K}} (3x + 2)\, \mathrm{d}x\mathrm{d}y$? — 20
5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = x^2 + y^2$ što leži iznad područja $D \ldots x^2 + y^2 \leq 2$. Nije potrebno računati površinu baze. — 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 15

$$y'''(t) + 2y''(t) + y'(t) + 2y(t) = 2, \quad y(0) = 2,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 0

6.

$$s^3Y(s) - s^2Y(0) - sY'(0) - Y''(0) + 2\left(s^2Y(s) - sY(0) - Y'(0)\right) + sY(s) - Y(0) + 2sY(s) = \frac{2}{s}$$

$$= s^3Y(s) - 2s^2 - 1 + 2s^2Y(s) - 4s + sY(s) - 2 + 2sY(s)$$

$$= Y(s)\left(s^3 + 2s^2 + 3s\right) = \frac{2}{s} - 2s^2 - 4s + 3$$

$$= \frac{2 - 2s^3 - 4s^2 + 3s}{s}$$

$$= \frac{-2s^3 - 4s^2 + 3s + 2}{s^4 + 2s^3 + 3s^2} = \frac{\quad}{s^2(s^2 + 2s + 3)}$$

$$= \frac{A}{s^2} + \frac{B}{s} + \frac{Cs + D}{(s^2 + 2s + 3)}$$

$$= A(s^2 + 2s + 3) + Bs(s^2 + 2s + 3) + (Cs + D)s^2$$

$$= As^2 + 2As + 3A + Bs^3 + 2Bs^2 + 3Bs + Cs^3 + Ds^2$$

1. $r=2 \quad T(0,3) \quad \int_{\partial K} xy \, ds$

$r \in [0,2]$

$\varphi \in [0, 2\pi]$

$x = r\cos\varphi$ ✗

$y = r\sin\varphi + 3$ ✗

$$\|r'\| = \sqrt{(-2\sin\varphi)^2 + (2\cos\varphi)^2}$$

$$\|r'\| = \sqrt{4\sin^2\varphi + 4\cos^2\varphi}$$

$$\|r'\| = \sqrt{4\underbrace{(\sin^2\varphi + \cos^2\varphi)}_{1}}$$

$$\|r'\| = \sqrt{4} = 2$$

DALJE... ?

Ime i prezime: ANTONIO VUJATOVIĆ **Broj indeksa:**

2.

$r = 3 \quad T(0,-3) \quad \int_{\partial K} (2x+2)\,dy$

$W = \begin{bmatrix} 3\cos t \\ -3\sin t \end{bmatrix}$ ✗

$x = 3\cos t$

$y = 3\sin t - 3$

$W' = \begin{bmatrix} -3\sin t \\ 0 \end{bmatrix}$

$r = \begin{vmatrix} 3\cos t \\ 3\sin t - 3 \end{vmatrix} = r' \begin{vmatrix} -3\sin t \\ 3\cos t \end{vmatrix}$

$\|r'\| = \sqrt{(-3\sin t)^2 + (3\cos t)^2}$

$= \sqrt{9\sin^2 t + 9\cos^2 t}$

$= \sqrt{9\,\underbrace{(\sin^2 t + \cos^2 t)}_{1}}$

$= \sqrt{9} = 3$

$= \int_0^{2\pi} 2(3\cos t + 2)$ ✗

$= 2\int_0^{2\pi} 3\cos t + 2$

$= 2 \cdot (3\sin t + 2t)\Big|_0^{2\pi} \cdot \|r'\|$

$= 24\pi$

6. $-2s^3-4s^2-3-2=$

$-2 = \overset{-5}{B} + C \qquad C = 3$

$-4 = A + 2B + D \qquad D = \frac{20}{3}$

$-3 = \overset{-\frac{4}{3}+3B}{2A} + 3B \quad \Rightarrow \frac{5}{3}B = -3 \Rightarrow \boxed{B=-5}$

$-2 = 3A \Rightarrow -\frac{2}{3}$

$$= -\frac{2}{3}\cdot\frac{1}{s^2} - 5\cdot\frac{1}{s} + 3\cdot\frac{s}{(s^2+2s+3)} + \frac{20}{3}\cdot\frac{1}{(s^2+2s+3)}$$

DALJE... ?

