MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: STIPE ŠPANJA **BROJ INDEKSA:** 17-2-0018-2010

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0,4)$. Izračunati $\int_{\partial K} (3x+3)\,ds$. 20
2. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(3,3)$. Izračunati $\iint_K (3x+2)\,dxdy$. 20
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 3^2$ za koji vrijedi $z \leq 1$. 15
4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2+y^2$, $z = 3$. 15
5. Zadana krivulja Γ s parametrizacijom $x = 3\cos t$, $y = 3\sin t$ i $z = t^2$, $t \in [-1,1]$. Još je zadano $f(x,y,z) = \sqrt{z}$. Izračunati: $\int_\Gamma f\,ds$. Pomoć: kod riješavanja integracije možeš iskoristiti supstituciju $2t+3 \mapsto u$. 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) + y''(t) + y'(t) + y(t) = t+1, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 85

1. $r = 3$
$T(0,4)$

$$\int_{\partial K} (3x+3)\,ds$$

$x^2 + (y-4)^2 = 9$

$\varphi \in [0, 2\pi]$

$$r(t) = \begin{pmatrix} r\cos t \\ r\sin t + 4 \end{pmatrix} = \begin{pmatrix} 3\cos t \\ 3\sin t + 4 \end{pmatrix}$$

$$r'(t) = \begin{pmatrix} -3\sin t \\ 3\cos t \end{pmatrix}$$

$$\|r'(t)\| = \sqrt{9\sin^2 t + 9\cos^2 t} = \sqrt{9(\sin^2 t + \cos^2 t)} = \sqrt{9} = 3$$

$$\int_0^{2\pi} 3(3\cos t + 3)\,dt = \int_0^{2\pi} (9\cos t + 9)\,dt = (9\sin t + 9t)\Big|_0^{2\pi}$$

$$= 18\pi$$

2. Krug $r = 3$

$T(3,3)$

$$\iint_K (3x+2)\,dx\,dy$$

$x = r\cos\varphi + 3$

$y = r\sin\varphi + 3$

$dx\,dy = r\,dr\,d\varphi$

$r \in [0,3]$

$\varphi \in [0, 2\pi]$

$$\iint_K (3x+2)\,dx\,dy = \int_0^{2\pi}\int_0^3 \left[3(r\cos\varphi + 3) + 2\right] r\,dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^3 (3r\cos\varphi + 9 + 2)\,r\,dr\,d\varphi = \int_0^{2\pi}\int_0^3 (3r\cos\varphi + 11)\,r\,dr\,d\varphi$$

$$= \int_0^{2\pi}\int_0^3 (3r^2\cos\varphi + 11r)\,dr\,d\varphi = \int_0^{2\pi} \left(\cancel{3}\frac{r^3}{\cancel{3}}\cos\varphi + 11\frac{r^2}{2}\right)\Big|_0^3 d\varphi$$

$$= \int_0^{2\pi} \left(27\cos\varphi + \frac{11}{2}\cdot 9\right) d\varphi = \int_0^{2\pi} \left(27\cos\varphi + \frac{99}{2}\right) d\varphi$$

$$= 27\sin\varphi + \frac{99}{2}\varphi \Big|_0^{2\pi} = 99\pi - (0+0) = 99\pi$$

4. $z = x^2 + y^2$ $\quad z = 3$

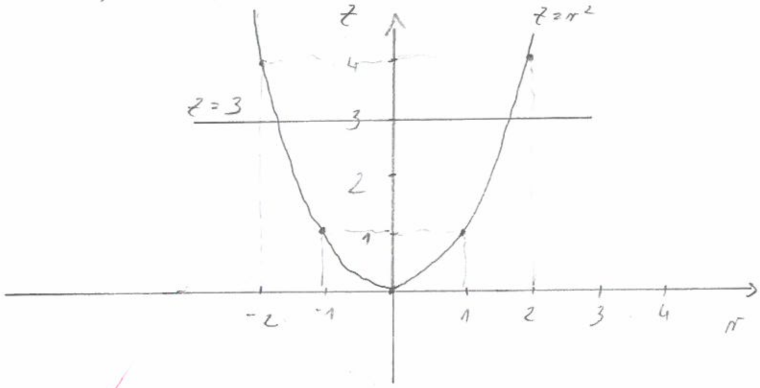
r	0	1	-1	2	-2
$z = r^2$	0	1	1	4	4

$z = r^2$

$r^2 = z$

$r^2 = 3$

$r = \sqrt{3}$



$r \in [0, \sqrt{3}]$

$\varphi = [0, 2\pi]$

$z \in [r^2, 3]$

$$V = \int_0^{2\pi} \int_0^{\sqrt{3}} \int_{r^2}^{3} r \, dz \, dr \, d\varphi$$

$$V = \int_0^{2\pi} \int_0^{\sqrt{3}} r(3 - r^2) \, dr \, d\varphi = \int_0^{2\pi} \int_0^{\sqrt{3}} (3r - r^3) \, dr \, d\varphi$$

$$V = \int_0^{2\pi} \left(3\frac{r^2}{2} - \frac{r^4}{4}\right)\Big|_0^{\sqrt{3}} d\varphi = \int_0^{2\pi} \left(3 \cdot \frac{3}{2} - \frac{9}{4}\right) d\varphi = \int_0^{2\pi} \left(\frac{9}{2} - \frac{9}{4}\right) d\varphi$$

$$= \int_0^{2\pi} \frac{9}{4} \, d\varphi = \frac{9}{4} \varphi \Big|_0^{2\pi} = \frac{9}{\cancel{4}} \cdot 2\pi = \frac{9}{2}\pi$$

Ime i prezime: STIPE ŠPANJA **Broj indeksa:** 17-2-0018-2010

6. ZADATAK NASTAVAK

$$Y(s) = \frac{0}{s} + \frac{1}{s^2} + \frac{1}{s+1} \quad \frac{0s+0}{s^2+1}$$

$$Y(s) = \frac{1}{s^2} + \frac{1}{s+1}$$

$$y(t) = t + e^{-t}$$

3. $x^2+y^2+z^2 = 3^2 \qquad z \leq 1$

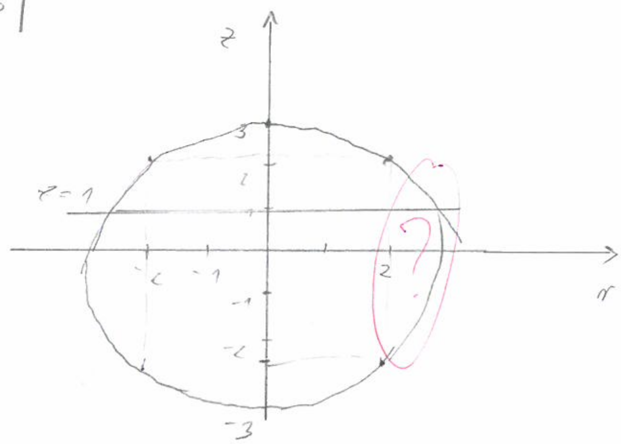
$$x^2+y^2+z^2 = 9$$

r	0	2	-2
$z=-\sqrt{9-r^2}$	-3	$-\sqrt{5}$	$-\sqrt{5}$

$$r^2+z^2=9$$

$$z^2 = 9-r^2$$

$$z = \pm\sqrt{9-r^2}$$



$$z \in \left[-\sqrt{9-r^2}, 1\right] \quad r \in [0,3] \quad \varphi \in [0, 2\pi]$$

$$V = \int_0^{2\pi} \int_0^3 \int_{-\sqrt{9-r^2}}^{1} r \, dz \, dr \, d\varphi$$

$$V = \int_0^{2\pi}\int_0^3 r\left(1+\sqrt{9-r^2}\right) dr\, d\varphi = \int_0^{2\pi}\int_0^3 \left(r\sqrt{9-r^2}+r\right) dr\, d\varphi$$

$$= \int_0^{2\pi} \left(-\frac{1}{3}\sqrt{(9-r^2)^3} + \frac{r^2}{2}\right)\Big|_0^3 d\varphi$$

$$= \int_0^{2\pi} \left[0 + \frac{9}{2} - \left(-\frac{1}{3}\cdot 27 + 0\right)\right] d\varphi$$

$$= \int_0^{2\pi} \frac{27}{2} \, d\varphi = \frac{27}{\cancel{2}} \cdot \cancel{2}\pi = 27\pi$$

$$\int r\sqrt{9-r^2}\,dr = \begin{pmatrix} 9-r^2 = t \\ -2r\,dr = dt \\ r\,dr = -\frac{1}{2}dt \end{pmatrix}$$

$$= -\frac{1}{2}\int \sqrt{t}\,dt$$

$$= -\frac{1}{2}\int t^{\frac{1}{2}}\,dt = -\frac{1}{2}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}}$$

$$= -\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\, t^{\frac{3}{2}}$$

$$= -\frac{1}{3}\sqrt{(9-r^2)^3}$$

Ime i prezime: STIPE ŠPANJA **Broj indeksa:** 17-2-0018-2010

6. $y'''(t) + y''(t) + y'(t) + y(t) = t+1$ $\quad y(0)=1 \quad y'(0)=0$ $\quad y''(0)=1$