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. IME I PREZIME: Luka Huljev BROJ INDEKSA: 58079-2009	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0, 3)$. Izračunati $\int_{\partial K} xy \, ds$. 20
2. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0, -3)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x + 2) \, dy$. 20
3. Izračunati $\displaystyle\int_{(3,-2)}^{(3,5)} \frac{dx}{2\sqrt{x}} + dy$ 10
4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\widehat{\partial K}} (3x + 2) \, dxdy$? 20
5. Koristeći plošni integral postaviti formulu za ploštinu dijela paraboloida $z = x^2 + y^2$ što leži iznad područja $D \ldots x^2 + y^2 \leq 2$. Nije potrebno računati površinu baze. 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) + 2y''(t) + y'(t) + y(t) = 2, \quad y(0) = 2, \, y'(0) = 0, \, y''(0) = 1.$$

Ukupno: 0

(1.) $r=2$ $T(0,3)$. $\int_{\partial K} xy\,ds$

$x = r\cos t + 0 \Rightarrow 2\cos t$ ✓

$y = r\sin t + 3 \Rightarrow 2\sin t + 3$ ✓

$r(t) = \begin{bmatrix} 2\cos t \\ 2\sin t + 3 \end{bmatrix}$

$r(t) = \begin{bmatrix} 2(-\sin t) \\ 2\cos t + 3 \end{bmatrix}$ ✗

$|r'(t)| = \sqrt{(2(-\sin t))^2 + (2\cos t) + 3^2}$ ✗

$= \sqrt{4\sin^2 t + 4\cos^2 t + 9}$

$= \sqrt{4} + 3$

$= 5\,dt$

$\int_0^{2\pi} (2\cos t)(2\sin t + 3)\,5dt$ ✗

2) $r = 3$ $\quad T(0,-3)$ $\quad \int_{\partial k}(2x+2)\,dy$

$x = r\cos\varphi \Rightarrow 3\cos\varphi$

$y = r\sin\varphi \Rightarrow 3\sin\varphi$

$dx\,dy = r\,dr\,d\varphi$

$$\int_0^{2\pi}\int_0^{-3}\left(2(3\cos\varphi)+2\right) r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^{-3}(6\cos\varphi+2)\,r\,dr\,d\varphi$$

$$\int_0^{2\pi}\left(6\int_0^{-3}\cos\varphi + 2\int_0^{-3} dr\right)d\varphi$$

$$\int_0^{2\pi}\left(6\sin + \Big/_0^{-3} + 2\Big/_0^{-3}\right)d\varphi$$

$$\int_0^{2\pi}(6\ {-18}\sin) - 6 -)\,d\varphi$$

4.) $a = 2$ $\quad \iint_{\partial k}(3x+2)\,dx\,dy$

$x = 2\cos\varphi$

$y = 2\sin\varphi$

$$\iint\left(3(2\cos\varphi)+2\right) r\,dr\,d\varphi$$

$$\iint(6\cos\varphi+2) = 0$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. IME I PREZIME: MARKO DANILOVIĆ BROJ INDEKSA: 0269045823	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r = 2$ sa centrom u točki $T(0,3)$. Izračunati $\int_{\partial K} xy \, \mathrm{d}s$. ~~20~~

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4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\widehat{\partial K}} (3x+2) \, \mathrm{d}x\mathrm{d}y$? ~~20~~

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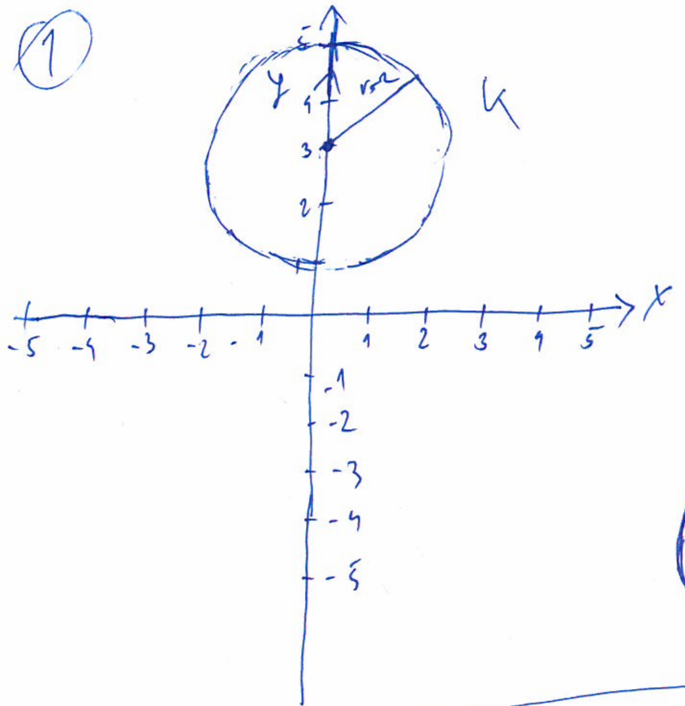
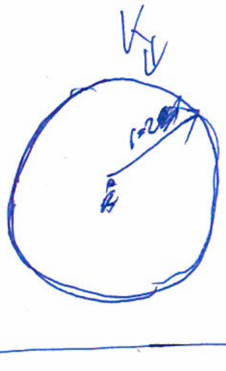
$$y'''(t) + 2y''(t) + y'(t) + y(t) = 2, \quad y(0) = 2, \, y'(0) = 0, \, y''(0) = 1.$$