$s^3Y(s) - s^2y(0) - \cancel{sy'(0)} - y''(0) + s^2Y(s) - sy(0) - \cancel{y'(0)} + sY(s) - y(0) + Y(s) = \frac{1}{s^2} + \frac{1}{s}$

$s^3Y(s) - s^2 - 1 + s^2Y(s) - s + sY(s) - 1 + Y(s) = \frac{1}{s^2} + \frac{1}{s}$

$s^3Y(s) + s^2Y(s) + sY(s) + Y(s) = \frac{1}{s^2} + \frac{1}{s} + s^2 + 1 + s + 1$

$Y(s)(s^3+s^2+s+1) = \frac{1+s+s^4+s^2+s^3+s^2}{s^2}$

$Y(s)(s^3+s^2+s+1) = \frac{s^4+s^3+2s^2+s+1}{s^2} \;/ : (s^3+s^2+s+1)$

$Y(s) = \frac{s^4+s^3+2s^2+s+1}{s^2(s^3+s^2+s+1)}$

$Y(s) = \frac{s^4+s^3+2s^2+s+1}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1} \;/ \cdot s^2(s+1)(s^2+1)$

$s^4+s^3+2s^2+s+1 = As(s+1)(s^2+1) + B(s+1)(s^2+1) + Cs^2(s^2+1) + (Ds+E)(s+1)s^2$

$s^4+s^3+2s^2+s+1 = As(s^3+s+s^2+1) + B(s^3+s+s^2+1) + Cs^4 + Cs^2 + (Ds+E)(s^3+s^2)$

$s^4+s^3+2s^2+s+1 = As^4 + As^2 + As^3 + As + Bs^3 + Bs + Bs^2 + B + Cs^4 + Cs^2 + Ds^4 + Ds^3 + Es^3 + Es^2$

$s^4+s^3+2s^2+s+1 = (A+C+D)s^4 + (A+B+D+E)s^3 + (A+B+C+E)s^2 + (A+B)s + B$

$A+C+D=1$

$A+B+D+E=1$

$A+B+C+E=2$

$A+B=1$

$\boxed{B=1}$

$\boxed{A=0}$

za $s+1=0$, $s=-1$

$\cancel{A-1} + 2 - \cancel{1} + \cancel{1} = C(1+1)$

$2 = 2C$

$\boxed{C=1}$

$0+1+D=1$

$\boxed{D=0}$

$0+1+0+E=1$

$\boxed{E=0}$

DALJE... ?

5. $x = 3\cos t$ $\quad t \in [-1, 1]$

$y = 3\sin t$ $\quad f(x, y, z) = \sqrt{z}$

$z = t^2$

$$r(t) = \begin{pmatrix} 3\cos t \\ 3\sin t \\ t^2 \end{pmatrix} \qquad r'(t) = \begin{pmatrix} -3\sin t \\ 3\cos t \\ 2t \end{pmatrix}$$

$$\| r'(t) \| = \sqrt{9\sin^2 t + 9\cos^2 t + 4t^2}$$

$$= \sqrt{9(\sin^2 t + \cos^2 t) + 4t^2} = \sqrt{9 + 4t^2}$$

$$\int_{-1}^{1} \left(\sqrt{t^2}\,\sqrt{9+4t^2}\right) dt = \int_{-1}^{1} \left(t\sqrt{9+4t^2}\right) dt$$

$$\begin{pmatrix} 9 + 4t^2 = u \\ 8t\,dt = du \\ t\,dt = \frac{1}{8}du \end{pmatrix}$$

$$\frac{1}{8}\int \sqrt{u}\,du$$

$$= \frac{1}{8}\int u^{\frac{1}{2}}\,du = \frac{1}{8}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$$

$$= \frac{1}{8}\cdot\frac{2}{3}\,u^{\frac{3}{2}} = \frac{2}{24}\sqrt{(9+4t^2)^3}$$

$$= \frac{1}{12}\sqrt{(9+4t^2)^3}$$

$$= \left(\frac{1}{12}\sqrt{(9+4t^2)^3}\right)\Big|_{-1}^{1}$$

$$= \frac{1}{12}\left(13\sqrt{13} - 13\sqrt{13}\right) = 0$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: MATE IVIĆ **BROJ INDEKSA:** 17-2-0008-2010

POPUNJAVA NASTAVNIK

Broj ↓ bodova

~~1.~~ Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0,4)$. Izračunati $\int_{\partial K} (3x+3)\,\mathrm{d}s$. 20

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$$y'''(t) + y''(t) + y'(t) + y(t) = t+1, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

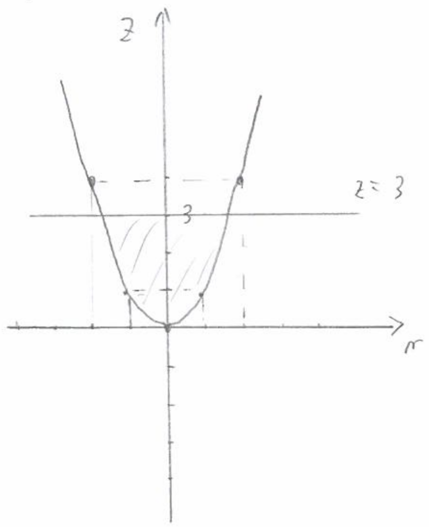
Ukupno: 70

4. VOLUMEN PARABOLOIDA $z = x^2+y^2$ $z=3$

$x^2+y^2 = r\,z$

$x^2+y^2 = r^2$

$z = r^2$



$\rightarrow r$	0	1	-1	2	-2
$\uparrow z = r^2$	0	1	1	4	4

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$dx\,dy\,dz = r\,dr\,dz\,d\varphi$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{3}]$

$z \in [r^2, 3]$

$z = r^2$

$3 = r^2$

$r^2 = 3 \;/\sqrt{\ }$

$r = \sqrt{3}$

$$V = \int_0^{2\pi}\int_0^{\sqrt{3}}\int_{r^2}^{3} r\,dz\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^{\sqrt{3}} rz\Big|_{r^2}^{3}\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^{\sqrt{3}} \left(r\cdot 3 - (r^3)\right)dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^{\sqrt{3}} (3r - r^3)\,dr\,d\varphi = \int_0^{2\pi}\left(3\cdot\frac{r^2}{2} - \frac{r^4}{4}\right)\Big|_0^{\sqrt{3}} d\varphi$$

$$\int_0^{2\pi}\left(3\cdot\frac{3}{2} - \frac{9}{4}\right) - (0-0)\,d\varphi =$$

$$\int_0^{2\pi}\left(\frac{9}{2} - \frac{9}{4}\right)d\varphi = \int_0^{2\pi}\left(\frac{18-9}{4}\right)d\varphi = \int_0^{2\pi}\frac{9}{4}\,d\varphi$$

$$V = \frac{9}{4}\varphi\Big|_0^{2\pi} = \frac{9}{\cancel{4}_2}\cdot\cancel{2}\pi = \frac{9}{2}\pi \,//$$

1) $r = 3 \qquad T\begin{pmatrix} x_0 & y_0 \\ 0, & 4 \end{pmatrix}$

$$\int_{\partial K} (3x+3)\, ds$$

$$r(t) = \begin{pmatrix} r\cos t \\ r\sin t \end{pmatrix} = \begin{pmatrix} 3\cos t + 0 \\ 3\sin t + 4 \end{pmatrix}$$

$$r'(t) = \begin{pmatrix} -3\sin t \\ 3\cos t \end{pmatrix}$$

$$\|r'(t)\| = \sqrt{(-3\sin t)^2 + (3\cos t)^2} =$$

$$= \sqrt{9\sin^2 t + 9\cos^2 t}$$

$$= \sqrt{9(\sin^2 t + \cos^2 t)}$$

$$= \sqrt{9}$$

$$= 3$$

$$\int_{\partial K} (3x+3)\, ds$$

$$\int_0^{2\pi} 3 \cdot [3 \cdot (3\cos t) + 3]\, dt =$$

$$\int_0^{2\pi} 3[9\cos t + 3]\, dt =$$

$$\int_0^{2\pi} [27\cos t + 9]\, dt =$$

$$27\sin t + 9t \Big|_0^{2\pi} = 27\sin(2\pi) + 18\pi - (\underbrace{27\sin(0)}_{=0} + 0)$$

$$= 0 + 18\pi - 0$$

$$= 18\pi$$ ✓

2. K KRUG RADIJUSA $r = 3$ $T\left(\overset{x_0}{3}, \overset{y_0}{3}\right)$ $\iint_K (3x+2)\,dx\,dy$

$x = r\cos\varphi + 3$
$y = r\sin\varphi + 3$
$dx\,dy = r\,dr\,d\varphi$
$r \in [0, 3]$
$\varphi \in [0, 2\pi]$

$$\iint_K (3x+2)\,dx\,dy \quad ✓$$

$$\int_0^{2\pi}\int_0^3 \left[3\cdot(r\cos\varphi + 3) + 2\right] r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^3 \left[3r\cos\varphi + 9 + 2\right] r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^3 \left[3r\cos\varphi + 11\right] r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^3 \left[3r^2\cos\varphi + 11r\right] dr\,d\varphi$$

$$\int_0^{2\pi} \left[\cancel{3}\frac{r^3}{\cancel{3}}\cos\varphi + 11\frac{r^2}{2}\right]\Big|_0^3 d\varphi = \int_0^{2\pi}\left[\left(27\cos\varphi + \frac{99}{2}\right) - 0\right] d\varphi$$

$$= \left[27\sin\varphi + \frac{99}{2}\varphi\right]\Big|_0^{2\pi} = \left(27\sin(2\pi) + \frac{\cancel{198}^{\,99}}{\cancel{2}_{\,1}}\pi\right) - (0 + 0)$$

$$= 0 + 99\pi$$
$$= 99\pi \quad // \quad ✓$$

Ime i prezime: MATE IVIĆ **Broj indeksa:** 17-2-0008-2010

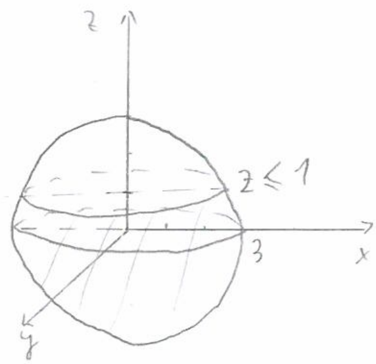
3. VOLUMEN DIJELA KUGLE $x^2+y^2+z^2=3^2$ $\quad z \leqslant 1$

$$x^2+y^2+z^2=9$$
$$x^2+y^2+z^2=R^2$$
$$R^2=9 \;/\sqrt{\ }$$
$$R=3$$

$$x=r\cos\varphi$$
$$y=r\sin\varphi$$
$$z=z$$
$$dx\,dy\,dz=r\,dr\,d\varphi\,dz$$

$$x^2+y^2+z^2=9$$
$$x^2+y^2=r^2$$

$$r^2+z^2=9$$
$$r^2=9-z^2 \;/\sqrt{\ }$$
$$r=\sqrt{9-z^2}$$



$$\varphi=[0,2\pi]$$
$$r=[0,\sqrt{9-z^2}]$$
$$z=[-3,1]$$

$$V=\int_0^{2\pi}\int_{-3}^{1}\int_0^{\sqrt{9-z^2}} 1\cdot(r\,dr\,dz\,d\varphi)$$

$$\int_0^{2\pi}\int_{-3}^{1}\int_0^{\sqrt{9-z^2}} r\,dr\,dz\,d\varphi=$$

$$\int_0^{2\pi}\int_{-3}^{1}\frac{r^2}{2}\Big|_0^{\sqrt{9-z^2}}dz\,d\varphi=\int_0^{2\pi}\int_{-3}^{1}\frac{\left(\sqrt{9-z^2}\right)^2}{2}\,dz\,d\varphi$$

$$\int_0^{2\pi}\int_{-3}^{1}\frac{9-z^2}{2}\,dz\,d\varphi=\int_0^{2\pi}\frac{9}{2}z-\frac{z^3}{6}\Big|_{-3}^{1}d\varphi=\int_0^{2\pi}\frac{9}{2}-\frac{1}{6}-\left(-\frac{27}{2}+\frac{27}{6}\right)d\varphi$$

$$\int_0^{2\pi}\left(\frac{9}{2}-\frac{1}{6}+\frac{27}{2}-\frac{27}{6}\right)d\varphi=\int_0^{2\pi}\left(\frac{\cancel{27}-1+81-\cancel{27}}{6}\right)d\varphi=\int_0^{2\pi}\left(\frac{\cancel{80}\,40}{\cancel{6}\,3}\right)d\varphi$$

$$\frac{40}{3}\varphi\Big|_0^{2\pi}=\frac{40}{3}\cdot 2\pi-0=\frac{80}{3}\pi$$

IME I PREZIME: MATE IVIĆ

BROJ INDEKSA: 17-2-0008-2010

⑥ $y'''(t) + y''(t) + y'(t) = t + 1 \quad y(0) = 1 \quad y'(0) = 0 \quad y''(0) = 1$

$s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + s^2 Y(s) - s y(0) - y'(0) + s Y(s) - y(0) + Y(s) = \frac{1}{s^2} + \frac{1}{s}$

$s^3 Y(s) - s^2 \cdot 1 - 0 - 1 + s^2 Y(s) - s \cdot 1 - 0 + s Y(s) - 1 + Y(s) = \frac{1}{s^2} + \frac{1}{s}$

$s^3 Y(s) + s^2 Y(s) + s Y(s) + Y(s) = \frac{1}{s^2} + \frac{1}{s} + s^2 + 1 + s + 1$

$Y(s)(s^3 + s^2 + s + 1) = \frac{1}{s^2} + \frac{1}{s} + s^2 + s + 2 \quad /:(s^3 + s^2 + s + 1)$

$Y(s) =$

0

(5) $r(t) = \begin{pmatrix} 3\cos t \\ 3\sin t \\ t^2 \end{pmatrix} \quad r'(t) = \begin{pmatrix} -3\sin t \\ 3\cos t \\ 2t \end{pmatrix}$

$$\|r'(t)\| = \sqrt{(-3\sin t)^2 + (3\cos t)^2 + (2t)^2}$$
$$= \sqrt{9\sin^2 t + 9\cos^2 t + 4t^2}$$
$$= \sqrt{9(\sin^2 t + \cos^2 t) + 4t^2}$$
$$= \sqrt{9 + 4t^2}$$

$$\int_{-1}^{1}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.		POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: ANTONIO MUŽANOVIĆ	**BROJ INDEKSA:** 17-2-0031-2010	

1. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0,4)$. Izračunati $\int_{\partial K} (3x+3)\,ds$. 20
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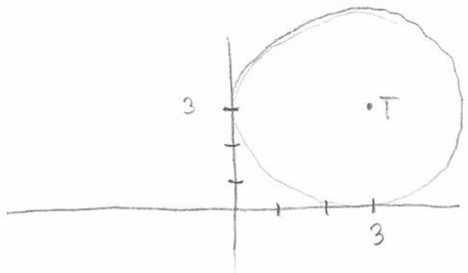
$$y'''(t) + y''(t) + y'(t) + y(t) = t+1, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 62

2. $r=3$ $\quad T(3,3)$ $\quad \iint_K (3x+2)\,dx\,dy$ ✓



$x = r\cos\varphi + 3$ $\quad \varphi \in [0, 2\pi]$

$y = r\sin\varphi + 3$ $\quad r \in [0,3]$ ✓

$$\iint_K (3x+2)\,dx\,dy = \int_0^{2\pi}\int_0^3 (3\cdot(r\cos\varphi+3)+2)\,r\,dr\,d\varphi = \int_0^{2\pi}\int_0^3 (3r\cos\varphi+9+2)\,r\,dr\,d\varphi =$$

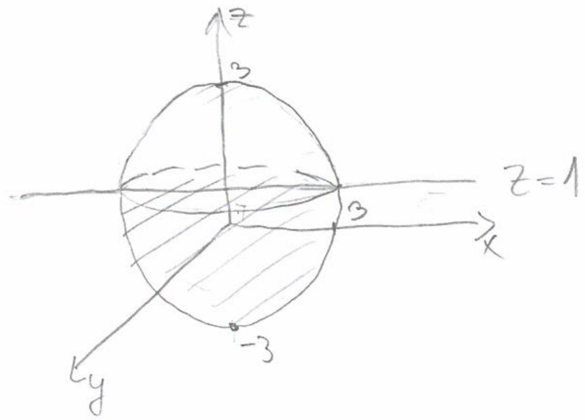
$$= \int_0^{2\pi}\int_0^3 (3r^2\cos\varphi + 11r)\,dr\,d\varphi = \int_0^{2\pi} 3\frac{r^3}{3}\cos\varphi + 11\frac{r^2}{2}\Big|_0^3 d\varphi = \int_0^{2\pi} r^3\cos\varphi + \frac{11}{2}r^2\Big|_0^3 d\varphi =$$

$$= \int_0^{2\pi}\left(3^3\cos\varphi + \frac{11}{2}3^2\right)d\varphi = \int_0^{2\pi}\left(27\cos\varphi + \frac{99}{2}\right)d\varphi = 2\pi\left[27\sin\varphi + \frac{99}{2}\varphi\right] = 99\pi$$ ✓