Ukupno: ~~0~~

1

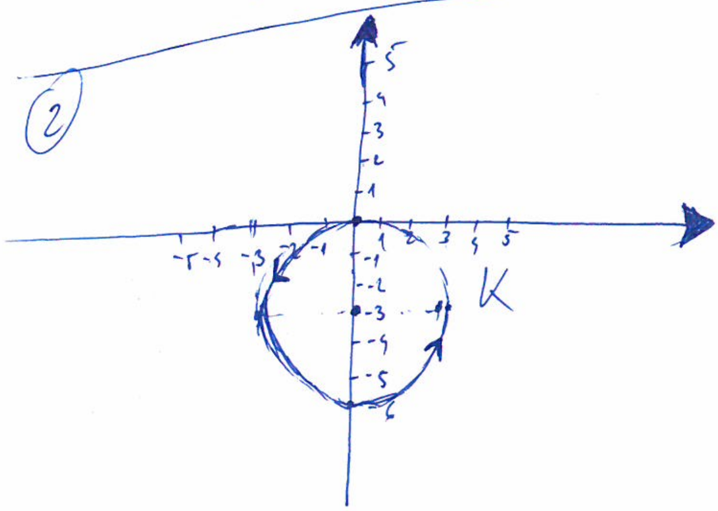


$\int_{\partial K} xy\,ds = ?$



$\int_{\partial K} xy\,ds =$ ~~$\int_{\partial K}$~~

2



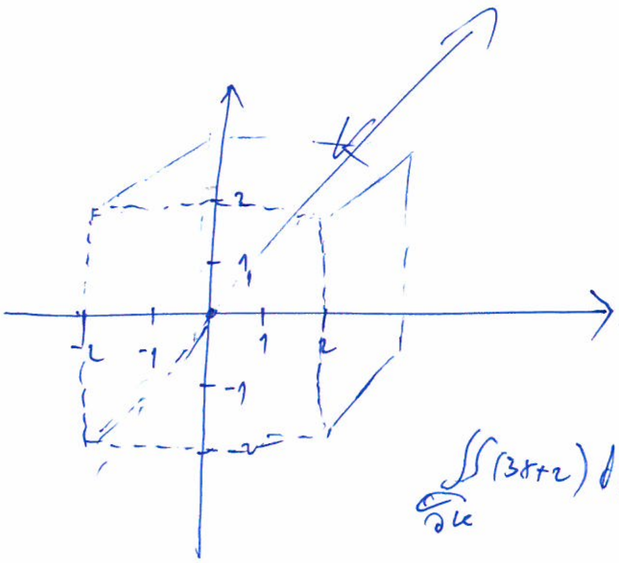
$\int_{\partial K} (2x+2)\,dy = ?$

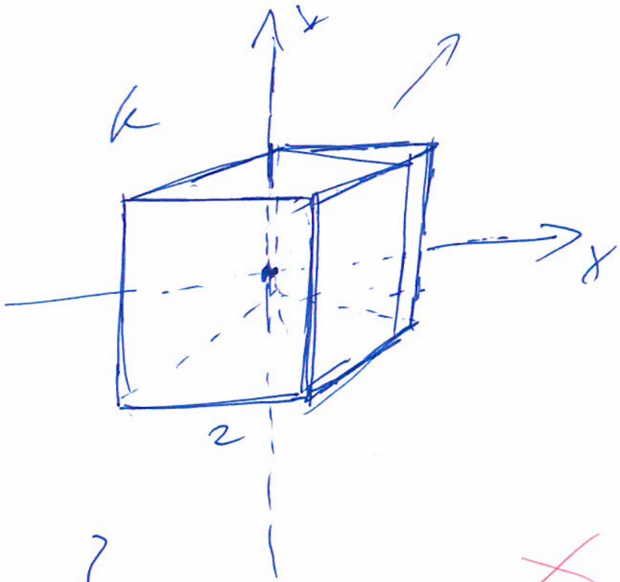
$\int_{\partial K} (2x+2)\,dy =$

(5) $\int_{(3,-2)}^{(3,5)} \frac{dx}{2\sqrt{x}} + dy = \int_{(3,-2)}^{(3,5)} (2\sqrt{x})^{-1} dx + dy = \int_{(3,-2)}^{(3,5)} \frac{dx}{2\sqrt{x}} + dy =$

$= \int_{(3,-2)}^{(3,5)} \frac{1}{2} \frac{dx}{\sqrt{x}} + dy = \frac{1}{2} \int_{(3,-2)}^{(3,5)} \frac{dx}{\sqrt{x}} + dy = \frac{1}{2} \int_{(3,-2)}^{(3,5)} \sqrt{x}^{-1} dx + dy \ldots$

(4)





$\iint_{\partial K} (3x+2)\, dx\, dy = ?$

$\iint_{\partial K} (3x+2)\, dx\, dy = \iint_{\partial K} 3x\,dx + 2\,dx + dy = 3 \iint_{\partial K} x\,dx + 2\,dx + d$

(6)

$y'''(t) + 2y''(t) + y't + y = 2$

$y''(t) + 2y'(t) + t + 1 = 2$

$y'(t) + 2t + 1 + 1 = 2$

$y(t) + 2 + 1 + 1 = 2$

$y(t) + 4 = 2$

$y(t) = 2 - 4$

$y(t) = -2$

$y(0) = 2, y'(0) = 0, y''(0) = 1$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	
IME I PREZIME: LUKA BROKIĆ	**BROJ INDEKSA:** 57826-2009

POPUNJAVA NASTAVNIK

Broj ↓ bodova

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Ukupno: $\cancel{0}$

6. $y'''(t) + 2y''(t) + y'(t) + y(t) = 2$

$y(0) = 2$

$y'(0) = 0$

$y''(0) = 1$

$s^3 Y(s) - s^2 y(0) - s\cancel{y'(0)}^{0} - y''(0) + 2s^2 Y(s) - sy(0) - \cancel{y'(0)}^{0} + sY(s) - sy(0) + y(0) = 2$

$s^3 Y(s) - 2s^2 - 1 + 2s^2 Y(s) - 2s + sY(s) - 2s - 2 = 2$

$Y(s)(s^3 + 2s^2 + s) = 5 + 2s^2 + 4s$

$Y(s) = \dfrac{2s^2 + 4s + 5}{s^3 + 2s^2 + s} = \dfrac{2s^2 + 4s + 5}{s(s^2 + 2s + 1)} = \dfrac{A}{s} + \dfrac{Bs + C}{s^2 + 2s + 1}$ $/ \cdot$ N/AZ