③ $x^2+y^2+z^2=3^2$

$z \leq 1$ $\quad T(0,0,0)$

$r=3$



$$\boxed{\begin{aligned} x &= r\cos\varphi \\ y &= r\sin\varphi \\ z &= z \end{aligned}}$$

$r \in [0,3]$

$\varphi \in [0,2\pi]$

$z \in [-3,1]$

$$V=\int_0^{2\pi}\int_0^3\int_{-3}^{1} r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_0^3 r(1+3)\,dr\,d\varphi = \int_0^{2\pi}\int_0^3 4r\,dr\,d\varphi = 4\cdot\int_0^{2\pi} \frac{r^2}{2}\Big|_0^3 d\varphi = 4\cdot\int_0^{2\pi}\frac{9}{2}\,d\varphi =$$

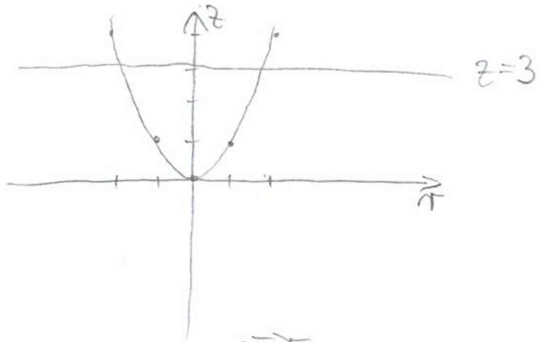
$$V = \cancel{4}^{2}\cdot\left[\frac{9}{2}\varphi\right]\Big|_0^{2\pi} = 2\cdot 18\pi = 36\pi \checkmark$$

④ $z=x^2+y^2 \quad x^2+y^2=r^2$

$z=3$

$r^2=z \Rightarrow r=\sqrt{z}$

$z=r^2$ $\quad \sqrt{3}^3=$



r	0	1	-1	2	-2
$z=r^2$	0	1	1	4	4

$\varphi \in [0,2\pi]$ ✓

$r \in [0,\sqrt{3}]$ ✓

$z \in [r^2,3]$ ✓

$$\cancel{V=\int_0^{2\pi}\int_0^{\sqrt{3}}\int_{r^2}^{3} r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{3}} r(3-r^2)\,dr\,d\varphi = \int_0^{2\pi}\int_0^{\sqrt{3}}(3r-3r^2)\,dr\,d\varphi =}$$

$$\cancel{=\int_0^{2\pi}\left(\frac{3r^2}{2}-\frac{3r^3}{3}\right)\Big|_0^{\sqrt{3}} d\varphi = \int_0^{2\pi}\left(\frac{9}{2}-\sqrt{27}\right)d\varphi = \left[\frac{9}{2}\varphi-\sqrt{27}\,\varphi\right]\Big|_0^{2\pi} = 9\pi-2\sqrt{27}\pi}$$

DRUGI NAČIN

$\varphi \in [0,2\pi]$

$r \in [0,\sqrt{z}]$ $\Rightarrow$

$z \in [0,3]$

$$V=\int_0^{2\pi}\int_0^{3}\int_0^{\sqrt{z}} r\,dr\,dz\,d\varphi = \int_0^{2\pi}\int_0^{3}\frac{r^2}{2}\Big|_0^{\sqrt{z}}dz\,d\varphi = \int_0^{2\pi}\int_0^{3}\frac{z}{2}\,dz\,d\varphi = \frac{1}{2}\int_0^{2\pi}\frac{z^2}{2}\Big|_0^3 d\varphi =$$

$$=\pi\cdot\left(\frac{3^2}{2}\right)=\frac{9\pi}{2} \checkmark$$

ANTONIO MUŽANOVIĆ

1.) $r = 3$

$T(0,4)$

$\int\limits_{\partial k}(3x+3)\,ds$

$r(t) = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi + 4 \end{pmatrix}, \; r(t) = \begin{pmatrix} 3\cos\varphi \\ 3\sin\varphi + 4 \end{pmatrix}$

$x = r\cos\varphi = 3\cos\varphi \qquad \varphi \in [0, 2\pi]$

$y = r\sin\varphi + 4 = 3\sin\varphi + 4 \qquad r \in [0, 3]$

$z = z$

$r'(t) = \begin{pmatrix} -3\sin\varphi \\ 3\cos\varphi \end{pmatrix}$

$\| r'(t) \| = \sqrt{9\sin^2\varphi + 9\cos^2\varphi} = \sqrt{9(\sin^2\varphi + \cos^2\varphi)} = \sqrt{9} = 3$

$\int\limits_0^{2\pi}\int\limits_0^3 3\cdot(3\cdot r\cos\varphi + 3)\,r\,dr\,d\varphi = \int\limits_0^{2\pi}\int\limits_0^3 (9r\cos\varphi + 9)\,r\,dr\,d\varphi = \int\limits_0^{2\pi}\int\limits_0^3 (9r^2\cos\varphi + 9r)\,dr\,d\varphi =$

$= \int\limits_0^{2\pi}\left(\overset{3}{\cancel{9}}\frac{r^3}{\cancel{3}_1}\cos\varphi + 9\frac{r^2}{2}\right)\Big|_0^3 d\varphi = \int\limits_0^{2\pi}\left(3\cdot 27\cos\varphi + \frac{9\cdot 9}{2}\right)d\varphi = \int\limits_0^{2\pi}\left(81\cos\varphi + \frac{81}{2}\right)d\varphi =$

$= \Big[\underset{= 0}{81\sin\varphi} + \frac{81}{2}\varphi\Big]\Big|_0^{2\pi} = 81\pi \checkmark$

5. $x=3\cos t$
$y=3\sin t$
$z=t^2$
$t\in[-1,1]$
$f(x,y,z)=\sqrt{z}$

$$\int_{\Gamma} f\cdot ds \Rightarrow \int_a^b f\cdot\|r'(t)\|\,dt$$

$\boxed{2t+3\to u}$

$$r(t)=\begin{pmatrix}3\cos t\\3\sin t\\t^2\end{pmatrix}\Rightarrow r'(t)=\begin{pmatrix}-3\sin t\\3\cos t\\2t\end{pmatrix}\Rightarrow \|r'(t)\|=\sqrt{9\sin^2 t+9\cos^2 t+4t^2}=$$
$$=\sqrt{9(\sin^2 t+\cos^2 t)+4t^2}=\sqrt{9+4t^2}=$$

$$\int_{-1}^{1}\sqrt{t^2}\cdot\sqrt{9+4t^2}\,dt=\int_{-1}^{1}t\cdot\sqrt{9+4t^2}\,dt=?$$

6. ANTONIO MUŽANOVIĆ

$$y'''(t)+y''(t)+y'(t)+y(t)=t+1 \qquad y(0)=1,\ y'(0)=0,\ y''(0)=1$$

$$s^3Y(s)-s^2\underbrace{y(0)}_{1}-s\underbrace{y'(0)}_{0}-\underbrace{y''(0)}_{1}+s^2Y(s)-s\underbrace{y(0)}_{1}-\underbrace{y'(0)}_{0}+sY(s)-\underbrace{y(0)}_{1}+Y(s)=\frac{1}{s^2}+\frac{1}{s}$$

$$s^3Y(s)-s^2-1+s^2Y(s)-s+sY(s)-1+Y(s)=\frac{1+s}{s^2}$$

$$Y(s)(s^3+s^2+s+1)=\frac{1+s}{s^2}+s^2+1+s+1 \quad \Big| \ s^2(s^3+s^2+s+1)$$

$$Y(s)(s^3+s^2+s+1)=\frac{1+s+s^4+s^2+s^3+s^2}{s^2}$$

$$Y(s)(s^3+s^2+s+1)=\frac{s^4+s^3+2s^2+s+1}{s^2}$$

$$Y(s)=\frac{s^4+s^3+2s^2+s+1}{s^2(s^3+s^2+s+1)}=\frac{s^4+s^3+2s^2+s+1}{s^2(s+1)(s^2+1)}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+1}+\frac{Ds+E}{s^2+1} \quad \Big/ \cdot s^2(s+1)(s^2+1)$$

$$s^4+s^3+2s^2+s+1=As(s+1)(s^2+1)+B(s+1)(s^2+1)+Cs^2(s^2+1)+(Ds^3+Es^2)(s+1)$$

ZA $s+1=0$

$s=-1$

$1-1+2-1+1=2C$

$2C=2$

$C=1$

$s=0$

$1=B(0+1)(0+1)$

$\boxed{B=1}$

$$s^4+s^3+2s^2+s+1=(As^2+As)(s^2+1)+(Bs+B)(s^2+1)+Cs^4+Cs^2+Ds^4+Ds^3+Es^3+Es^2$$

$$s^4+s^3+2s^2+s+1=As^4+As^2+As^3+As+Bs^3+Bs+Bs^2+B+Cs^4+Cs^2+Ds^4+Ds^3+Es^3+Es^2$$

$$\cancel{s^4+s^3+2s^2+s+1=s^4(A+C)+s^3(A+B+D+E)+s^2(A+B+C+D+E)+s(A+B)+B}$$

$\boxed{B=1}$

$$s^4+s^3+2s^2+s+1=s^4(A+C+D)+s^3(A+B+D+E)+s^2(A+B+C+E)+s(A+B)+B$$

$A+C+D=1$

$A+B+D+E=1$

$A+B+C+E=2$

$A+B=1$

$\boxed{B=1}$

$\boxed{A=0}$

$0+1+C+E=2$

$C+E=1$

$C=1-E$

$0+1+D+E=1$

$D=-E$

$0+1-E-E=1$

$1-2E=1$

$-2E=0$

$\boxed{E=0}$ $\boxed{D=0}$

$\boxed{C=1}$

$$Y(s)=\frac{1}{s^2}+\frac{1}{s+1}$$

$$y(t)=t+e^{-t}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: VANJA HRASTIĆ - CAR **BROJ INDEKSA:** 17-1-0036-2016

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0,4)$. Izračunati $\int_{\partial K} (3x+3)\,ds$. 20
2. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(3,3)$. Izračunati $\iint_K (3x+2)\,dxdy$. 20
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 3^2$ za koji vrijedi $z \le 1$. 15
4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2+y^2$, $z = 3$. 15
5. Zadana krivulja Γ s parametrizacijom $x = 3\cos t$, $y = 3\sin t$ i $z = t^2$, $t \in [-1,1]$. Još je zadano $f(x,y,z) = \sqrt{z}$. Izračunati: $\int_\Gamma f\,ds$. Pomoć: kod riješavanja integracije možeš iskoristiti supstituciju $2t+3 \mapsto u$. 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) + y''(t) + y'(t) + y(t) = t+1, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 70

(1)

$r = 3$, $T(0,4)$, $\int_{\partial K} (3x+3)ds$

$$r = \begin{bmatrix} 3\cos t \\ 3\sin t + 4 \end{bmatrix} \qquad \|r'\| = \sqrt{(-3\sin t)^2 + (3\cos t)^2}$$

$$r' = \begin{bmatrix} -3\sin t \\ 3\cos t \end{bmatrix} \qquad \|r'\| = \sqrt{9\sin^2 t + 9\cos^2 t}$$

$$\|r'\| = \sqrt{9}$$

$$\|r'\| = 3$$

$$= \int_0^{2\pi} 3\cdot(3\cos t) + 3\cdot 3\,dt$$

$$= \int_0^{2\pi} 9\cos t + 9\,dt$$

$$= 9\int_0^{2\pi} \cos t\,dt + 9\int_0^{2\pi} dt$$

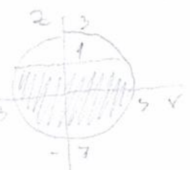
$$= \cancel{9\sin t\Big|_0^{2\pi}} + 9t\Big|_0^{2\pi} \Rightarrow 9\cdot 2\pi = 18\pi$$

(3)

$x^2+y^2+z^2 = 3^2$ $\quad z \le 1$

$r^2 + z^2 = 3^2$

$r = \sqrt{9-z^2}$



$z\,(-3, 1)$

$t\,(0, 2\pi)$

$r\,(0, \sqrt{9-z^2})$

$$\int_0^{2\pi}\int_{-3}^{1}\int_0^{\sqrt{9-z^2}} 1\,r\,dr\,dz\,dt \Rightarrow \int_0^{2\pi}\int_{-3}^{1} \frac{1}{2}r^2\Big|_0^{\sqrt{9-z^2}} \Rightarrow \int_0^{2\pi}\int_{-3}^{1} \frac{1}{2}\left(\sqrt{9-z^2}\right)^2 \Rightarrow \int_0^{2\pi}\int_{-3}^{1} \frac{9-z^2}{2}\,dz\,dt$$

$$\int_0^{2\pi}\frac{1}{2}\int_{-3}^{1} 9 - z^2\,dz\,dt \Rightarrow \pi\int_{-3}^{1} 9 - z^2\,dz = \underbrace{9\int_{-3}^{1} dz}_{9z} - \underbrace{\int_{-3}^{1} z^2\,dz}_{\frac{1}{3}z^3} \Rightarrow \underbrace{(9\cdot 1 - 9\cdot(-3))}_{36} - \underbrace{\left(\frac{1}{3}1^3 - \frac{1}{3}\cdot(-3)^3\right)}_{\frac{28}{3}}$$

$2\pi \cdot \frac{1}{2} = \pi$

$$= 36 - \frac{28}{3} = \frac{80}{3}\cdot\pi = \boxed{\frac{80}{3}\pi}$$

(2) $r = 3$, $T(3,3)$, $\iint\limits_K (3x+2)\,dx\,dy$

$r(0,3)$
$f(0,2\pi)$

$x = r\cos f = \left[r\cos f + 3\right]$
$y = r\sin f = \left[r\sin f + 3\right]$

$$\int_0^{2\pi}\int_0^3 3\cdot(r\cos f+3)+2\cdot r\,dr\,df$$

$$\int_0^{2\pi}\int_0^3 (3r\cos f + \underbrace{9+2}_{11})\cdot r\,dr\,df$$

$$\int_0^{2\pi}\int_0^3 3r^2\cos f + 11r\,dr\,df$$

$$\int_0^{2\pi}\int_0^3 3r^2\cos f\,dr\,df + \int_0^{2\pi}\int_0^3 11r\,dr\,df$$

$$\int_0^{2\pi} 3\cos f \int_0^3 r^2\,dr + \int_0^{2\pi} 11\cdot\int_0^3 r\,dr$$

$$\int_0^{2\pi} 3\cos f\cdot\frac{1}{3}r^3\Big|_0^3 + \int_0^{2\pi} 11\cdot\frac{1}{2}r^2\Big|_0^3$$

$$\int_0^{2\pi} 3\cos f\cdot 9\,df + \int_0^{2\pi} 11\cdot\frac{9}{2}\,df$$

$$27\int_0^{2\pi}\cos f\,df + \frac{99}{2}\int_0^{2\pi} df$$

$$\underbrace{27\sin(2\pi) - 27\cdot\sin(0)}_{0} + \frac{99}{2}\cdot 2\pi - \cancel{\frac{99}{2}\cdot 0}^{\,0}$$

$= \underline{99\pi}$ ✓

Ime i prezime: VANJA HRASTIĆ-CAR **Broj indeksa:** 17-1-0036-2010

(4) $x^2+y^2=z$, $z=3$

$r^2=z$

$r=\sqrt{z}$

$z(0,3)$

$r(0,\sqrt{z})$

$\varphi(0,2\pi)$

$x=r\cos\varphi$

$y=r\sin\varphi$

$$\int_0^{2\pi}\int_0^{3}\int_0^{\sqrt{z}} 1\, r\,dr\,dz\,d\varphi$$

$$\int_0^{2\pi}\int_0^{3} \frac{1}{2}r^2\Big|_0^{\sqrt{z}} = \int_0^{2\pi}\frac{1}{2}\int_0^{3} z\,dz\,d\varphi = \int_0^{2\pi}\frac{1}{2}\cdot\frac{1}{2}z^2\Big|_0^{3} = \frac{1}{2}\cdot\left(\frac{1}{2}3^2\right) = \int_0^{2\pi}\frac{9}{4}\,d\varphi$$

$$= \frac{9}{4}\cdot\varphi\Big|_0^{2\pi} = \frac{9\pi}{2}$$

(5) $x=3\cos t$, $y=3\sin t$, $z=t^2$

$$r=\begin{bmatrix}3\cos t\\ 3\sin t\\ t^2\end{bmatrix} = r'=\begin{bmatrix}-3\sin t\\ 3\cos t\\ 2t\end{bmatrix}$$

$\|r'\| = \sqrt{9\sin^2 t+9\cos^2 t+4t^2}$

$\|r'\| = \sqrt{9+4t^2}$

$\|r'\| = 3+2t$

$f(x,y,z)=\sqrt{z} = \sqrt{t^2}$

$= t$

$$\begin{bmatrix}3+2t = u\\ t\,dt = du\end{bmatrix}$$

$$f\cdot\|r'\| = \int_{-1}^{1} t\cdot(3+2t)\,dt$$

$$= \int_{-1}^{1} u\,du = \frac{1}{2}u^2\Big|_{-1}^{1} = \frac{1}{2}(3+2t)^2\Big|_{-1}^{1} = \frac{1}{2}\cdot\Big(\underbrace{(3+2\cdot 1)^2}_{25} - \underbrace{(3+2\cdot(-1))^2}_{1}\Big)$$

$$= \frac{1}{2}\cdot 24 = \frac{24}{2} = 12$$

⑥ $y'''(t) + y''(t) + y'(t) + y(t) = t + 1$ $y(0) = 1,\ y'(0) = 0,\ y''(0) = 1$

$$\cancel{s^3 y(s)} - \cancel{s^2 y(0)}^{-s^2} - s\,y'(0)^{0} - y''(0)^{-1} + \cancel{s^2 y(s)} - s\,y(0)^{-s} - \cancel{y'(0)}^{0} + \cancel{s\,y(s)} - \cancel{y(0)}_{-1} + \cancel{s\,y(s)} =$$

$$= \frac{1}{s^2} + \frac{1}{s} \Big\} \left[\frac{1+s}{s^2}\right]$$

$$y(s)\left[s^3 + s^2 + 2s\right] = \frac{1+s}{s^2} + \overbrace{s^2 + 1 + s + 1}^{s^2+s+2} \Big\} \frac{1+s+s^2(s^2+s+2)}{s^2} = \frac{1+s+s^4+s^3+2s^2}{s^2}$$

$$\frac{s^4+s^3+2s^2+s+1}{s^2(s^3+s^2+2s)} = \frac{s^4+s^3+2s^2+s+1}{s^3(s^2+s+2)} =$$

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+s+2} =$$

$$= As^2(s^2+s+2) + Bs(s^2+s+2) + Cs^2 + Cs + 2C + Ds^4 + Es^3$$

$$As^4 + \underline{As^3} + \underline{2As^2} + \underline{Bs^3} + \underline{Bs^2} + \boxed{2Bs} + \underline{Cs^2} + \boxed{Cs} + 2C + Ds^4 + \underline{Es^3}$$

$s^4 = A + D = 1 \Rightarrow \frac{5}{8} + D = 1 \Rightarrow \boxed{D = \frac{3}{8}}$

$s^3 = A + B + E = 1 \Rightarrow \frac{5}{8} + \frac{1}{4} + E = 1 \Rightarrow \boxed{E = \frac{1}{8}}$

$s^2 = 2A + B + C = 2 \Rightarrow 2A + \frac{1}{4} + \frac{1}{2} = 2 \Rightarrow 2A = \frac{5}{4} \Rightarrow \boxed{A = \frac{5}{8}}$

$s = 2B + C = 1 \Rightarrow 2B + \frac{1}{2} = 1 \Rightarrow 2B = \frac{1}{2}$ $\boxed{B = \frac{1}{4}}$

$1 = 2C \Rightarrow 2C = 1$ $\boxed{C = \frac{1}{2}}$

$$y(s) = \frac{5}{8}\cdot\frac{1}{s} + \frac{1}{4}\cdot\frac{1}{s^2} + \frac{1}{2}\cdot\frac{1}{s^3} + \frac{\frac{3}{8}s + \frac{1}{8}}{s^2+s+2}$$

$$y(t) = \frac{5}{8} + \frac{1}{4}t + \frac{1}{4}t^2 + \underbrace{\frac{3}{8}\frac{s}{s^2+s+2}}_{①} + \underbrace{\frac{1}{8}\frac{1}{s^2+s+2}}_{②}$$

① $\frac{s}{s^2+s+2} = \frac{s}{(s+1)^2 - 1^2} \Rightarrow e^{t}\cos(-t)$

② $\frac{1}{(s+1)^2 - 1^2} \Rightarrow -e^{-t}\sin(-t)$

$$y(t) = \frac{5}{8} + \frac{1}{4}t + \frac{1}{4}t^2 + \frac{3}{8}e^{t}\cos(-t) + \frac{1}{8}\cdot\left(-e^{-t}\sin(-t)\right)$$

$$= \frac{5}{8} + \frac{1}{4}t + \frac{1}{4}t^2 + \frac{3}{8}e^{t}\cos(-t) - \frac{1}{8}e^{-t}\sin(-t)$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano. IME I PREZIME: KRISTINA POŽARINA BROJ INDEKSA: 17-2-0021-2010	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0,4)$. Izračunati $\int_{\partial K} (3x+3)\,\mathrm{d}s$. ~~20~~

2. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(3,3)$. Izračunati $\iint_K (3x+2)\,\mathrm{d}x\mathrm{d}y$. ~~20~~ 15

3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 3^2$ za koji vrijedi $z \le 1$. ~~15~~

4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2+y^2$, $z = 3$. 15

5. Zadana krivulja Γ s parametrizacijom $x = 3\cos t$, $y = 3\sin t$ i $z = t^2$, $t \in [-1,1]$. Još je zadano $f(x,y,z) = \sqrt{z}$. Izračunati: $\int_\Gamma f\,\mathrm{d}s$. Pomoć: kod riješavanja integracije možeš iskoristiti supstituciju $2t+3 \mapsto u$. ~~15~~

6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~15~~

$$y'''(t) + y''(t) + y'(t) + y(t) = t+1, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: 30

~~1.) $r=3$~~

~~$T(0,4)$~~

~~$\int_{\partial K} (3x+3)\,ds$~~

~~$dx\,dy = r\,dr\,d\varphi$~~

~~$x = r\cos\varphi$~~

~~$y = r\sin\varphi + 4$~~

~~$r \in [0,3]$~~

~~$\varphi \in [0,2\pi]$~~

~~$\int_0^{2\pi}\int_0^3 \left((3 \cdot 3r\cos\varphi) + 3\right) \cdot r\sqrt{3}\, r\,dr\,d\varphi =$~~

~~$r(t) = \begin{bmatrix} 3r\cos t \\ 3\sin t \end{bmatrix}$~~

~~$r'(t) = \begin{bmatrix} -3r\sin t \\ 3r\cos t \end{bmatrix}$~~

~~$\|r'(t)\| = \sqrt{(-3r\sin t)^2 + (3r\cos t)^2} =$~~

~~$= \sqrt{3r^2\sin^2 t + 3r^2\cos^2 t} =$~~

~~$= \sqrt{3r^2(\sin^2 t + \cos^2 t)} =$~~

~~$\sqrt{3r^2} = r\sqrt{3}$~~

2. $r = 3$

$T(3, 3)$

$\iint (3x + 2)\, dx\, dy$

$x^2 + y^2 + z^2 = 3$

$(r\cos\varphi)^2 + (r\sin\varphi)^2 + z^2 = 3$

$r^2 \underbrace{(\cos^2\varphi + \sin^2\varphi)}_{1} + z^2 = 3$

$r^2 + z^2 = 3$

$z^2 = 3 - r^2 \quad / \sqrt{\ }$

$z = \sqrt{3 - r^2}$

$x = r\cos\varphi + 3$

$y = r\sin\varphi + 3$

$z = 2$

IME I PREZIME: KRISTINA POŽARWA **BROJ INDEKSA:** 17-2-0021-2010

1.) $r=3$

$T(0,4)$

$\int_{\partial K} (3x+3)\,ds$

$x = r\cos\varphi$

$y = r\sin\varphi$

$r(t) = \begin{vmatrix} 3\cos t \\ 3\sin t \end{vmatrix}$ $r'(t) = \begin{vmatrix} -3\sin t \\ 3\cos t \end{vmatrix}$

$r \in [0,3]$

$\varphi \in [0, 2\pi]$

$\|r'(t) = \sqrt{(-3\sin t)^2 + (3\cos t)^2} =$

$= \sqrt{3\sin^2 t + 3\cos^2 t} = \sqrt{3(\underbrace{\sin^2 t + \cos^2 t}_{1})} =$

$\|r'(t)| = \sqrt{3}$

$\int_0^4 \sqrt{3}\,dt = \sqrt{3}t\Big|_0^4 = 4\sqrt{3}$

$\int_0^{2\pi}\int_0^3 (3\cos\varphi + 3)\cdot 4\sqrt{3}\, r\,dr\,d\varphi$

IME I PREZIME: KRISTINA POŽARINA **BROJ INDEKSA:** 17-2-0021-2010

5) $x = 3\cos t$ $\quad t \in [-1, 1]$

$y = 3\sin t$

$z = t^2$

$$r(t) = \begin{vmatrix} 3\cos t \\ 3\sin t \\ t^2 \end{vmatrix} \qquad r'(t) = \begin{vmatrix} -3\sin t \\ 3\cos t \\ 2t \end{vmatrix}$$

$$\|r' t\| = \sqrt{(-3\sin t)^2 + (3\cos t)^2 + (2t)^2} = \sqrt{3\sin^2 t + 3\cos^2 t + 4t^2} =$$

$$= \sqrt{3(\sin^2 t + \cos^2 t) + 4t^2} = \sqrt{3+4t^2}$$

$$\int_{-1}^{1} \sqrt{3+4t^2}\, dt = \begin{vmatrix} 3+4t = u \\ 4dt = du \,/:4 \\ dt = \frac{du}{4} \end{vmatrix} = \int_{-1}^{1} \sqrt{u}\, \frac{du}{4} = \frac{1}{4}\int_{-1}^{1} u^{\frac{1}{2}}\, du =$$

$$= \frac{1}{4} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \Bigg|_{-1}^{1} = \frac{1}{4} \cdot \frac{3u^{\frac{3}{2}}}{2} \Bigg|_{-1}^{1} = \frac{1}{4} \cdot \frac{3}{2}\, u^{\frac{3}{2}} \Bigg|_{-1}^{1} = \frac{3}{8} \sqrt{(3+4t^2)^3} \Bigg|_{-1}^{1}$$

$$= \frac{3}{8}\left(\sqrt{(3+4)^3} - \sqrt{(3+4)^3}\right) = \emptyset$$

4.) $z = x^2 + y^2 \quad z = 3$

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$z = r^2$

$r^2 = z \;/\sqrt{\ }$

$r = \sqrt{z}$

$z \in [0, 3]$

$r \in [0, \sqrt{z}]$

$\varphi \in (0, 2\pi]$

$$\int_0^{2\pi}\int_0^{3}\int_0^{\sqrt{z}} r\,dr\,dz\,d\varphi = \int_0^{2\pi} d\varphi \int_0^{3} dz \int_0^{\sqrt{z}} r\,dr = \int_0^{2\pi} d\varphi \int_0^{3} dz \left(\frac{r^2}{2}\right)\Bigg|_0^{\sqrt{z}}$$

$$= \frac{1}{2}\int_0^{2\pi} d\varphi \int_0^{3} z\,dz = \frac{1}{2}\int_0^{2\pi} d\varphi \left(\frac{z^2}{2}\right)\Bigg|_0^{3} = \frac{1}{2}\cdot\frac{1}{2}\int_0^{2\pi} 9\,d\varphi =$$

$$= \frac{9}{4}\int_0^{2\pi} d\varphi = \frac{9}{\cancel{4}}\cdot\cancel{2\pi} = \frac{9\pi}{2}$$

2.) $r = 3$

$T(3,3) \quad \iint_K (3x+2)\,dx\,dy$

$x = r\cos\varphi + 3$

$y = r\sin\varphi + 3$

$dx\,dy = r\,dr\,d\varphi$

$r \in [0, 3]$

$\varphi \in [0, 2\pi]$

$$\int_0^{2\pi}\int_0^{3} \big(3(r\cos\varphi + 3) + 2\big) r\,dr\,d\varphi = \int_0^{2\pi}\int_0^{3} 3r^2\cos\varphi + 11\,r\,dr\,d\varphi =$$

$$= \underbrace{3\int_0^{2\pi}\cos\varphi\,d\varphi \int_0^{3} dr}_{I = 0} + \underbrace{11\int_0^{2\pi} d\varphi \int_0^{3} r^2\,dr}_{II}$$

$I + II = 198\pi$

$I = 9\int_0^{2\pi}\cos\varphi\,d\varphi = 9\sin\varphi\Big|_0^{2\pi} =$

$= 9(\sin 2\pi - \sin 0) = \varnothing$

$$II = 11\int_0^{2\pi} d\varphi \left(\frac{r^3}{3}\right)\Bigg|_0^{3} = \frac{11}{3}\int_0^{2\pi} 27\,d\varphi = \frac{11}{\cancel{3}_1}\cdot \cancel{27}_9\,\varphi\Big|_0^{2\pi} = 99\cdot 2\pi = 198\pi$$

Ime i prezime: KRISTINA POŽARINA **Broj indeksa:** 17-2-0021-2010

3.) $x^2+y^2+z^2=3^2$ $\quad z \leq 1$

$x = r\cos\varphi$ $\quad z \in [0,1]$

$y = r\sin\varphi$ $\quad \varphi \in [0,2\pi]$

$z = z$ $\quad r \in [0,\sqrt{9-z^2}]$

$r^2+z^2=9$

$r^2 = 9-z^2 \,/\sqrt{\ }$

$r=\sqrt{9-z^2}$

$$\int_0^{2\pi}\int_0^1\int_0^{\sqrt{9-z^2}} r\,dr\,dz\,d\varphi = \int_0^{2\pi} d\varphi \int_0^1 dz \int_0^{\sqrt{9-z^2}} r\,dr =$$

$$= \int_0^{2\pi} d\varphi \int_0^1 dz \left(\frac{r^2}{2}\right)\Big|_0^{\sqrt{9-z^2}} = \frac{1}{2}\int_0^{2\pi} d\varphi \int_0^1 \left(\left(\sqrt{9-z^2}\right)^2 - 0^2\right) dz =$$

$$= \frac{1}{2}\int_0^{2\pi} d\varphi \int_0^1 (9-z^2)\,dz =$$

$$= \frac{1}{2}\left[\int_0^{2\pi} d\varphi \int_0^1 9\,dz - \int_0^{2\pi} d\varphi \int_0^1 z^2 dz\right] = \frac{1}{2}\left[9\int_0^{2\pi} d\varphi\, z\Big|_0^1 - \int_0^{2\pi} d\varphi \left(\frac{z^3}{3}\right)\Big|_0^1\right] =$$

$$= \frac{1}{2}\left[9\cdot\varphi\Big|_0^{2\pi} - \frac{1}{3}\cdot\varphi\Big|_0^{2\pi}\right] = \frac{1}{2}\left[18\pi - \frac{2\pi}{3}\right] =$$

$$= 27{,}23$$

6.) $y'''(t) + y''(t) + y'(t) + y(t) = t + 1$ $y(0) = 1$ $y''(0) = 1$

$y'(0) = 0$

$$s^3 Y(s) - s^2 \overset{1}{y(0)} - \cancel{s y'(0)} - \overset{1}{y''(0)} + s^2 Y(s) - s\overset{1}{y(0)} - \cancel{y'(0)} + sY(s) - \overset{1}{y(0)} + Y(s) =$$

$$= \frac{1}{s^2} + \frac{1}{s}$$

$$s^3 Y(s) + s^2 Y(s) + sY(s) + Y(s) = \frac{1}{s^2} + \frac{1}{s} + s^2 + 1 + s + 1$$

$$Y(s)\left(s^3 + s^2 + s + 1\right) = \frac{1}{s^2} + \frac{1}{s} + s^2 + s + 2$$

$$Y(s)\left(s^3 + s^2 + s + 1\right) = \frac{1 + s + s^4 + s^3 + 2s^2}{s^2} \Big/ : \left(s^3 + s^2 + s + 1\right)$$

$$Y(s) = \frac{\dfrac{1 + s + s^4 + s^3 + 2s^2}{s^2}}{\dfrac{s^3 + s^2 + s + 1}{1}} = \frac{s^4 + s^3 + 2s^2 + s + 1}{s^5 + s^4 + s^3 + s^2} = \frac{s^4 + s^3 + 2s^2 + s + 1}{s^2(s^3 + s^2 + s + 1)} =$$

$$= s^2(s+1)(s^2+1)$$

$s^3 + s + s^2 + 1$

$$Y(s) = \frac{s^4 + s^3 + 2s^2 + s + 1}{s^2(s+1)(s^2+1)}$$

$s^3 + s$

$$\frac{s^4 + s^3 + 2s^2 + s + 1}{s^2(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds + E}{s^2+1} \Big/ \cdot s^2(s+1)(s^2+1)$$

$$s^4 + s^3 + 2s^2 + s + 1 = As(s+1)(s^2+1) + B(s+1)(s^2+1) + Cs^2(s^2+1) + (Ds + E)(s^2(s+1))$$

$$s^4 + s^3 + 2s^2 + s + 1 = As^4 + As^3 + As^2 + As + Bs^3 + Bs^2 + Bs + B + Cs^4 + Cs^2 + Ds^4 + Ds^2 + Es^3 + \ldots$$

$1 = A + C + D$

$1 = A + B + E$

$2 = A + B + C + D$

$1 = A + B + E$

$1 = B$

$B = 1$

$2 = A + 1 + C + D$

$1 = A + C + D$

$1 = A + C + D \quad / \cdot (-1)$

$$\left.\begin{aligned} 1 &= A + C + D \\ -1 &= -A - C - D \end{aligned}\right\} +$$

$0 = 0$

$A = 0$

$C = 0$

$D = 0$

$B = 1$

$1 = 0 + 0 + E$

$E = 1$

PROVJERA?

$$Y(s) = \frac{1}{s^2} + \frac{1}{s^2+1} = t + \sin t$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.	POPUNJAVA NASTAVNIK Broj ↓ bodova
IME I PREZIME: MARKO BAMBIĆ BROJ INDEKSA: 54952-2007	

1. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(0,4)$. Izračunati $\int_{\partial K} (3x+3)\,ds$. 20
2. Neka je K krug radijusa $r = 3$ sa centrom u točki $T(3,3)$. Izračunati $\iint_K (3x+2)\,dxdy$. 20
3. Prijelazom na cilindrične koordinate izračunati volumen dijela kugle $x^2+y^2+z^2 = 3^2$ za koji vrijedi $z \leq 1$. 15
4. Izračunati volumen paraboloida omeđenog plohama: $z = x^2+y^2$, $z = 3$. 15
5. Zadana krivulja Γ s parametrizacijom $x = 3\cos t$, $y = 3\sin t$ i $z = t^2$, $t \in [-1,1]$. Još je zadano $f(x,y,z) = \sqrt{z}$. Izračunati: $\int_\Gamma f\,ds$. Pomoć: kod riješavanja integracije možeš iskoristiti supstituciju $2t+3 \mapsto u$. 15
6. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 15

$$y'''(t) + y''(t) + y'(t) + y(t) = t+1, \quad y(0) = 1,\, y'(0) = 0,\, y''(0) = 1.$$

Ukupno: 0

4. $z = x^2+y^2$
$z = 3$

$$r^2 = z$$
$$r = \sqrt{z}$$
$$r \in |0, \sqrt{z}|$$
$$\varphi \in |0, 2\pi|$$
$$z \in |0, 4|$$

$$V = \int_0^{2\pi} d\varphi \int_{z_1}^{z_2} dz \int_{r_1}^{r_2} r\,dr$$

$$V = \int_0^{2\pi} d\varphi \int_0^4 dz \int_0^{\sqrt{z}} r\,dr = \varphi\Big|_0^{2\pi} \cdot z\Big|_0^4 \frac{r^2}{2}\Big|_0^{\sqrt{z}} =$$

$$= 2\pi \cdot \int_0^4 dz \cdot \left(\frac{r^2}{r}\right)\Big|_0^{\sqrt{z}} = 2\pi \cdot \int_0^4 dz \frac{(\sqrt{z})^2}{2}$$

$$= 2\pi \cdot \int_0^4 dz \cdot \frac{z}{2} = \frac{1}{2} \cdot 2\pi \cdot \int_0^4 z\,dz$$

$$= \pi \cdot \frac{z^2}{2}\Big|_0^4 = \pi \cdot \frac{4^2}{2} = 8\pi$$

3. $x^2+y^2+z^2=3^2$ $\qquad z \leq 1$

$R^2=3^2=9$

$R=\sqrt{9}$

$R=3$

$\varphi \in$

2. POLARNE KOORDINATE:

$T(3,3)$

$x=r\cos\varphi+p$

$y=r\sin\varphi+q$

$x=r\cdot\cos\varphi+3$

$y=r\cdot\cos\varphi+3$

$$\int_0^{2\pi}\int_0^1 (3x+2)\,dx\,dy=\int_0^{2\pi}\int_0^1 (3\cdot(r\cos\varphi+3)+2)\cdot r\,dr\,d\varphi=$$

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$$\int_0^{2\pi}\int_0^1 (3r\cos\varphi+3+2)\,r\,dr\,d\varphi=\int_0^{2\pi}\int_0^1 (3r^2\cos\varphi+3r+2r)\,dr\,d\varphi=$$

$$\int_0^{2\pi}\left(3\cdot\frac{r^3}{3}\cos\varphi+3\cdot\frac{r^2}{2}+2\cdot\frac{r^2}{2}\right)\Bigg|_0^1=\int_0^{2\pi}\left(3\cdot\frac{1}{3}\cos\varphi+3\cdot\frac{1^2}{2}+2\cdot\frac{1^2}{2}\right)d\varphi=$$

$$=\int_0^{2\pi}\left(\cos\varphi+\frac{3}{2}+2\right)d\varphi=\int_0^{2\pi}\left(\cos\varphi+\frac{5}{2}\right)d\varphi=\left(\sin\varphi+\frac{5}{2}\varphi\right)\Bigg|_0^{2\pi}=\left(\sin 2\pi+\frac{5}{2}2\pi\right)-$$

$$\cancel{\left(\sin\cdot 0+\frac{5}{2}\cdot 0\right)}=0+\frac{5}{\cancel{2}}\cdot\cancel{2}\pi=\underline{\underline{5\pi}}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. Pišite dvostrano.

IME I PREZIME: Marko Ugarinić **BROJ INDEKSA:** 54949-2007

POPUNJAVA NASTAVNIK
Broj ↓ bodova

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$$y'''(t) + y''(t) + y'(t) + y(t) = t+1, \quad y(0) = 1,\ y'(0) = 0,\ y''(0) = 1.$$

Ukupno: $\cancel{0}$

(2) $r = 3 \quad T(\overset{x_0}{3}, \overset{y_0}{3})$

$x = r\cos\varphi\, x_0 = r\cos 3$

$y = r\sin y_0 = r$

$dxdy = r\,dr\,d\varphi$

$r \in [0,3] \quad \varphi \in [0, 2\pi]$

$\iint (3x+2)\,dxdy = \int_0^{2\pi}\int_0^3 (3(r\cos\varphi) + 2)\,r\,dr\,d\varphi$

$\int_0^{2\pi}\int_0^3 3r\cos\varphi + 9 + 2 \; r\,dr\,d\varphi = \int_0^{2\pi}\int_0^3 \frac{3r^2}{2}\cos\varphi + 9r + 2r\,dr\,d\varphi$

$\int_0^{2\pi} \frac{3r^2}{2}\cos\varphi + 9r + 2r \Big|_0^3 d\varphi = \int_0^{2\pi} \frac{3}{2}\cdot 9\cos\varphi + 9\cdot 3 + 2\cdot 3 - \frac{3}{2}\cdot 0\cos\varphi + 9\cdot 0 + 2\cdot 0\, d\varphi$

$\int_0^{2\pi} \frac{27}{2}\cos\varphi + 27 + 6 - 1\,d\varphi = \int_0^{2\pi} \frac{27}{2}\cos\varphi + 32\,d\varphi = \frac{27}{2}\sin\varphi + 32\varphi \Big|_0^{2\pi} = \Rightarrow$

$$= \underbrace{\frac{2^7}{8} \cdot 2^4 \sin 2\pi}_{=0} + 32 \cdot 2\pi - 0 = 64\pi$$

Ime i prezime: Marko Ugrinić **Broj indeksa:** 51949-2007

(1) $r=3$ $r\in[0,3]$

$T(0,4)$ $\varphi\in[0,2\pi]$

$$\|r\| = \begin{vmatrix} r\cos\varphi \\ r\sin\varphi+4 \end{vmatrix} = \begin{vmatrix} 3\cos\varphi \\ 3\sin\varphi+4 \end{vmatrix} \quad \|r'\| = \begin{vmatrix} -3\sin\varphi \\ 3\cos\varphi+4 \end{vmatrix}$$

$$\|r'(t)\| = \sqrt{(-3\sin\varphi)^2 + (3\cos\varphi+4)^2}$$

$$= \sqrt{9\sin^2\varphi + 9\cos^2\varphi + 16}$$

$$= \sqrt{9\cdot 1 + 16} = \sqrt{25} = 5$$

$$\int_k (3x+3)\,ds = \int_0^{2\pi}\int_0^3 5\big(3(3\cos\varphi)+3\big)\,r\,dr\,d\varphi$$

ok

$$\int_0^{2\pi}\int_0^3 5\,(9\cos\varphi+3\,r\,dr\,d\varphi = \int_0^{2\pi}\int_0^3 45\cos\varphi+15\,r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^3 45r\cos\varphi+15r\,dr\,d\varphi = \int_0^{2\pi} \frac{45}{2}r^2\cos\varphi + \frac{15}{2}r^2\Big|_0^3 d\varphi$$

$$\int_0^{2\pi} \frac{45}{2}\cdot 9\cos\varphi + \frac{15}{2}\cdot 9 - \frac{45}{2}\cdot 0\cos\varphi\ \frac{15}{2}\cdot 0\,d\varphi$$

$$\int_0^{2\pi} \frac{405}{2}\cos\varphi + \frac{135}{2} - 1\,d\varphi = \int_0^{2\pi} \frac{405}{2}\cos\varphi + \frac{133}{2}\,d\varphi$$

$$\frac{405}{2}\varphi\cos\varphi + \frac{133}{2}\varphi\Big|_0^{2\pi} = \underbrace{\frac{405}{2}\sin 2\pi}_{=0} + \frac{133}{2}2\pi - 0 = 133\pi$$

⑤ $\left| \begin{array}{l} x = 3\cos t \\ y = 3\sin t \\ z = t^2 \end{array} \right|$ $f(x, y, z) = \sqrt{z}$

$t \in [-1, 1]$ $z \in [0, \sqrt{z}]$

$\|\gamma\|' = \sqrt{a^2 + b^2 + c^2} = \sqrt{(3\cos t)^2 + (3\sin t)^2 + (t^2)^2}$

$\|\gamma\|' = \sqrt{9 \cdot 1 (t^2)^2} = 3t^2$

×